DIONYSIUS LARDNER

HAND-BOOK OF NATURAL PHILOSOPHY

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ELEMENTARY COURSE OF OPTICS

PREFACE.

THIS work is intended for all who desire to attain an accurate knowledge of Physical Science, without the profound methods of Mathematical investigation. Hence the explanations are studiously popular, and everywhere accompanied by diversified elucidations and examples, derived from common objects, wherein the principles are applied to the purposes of practical life.

It has been the Author's especial aim to supply a manual of such physical knowledge as is required by the Medical and Law Students, the Engineer, the Artisan, the superior classes in Schools, and those who, before commencing a course of Mathematical Studies, may wish to take the widest and most commanding survey of the field of inquiry upon which they are about to enter.

Great pains have been taken to render the work complete in all respects, and co-extensive with the actual state of the Sciences, according to the latest discoveries.

Although the principles are here, in the main, developed and demonstrated in ordinary and popular language, mathematical symbols are occasionally used to express results more clearly and concisely. These, however, are never employed without previous explanation.

The present edition has been augmented by the introduction of a vast number of illustrations of the application of the various branches of Physics to the Industrial Arts, and to the practical business of life. Many hundred engravings have also been added to those, already numerous, of the former edition.

For the convenience of the reader the series has been divided into Four Treatises, which may be obtained separately.

Mechanics	One Volume.
Hydrostatics, Pneumatics, and Heat	One Volume.
Optics	One Volume.
Electricity, Magnetism, and Acoustics	One Volume.

The Four Volumes taken together will form a complete course of Natural Philosophy, sufficient not only for the highest degree of School education, but for that numerous class of University Students who, without aspiring to the attainment of Academic honours, desire to acquire that general knowledge of these Sciences which is necessary to entitle them to graduate, and, in the present state of society, is expected in all well educated persons.

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ELEMENTARY COURSE

OF

OPTICS.

CHAPTER I.

LUMINOUS AND NON-LUMINOUS BODIES. — TRANSPARENCY. — OPACITY.

1. Light is the physical agent by which the external world is rendered manifest to the sense of sight. Opinion has long been divided as to its nature; one party has regarded it as a specific fluid, another as the effect of undulation. The former consider that the eye is affected by light as the sense of smell is affected by the odoriferous effluvia; the latter maintain that light is to the eye what sound is to the ear. Before these theories, however, can be understood, or their claims to adoption be appreciated, it will be necessary that the chief properties of light, and the phenomena consequent upon them, be explained.

2. **Bodies luminous and non-luminous.** — In relation to the production of light, bodies are considered as luminous and non-luminous. Luminous bodies, or luminaries, are those which are original sources of light; such, for example, as the sun, the flame of a lamp or candle, metal rendered red hot, the electric spark, lightning, and so forth. Luminaries are necessarily always visible when present, provided the light they emit be strong enough to excite the eye. Non-luminous bodies are those which themselves produce no light, but which may be rendered temporarily luminous when placed in the presence of luminous bodies. These cease, however, to be luminous, and therefore visible, the moment the luminary from which they borrow their light is removed. Thus the sun, placed in the midst of the planets, satellites, and

comets, renders these bodies luminous and visible; but when any of them are removed from the solar influence by the interposition of any object not pervious by light, they cease to be visible, as is manifest in the case of lunar eclipses, when the globe of the earth is interposed between the sun and moon, and the latter object is therefore deprived of light. A candle or lamp placed in the room renders the walls, furniture, and surrounding objects temporarily luminous, and therefore visible; but if the candle be screened by any object not pervious to light, those parts of the room from which light is intercepted would become invisible, did they not receive some light from the other parts of the room still illuminated. If, however, the candle or lamp be completely covered, all the objects in the room become invisible.

3. **Transparency and opacity.** — In relation to the propagation of light, bodies are considered as transparent and opaque. Bodies through which light passes freely are called transparent, because the eye placed behind them will see such light through them. Bodies, on the contrary, which do not admit light to pass through them, are called opaque; and such bodies consequently render a luminary invisible if interposed between it and the eye.

Transparency and opacity exist in various bodies in different degrees. Glass, air, and water are examples of very transparent bodies. The metals, stone, earth, wood, &c. are examples of opaque bodies. Correctly speaking, no body is perfectly transparent or perfectly opaque.

4. There is no substance, however transparent, which does not intercept some portion of light, however small. The light is thus intercepted in two ways: first, when it falls upon the surface of any body or medium, a portion is arrested, and either absorbed upon the surface, or reflected back from it; the remainder passes through the body or medium, but in so passing more or less is absorbed, and this increases according to the extent of the medium through which the light passes. Analogy, therefore, justifies the conclusion that there is no transparent medium which, if sufficiently extensive, would not absorb all the light which passes into it.

A very thin plate of glass is almost perfectly transparent; a thicker is less so, and according as the thickness is increased, the transparency will be diminished. The distinctness with which objects are seen through the air diminishes as their distance increases, because more or less of the light transmitted from them is absorbed in its progress through the atmosphere. This is the case with the sun, moon, and other celestial objects, which, when seen near the horizon, are more dim, however clear the atmosphere may be, than when seen in the zenith. In the former case the light transmitted from them passes through a greater mass of atmosphere, and more of it is absorbed. According to Bouguer, sea water at about the depth of 700 feet would lose all its transparency, and the atmosphere would be impervious to the sun's light if it had a depth of 700 miles.

5. The transparency of the same substance varies according to the density of its structure, the transparency generally increasing with the density. Thus, charcoal is opaque; but if the same charcoal be converted into a diamond, which it may be, without any change of the matter of which it is composed, it will become transparent.

Bodies are said to be imperfectly transparent, or semi-transparent, when light passes through them so imperfectly that the forms and colours of the objects behind them cannot be distinguished. Ground glass, paper, and thin tissues in general, foggy air, the clouds, horn, and various species of shell, such as tortoise-shell, are examples of this.

The degrees of this imperfect transparency are infinitely various; some substances, such as horn, being so nearly transparent as to render the form of a luminous object behind it indistinctly visible. Porous bodies, which are imperfectly transparent, usually have their transparency increased by filling their pores with some transparent liquid. Thus paper, which is imperfectly transparent, is rendered much more transparent by saturating it with oil, or by wetting it with any liquid. The variety of opal called hydrophane is white and opaque when dry, but when saturated with water it becomes transparent. Ground glass is rendered more transparent by pouring oil upon it. Two plates of ground glass placed one upon the other are very imperfectly transparent; but if the space between them be filled with oil, and their external surfaces be rubbed with the same liquid, they will be rendered nearly transparent.

6. Bodies, however opaque, lose their perfect opacity when reduced to the form of extremely attenuated laminæ. Gold, one of the most dense of metals, is, in a state of ordinary thickness, perfectly opaque; but if it be reduced to the form of leaf-gold by the process of the gold-beater, and attached to a plate of glass, light will pass partially through it, and to an eye placed behind it, it will appear of a greenish colour. Other metals, when equally attenuated, show the same imperfect opacity.

CHAP. II.

RECTILINEAR PROPAGATION OF LIGHT. — RADIATION. — SHADOWS AND PENUMBRÆ. — PHOTOMETRY.

7. **Rectilinear propagation of light.** — One of the first properties recognised in light by universal observation and experience is, that when transmitted through a uniform medium, it maintains a rectilinear course.

A luminous point is a centre from which light issues in every direction through the surrounding space in straight lines. This effect of rectilinear propagation in all directions from a common centre is called *radiation.*

Any straight line along which light is transmitted is called a *ray of light.* Any point from which rays of light radiate through the surrounding space, is called a *luminous point.*

The rectilinear propagation of light is established by numerous examples, and by a vast variety of effects, of which it affords the explanation. If any opaque object be interposed in a right line between the eye and a luminous point, the luminous point will cease to be visible; but if the opaque object be removed in the slightest degree from the direct line between the eye and the luminous point, the latter will become immediately visible.

This law, in its strictest sense, may be verified by the following experiment. Let three discs be pierced, each with a small hole, and let them be attached to a straight rod, in such a manner that the three holes shall be precisely in the same straight line, and, consequently, at the same distance from the rod. If a light be placed behind one of the extreme discs, and the eye behind the others, the light will be visible. The ray, therefore, which renders it visible, must pass successively through the holes in the two extreme discs, and in the intermediate disc; but if the intermediate disc be slightly moved on either side, or upwards or downwards, or, in a word, have its position deranged in any manner, so that a thread stretched between the holes in the extreme discs would not pass through the hole in the intermediate disc, then the light will be no longer visible.

8. **Aim depends on this.** — The rectilinear propagation of light supplies the means of directing all forms of artificial instruments at a distant object. These instruments and the particular form given to the expedients by which they are aimed, are extremely various, being extensively used in gunnery, surveying, and in practical astronomy.

9. **Sights.** — One of the earliest and most simple of these expedients consisted in two narrow slits formed in pieces of card or thin metal fixed at right angles to an oblong board, as shown in *fig.* 1.

Fig. 1.

The observer, placing his eye behind one of these slits, adjusted the board so that the object aimed at was seen through the other slit; a straight line drawn along the board, between the two slits, would in that case be the line of direction of the object.

An improvement on this expedient was adopted, in which a thin upright wire, B, *fig.* 2., was substituted for the slit, more remote from the eye. The observer, placing his eye behind the slit A, so directed the board, that the object aimed at was divided along the middle of its breadth by the pin B, or was *covered*, as it was technically called.

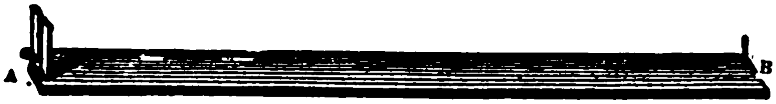

Fig. 2.

10. **Rifle shooting.** — This method of aim is still preserved in the improved rifle. Upon the top of the barrel and near its breech is placed at right angles to it a flat plate of metal, A, *fig.* 3., in which the slit is made, and near the end of the barrel a pin or knob is placed, so that the marksman, placing his eye in the usual manner, sees the knob B through the slit A, and holds the gun so that the knob covers the object aimed at.

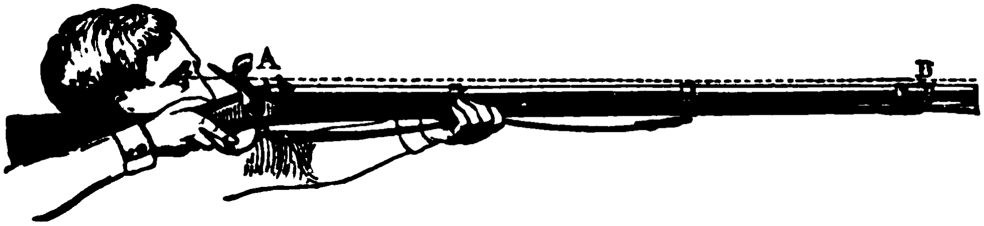

Fig. 3.

But, since while the ball is projected from the gun, it is also affected by gravity, it will necessarily fall more or less in the interval between the moment it leaves the barrel and that at which it arrives at a vertical line passing through the object. It would, therefore, strike that line at a point below that to which the line of aim was directed. To remedy this, a contrivance has been adopted, by which the marksman can change the elevation of the slit, according to the computed distance of the object, so that the line of aim, A B, *fig.* 4., shall be inclined to the axis of the barrel.

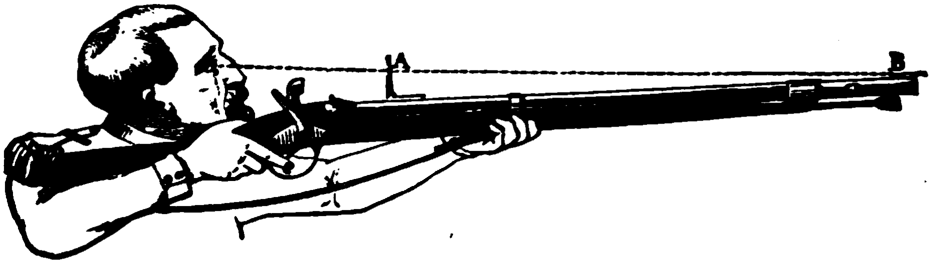

Fig. 4.

The latter will then be directed above the line of aim to such an extent that the ball, in moving from the barrel to the object, will describe a curve concave downwards, first rising and then falling, thus compensating for the effects of gravity, and hitting the object point blank.

11. **Astronomical instruments.**—In most astronomical observations, the direction of the objects which are observed is referred to the zenith, that is, the point of the heavens directly over the head of the observer, to which a plumb-line, if continued upwards, would be directed. Before the invention of telescopes, such observations were made by taking the direction of the object by means similar to those explained above.

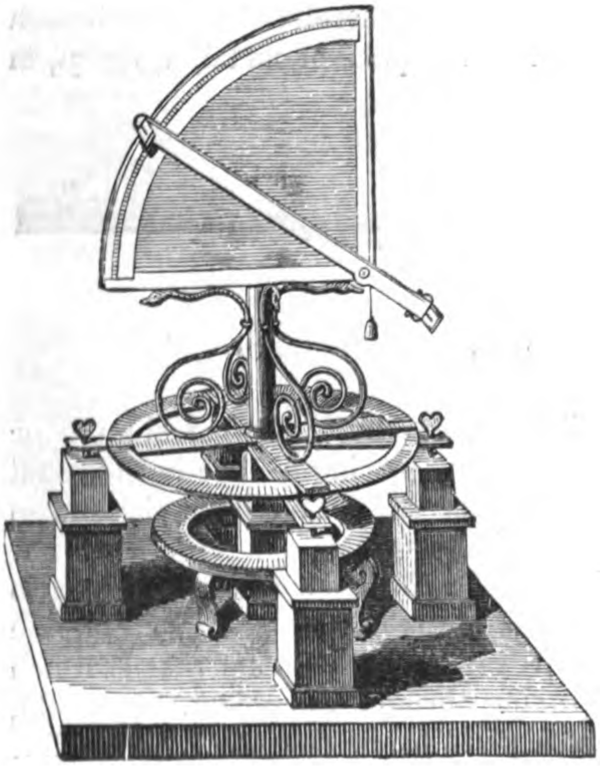

Fig. 5.

12. **Quadrant.** — An instrument called a quadrant, in the form in which it was used by the celebrated Danish astronomer, Tycho Brahe, is shown in *fig.* 5. It consisted of a graduated metal arch connected with two radii, the arch being 90°, and the radii, consequently, at right angles to each other. The instrument was adjusted by means of a plumb-line, so that one radius was vertical and the other horizontal. It was supported on a vertical pillar, which stood in the centre of a graduated

horizontal circle and was capable of turning horizontally, so that the plane of the quadrant could be directed to any point of the horizon.

An oblong slip of board, such as that shown in *fig.* 1., was attached to the quadrant, one end turning on a pin at its centre, while the other moved upon its arc. The observer, having turned the quadrant by the horizontal motion in the direction of the object to be observed, placing his eye behind the slit at the centre, moved the oblong board along the arc until he could see the object through the remote slit. The arc of the quadrant between the remote slit and the vertical radius would then be the angular distance of the object from the zenith, and the arc between it and the horizontal radius would be its angular distance from the horizon.

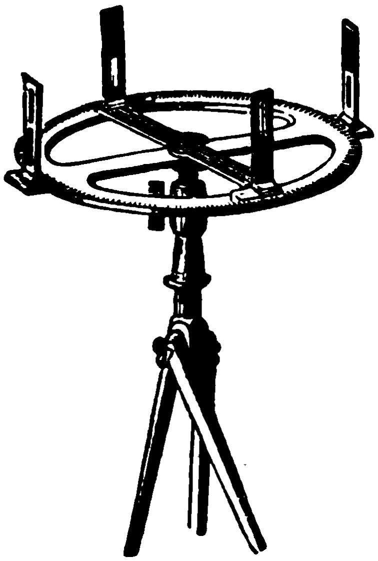

Fig. 6.

13. **Levelling instrument.** — The same expedient is still used in the ruder sort of surveying operations where great precision is not required. The form of instrument generally used for this purpose consists of a horizontal circle, supported on a tripod carrying upright pieces, having slits with vertical wires passing along them as shown in *fig.* 6.

The method of using this instrument will be sufficiently obvious without further explanation.

14. **Pencil of rays.**—Any collection of rays having a luminous point as their common origin, and included within the surface of a cone or any other regular limit, is called a *pencil of rays.* The point from which such rays diverge, and which is the apex of the cone, is called the focus of the pencil.

When the surface of any object receives light from a luminous point, it is customary to consider each portion of such surface as the base of a pencil of rays, the focus of which is the luminous point, so that the illuminated surface of any body is considered as composed of the bases of a number of pencils of rays having the luminous point as their common focus.

When rays radiate from a luminous point in this manner, they are called *divergent.* But cases will be shown hereafter, in which such rays may be so changed in their direction that, instead of diverging from the same point, they will converge to a common point. In this case the rays are called *converging* rays, the pencils *converging* pencils, and the point towards which the rays converge, and at which they would meet if not intercepted, is called the *focus of the pencil.*

15. **Shadows.** — When light radiating from a luminous point through the surrounding space encounters an opaque body, it will be excluded from the space behind such body. The space from which it is thus excluded is called the *shadow* of the opaque body.

This term *shadow* is sometimes applied, not to the space from which the light is thus excluded, but to a section of such space formed upon the surface of some body placed behind the opaque body which intercepts the light. Thus, the floor or wall of a room intersecting the space from which light is excluded by an opaque body placed between such wall or floor and a luminary will exhibit a dark figure, resembling more or less in outline the body which intercepts the light.

If a straight line be imagined to be drawn from the luminous point to the boundary of the opaque body, and to be continued beyond it indefinitely, such line being imagined to be moved round the opaque body, following its limits and its form, that part of the line which is beyond the body will pass through a surface which will form the limits of the shadow of such body, or of the space from which it excludes the light. If such line, however, encounter a wall, screen, or other surface, it will trace upon such surface the limits of the shadow, in the common acceptation of that term.

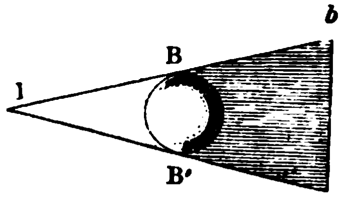

Fig. 7.

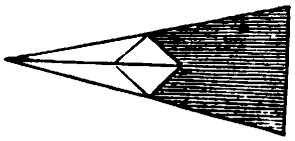

Fig. 9.

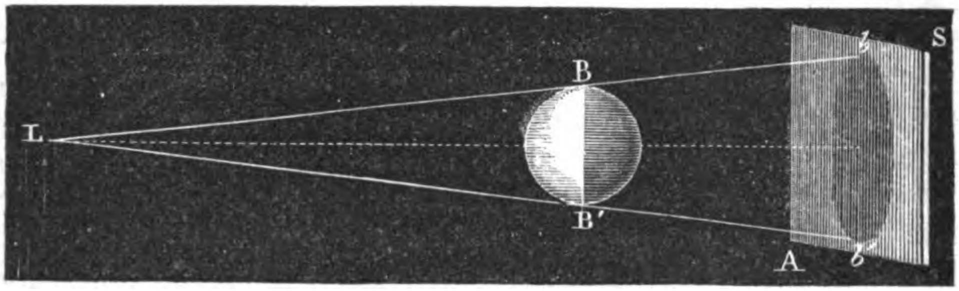

Fig. 8.

If the opaque object be a sphere, whose section, taken at right angles to the direction of the luminous pencil, is a circle, the shadow will be a truncated cone. Thus, in *fig.* 7., if L represent a luminous point, B B′ being an opaque spherical body, the light will be excluded by B B′ from all that part of a cone whose vertex is at L, which is included between the lines B *b* and B′ *b′*. The effect is rendered more apparent in the perspective diagram given in *fig.* 8.

If the opaque object receiving the light from the point L have any other form (*fig.* 9.), the form of its shadow will depend on that of a section of it made by a plane at right angles to the direction of the rays. Thus, if that section be square, the form of the shadow projected upon a screen will also be square, and the space from which the light will be excluded will be a truncated quadrangular pyramid.

There is, however, no luminary which, strictly speaking, is a luminous point. All luminous objects have a certain definite surface of more or less extent, and consist therefore of an infinite number of luminous points. Now each luminous point of such a body is the focus of an independent pencil of luminous rays, and each such pencil encountering the opaque object will produce an independent shadow.

16. This gives rise to phenomena which it is necessary here more fully to explain. Let C D, (*fig.* 10.), represent the section of an opaque object, and let B A represent the section of a luminary. B A will then consist of a line of luminous points, from each of which a pencil of rays will issue. The pencil which issues from the point B, will encounter the object C D, and the extreme rays of the pencil grazing the edge of the object, will proceed in the direction C B′ and D B″, being the continuation of the lines B C and B D. Now it is evident that the light proceeding from the point B will be excluded from the space included between the lines C B′ and D B″.

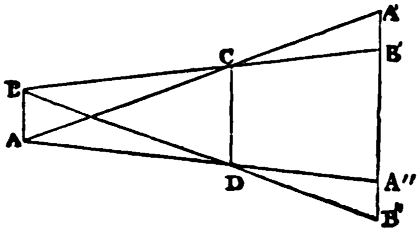

Fig. 10.

In like manner it may be shown that the light issuing from the point A will be excluded from the space included between the lines C A′ and D A″. It will also be easily perceived that the light proceeding from all the luminous points from A to B will be excluded from the space included between the lines C B′ and D A″; while more or less of such light, according to the position of the luminous points, will enter the space included between the lines C A′ and C B′, and the lines D A″ and D B″ respectively. The space, therefore, included between the lines C B′ and D A″, from which the entire light of the luminary A B is excluded, is called the *umbra*, or absolute shadow; while the spaces included between C A′ and C B′ and between D A″ and D B″, from which the light of the luminary A B is only partially excluded, is called the *penumbra*, or imperfect shadow.

If a screen be fixed behind the body C D, the shadow and penumbra will be cast upon it, and will be perceptible. At B′ and A″, the boundaries between the shadow and the penumbra, the limit of shadow will be scarcely discernible, and the shadow will become gradually less dark, proceeding from such points to the points A′ and B″, which are the limits of the penumbra. The points A′ and B″ respectively receive light from all the points between A and B, but a point below A′ receives no light from the point A, or from the points immediately above it.

In like manner the points immediately above B″ receive no light from the point B, or the points immediately below it; and as we proceed onwards along the penumbra, the nearer we approach to the limits B′ and A″, the less will be the number of luminous points of the luminary A B from which light will be received. Hence it is, that the obscurity of the penumbra augments by degrees in proceeding from its outward limits to the limits of the umbra, where the obscurity becomes complete.

The effect of the penumbra is rendered more apparent by the perspective diagram (*fig.* 11.), where S S′ is the luminous, M M′ the opaque body, *m m′* the shadow, and the faint band outside it the penumbra.

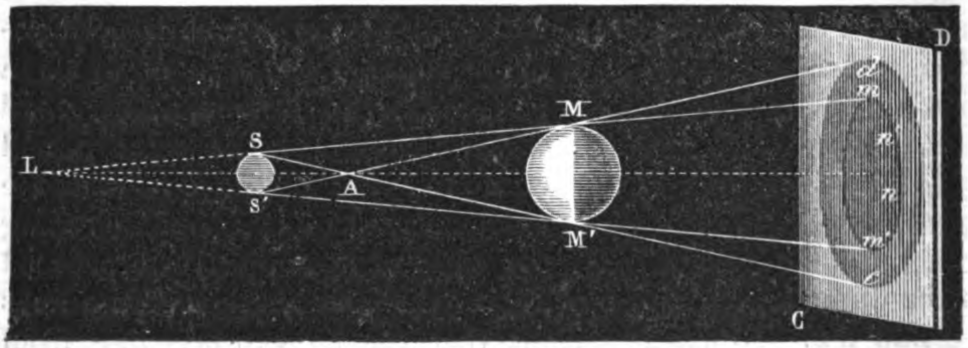

Fig 11.

17. When an object is placed with its principal plane parallel to the plane of a screen, both being at right angles to the pencil of rays which proceeds from the luminary, the outline of the shadow will resemble the outline of the object; but if the pencil falls obliquely on the object, or if the screen be not parallel to it, then the form and dimensions of the shadow will be distorted, the relative proportions and directions being different from those of the object.

When the sun is near the horizon, the shadow of an object standing vertically, which is cast upon a vertical wall, will present the form of the object with but little distortion, but the shadow which is cast upon the level ground will be disproportionally elongated.

18. *The intensity of light which issues from a luminous point diminishes in the same proportion as the square of the distance from such point increases.*

This is a common property of all radiation. The intensity of the light at any point is in the direct proportion of the number of rays which fall upon a surface of given magnitude, or in the inverse proportion of the surface over which a given number of rays are diffused.

Now let us suppose a luminous point radiating in all directions round it to be the centre of a sphere. Let two spheres be imagined, having the luminous point as a common centre, the radius of one being double the radius of the other. The surface of the greater sphere will be therefore twice as far from the luminous point as the surface of the lesser sphere; and since the surfaces of spheres are in the ratio of the squares of their radii, the surface of the greater sphere will be four times that of the lesser. Now since all the light issuing from the luminous point is diffused over the surface of each sphere, it is clear that its density on the surface of the lesser sphere will be greater than its density on the surface of the greater sphere, in the exact proportion of the magnitude of the surface of the greater sphere to the magnitude of the surface of the lesser sphere, that is, in the present example, as 4 to 1. In general it is evident, therefore, that the superficial space over which the rays issuing from a luminous point are diffused, is in the inverse proportion of the squares of the distances from the luminous point.

If, therefore, any opaque surface be presented at right angles to the rays proceeding from a luminous point, the intensity of the illumination which it receives will be increased in the same proportion as the square of its distance from its luminous point is diminished.

Since, then, the intensity of the light proceeding from each luminous point is inversely as the square of the distance from such point, it follows that the intensity of the light proceeding from any luminary will depend conjointly on, first, the number of luminous points upon the luminary, or, what is the same, the magnitude of the luminous surface; secondly, on the intensity of the light of each luminous point composing such surface; and thirdly, upon the distance from the luminary at which the illuminated object is placed.

19. The absolute brilliancy of each luminous point composing any luminous object is called the absolute intensity of its light. Let this be expressed by i. Let the number of luminous points composing it, or the magnitude of its luminous surface, be expressed by s, and let the distance of the illuminated object from the lumi-

white screen fixed in a vertical position, having a small opaque rod placed at a short distance from it, also in a vertical position. The screen, rod, and the two lights whose powers are to be compared are so placed relatively to each other, that the two shadows of the rod formed by the two luminaries on the screen shall just touch without overlaying each other. Under these circumstances, it is evident that the space on the screen occupied by the shadow proceeding from each luminary, will be illuminated by the other luminary Thus, two spaces on the screen are exhibited in juxtaposition, each of which is illuminated by one of the luminaries independent of the other. It will at first be found that these two spaces will be unequally bright. The position of the luminaries, or of the screen or rod, must then, one or all, be changed until the two shadows, being still kept in juxtaposition, appear to be equally bright, so as to present a uniform shadow. Let the distance of the two luminaries from the shadows be then measured, and it will follow, according to the principle that has been already established, that the intensity of the two luminaries will be as the squares of these distances.

If in this case the two luminaries have equal luminous surfaces, their absolute intensities will be in the ratio of the squares of their distances; but if either luminous surface be unequal, the squares of the distances will represent the proportion, not of their absolute intensities, but of the products of their absolute intensity multiplied by their luminous surface.

24. **Rumford's photometer.** — This is sometimes constructed as shown in *fig.* 13., consisting of two plates of ground glass, having an opaque partition between them. Two opaque vertical rods, S and S′, are placed opposite these plates, and the two luminaries C and L are placed opposite each rod, in such a position that

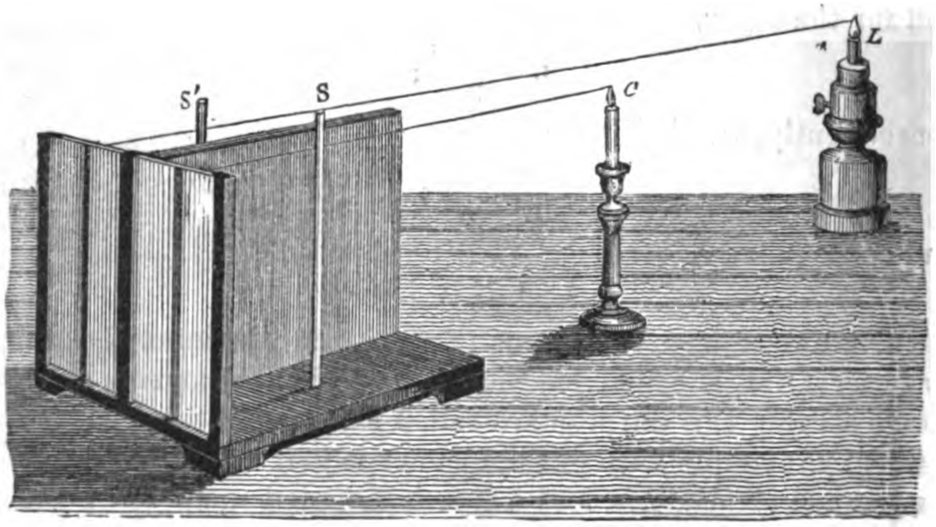

Fig. 13.

rays, and gradually diminishes until the surface is in the direction of the rays, when it ceases altogether to be illuminated.

21. If two luminaries, having equal luminous surfaces at equal distances from the same white opaque surface, placed at the same angle with the rays, shed lights of equal brightness on such surface, it follows that their absolute intensities must be equal.

In that case, the distances and the luminous surfaces being respectively equal, there is no other condition which can affect the illumination, except the intensity of the light proceeding from each luminous point; and since, therefore, the illuminations are equal, these intensities must be equal.

If, on the contrary, two such luminaries so placed produce different degrees of illumination on the same surface, their absolute intensities must be different, and must be in the proportion of the illuminations they produce. If in this case that luminary which produces the more feeble illumination be moved towards the illuminated object, until its proximity is increased, so that it produces an illumination equal to that of the other luminary, then the absolute intensity of the two luminaries will be as the squares of their distances. This may be demonstrated as follows: —

Let B express the brilliancy of the illumination produced by the two luminaries. Let S express the common magnitude of their luminous surfaces. Let I and I' express their intensities, and let D and D' express those distances which render their illuminations equal; we shall then have for the one

$$B = \frac{I \times S}{D^2},$$

and for the other,

$$B = \frac{I' \times S}{D'^2};$$

consequently, we shall have

$$\frac{I}{D^2} = \frac{I'}{D'^2};$$

and consequently,

$$I : I' :: D^2 : D'^2.$$

22. **Photometry.** — The art of measuring the intensity of light by observation is called *photometry*, and the instruments or expedients serving this purpose are called *photometers*.

The most simple form of photometer is that which may be called the method of shadows, and which is founded upon the principle which has just been demonstrated, — that with equal illumination the intensity of the light is directly as the square of the distance of the luminary.

23. **Photometer by shadows.** — This photometric apparatus, the invention of which is due to Count Rumford, consists of a

a continuous line of light, as a lighted stick does when whirled in a circle. Two similar looped curves, as shown at C D, in *fig.* 15., will thus be produced, each consisting of a luminous line. The distances of the light from the ball B must then be so adjusted, that the two luminous curves shall appear with precisely equal brightness. In that case, the relative intensities of the lights will be found by the method explained above.

26. **Ritchie's photometer.** — Another photometer, on a simple and beautiful principle, proposed by the late Professor Ritchie, and represented in *fig.* 16., consists of a rectangular box about an inch and a half or two inches wide, and eight inches long, open at both ends, and blackened in the middle. In the centre of its length are two surfaces placed at right angles with each other, and at an angle of 45° with the bottom of the box. Upon these surfaces white paper is pasted. A round hole is made in the top of the box immediately over the line formed by the edges of the paper, so that an eye looking in at this hole may see equally the two surfaces of paper. To compare two lights, the instrument is placed in such a manner before them that each may illuminate one of the pieces of paper. The distance of the lights from the surfaces of the paper are then to be so adjusted by successive trials that the two surfaces of paper shall appear to the eye of uniform brightness. In that case, the illumination of the surfaces being the same, the illuminating powers of the luminaries will be in the same proportion as the squares of their distances from the paper, the principle of this being the same as that of the photometer of Count Rumford.

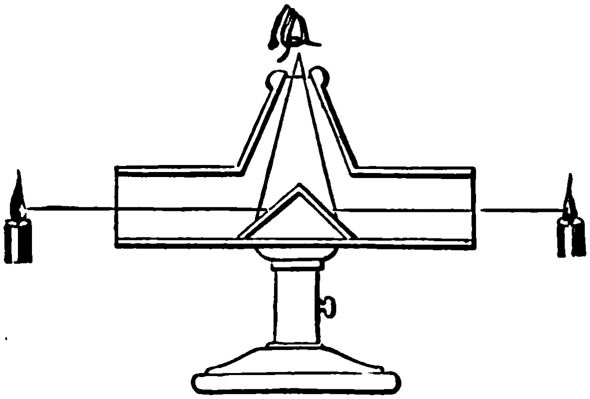

Fig. 16.

In this and all similar experiments, the colour of the light exercises a material influence on the results; and the comparative brilliancy cannot be ascertained with any precision, unless the two luminaries give light of nearly the same colour.

When it is desired to ascertain the absolute intensities of the lights, it is, as has been stated, necessary to expose equal illuminating surfaces to the photometric apparatus; but as it is not always easy to produce luminaries having surfaces exactly equal, this object may be attained by the following expedient: — Let two opaque screens, having holes in them of exactly equal magnitude, be placed near and exactly opposite to the middle of each luminous surface. The rays of light which pass through the two apertures will in such case proceed from equal portions of the surfaces of the two luminaries, and the result of the experiment will therefore show the absolute intensities.

27. **Intensity of solar light.** — The sun produces the most intense illumination with which we are acquainted. This arises partly from the absolute intensity of that luminary, and partly from the vast extent of his luminous surface. The diameter of the sun is very near a million of miles, and consequently, being a sphere, the superficial extent of his surface is about three billions of square miles; but as one half the surface only is presented to us at any one time, the magnitude of it will be a billion and a half of square miles.

28. **Electric light.** — The most brilliant artificial light yet produced is inferior to the splendour of solar light in an incredible proportion. The brightest artificial lights are those produced by the contact of charcoal points, through which a galvanic current passes, and by lime submitted to the heating power of the oxy-hydrogen blowpipe. These lights, when projected on the disc of the sun, appear, nevertheless, as black spots.

CHAP. III.

REFLECTION OF LIGHT.

29. **Reflection varies according to the quality of the surface.** — When rays of light encounter the surface of an opaque body, they are arrested in their progress, such surfaces not being penetrable by them. A certain part of them, more or less according to the quality of the surface and the nature of the body, is absorbed, and the remaining part is driven back into the medium from which the rays proceed. This recoil of the rays from the surface on which they strike is called *reflection*, and the light thus returning into the same medium from which it had arrived, is said to be *reflected*.

The manner in which the light is reflected from such a surface varies according as the surface is polished or unpolished, and according to the degree to which it is polished.

We shall consider three cases: 1st, that of a surface absolutely unpolished; 2ndly, that of a surface perfectly polished; and 3rdly, that of a surface imperfectly polished.

30. **Reflection from unpolished surfaces.**—If light fall upon a uniformly rough surface of an opaque body, each point of such surface becomes the focus of a pencil of reflected light, the rays of such pencil diverging equally in all directions from such focus.

The pencils which thus radiate from the various points are those which render the surface visible. If the light were not thus reflected indifferently in all directions from each point of the surface, the surface would not be visible, as it is from whatever point it may be viewed.

The light which is thus reflected from the various points upon the surface of any opaque body, has the colour which is commonly imputed to the body. The conditions, however, which determine the colour of bodies will be fully explained hereafter; for the present, it will be sufficient to establish the fact that each point of the surface of an opaque body which is illuminated is an independent focus from which light radiates, having the colour proper to such point, by which light each such point is rendered visible.

31. **Irregular reflection.** — This mode of reflection, by which the forms and qualities of all external objects are rendered manifest to sight, has been generally denominated, though not as it should seem with strict propriety, the irregular reflection of light. There is, nevertheless, nothing irregular in the character of the phenomena. The direction of the reflected rays is independent of each of the incident rays; but such direction obeys the common law of radiation.

The existence of these radiant pencils proceeding from the surface of any illuminated object, and their independent propagation through the surrounding space, may be rendered still more manifest by the following experiment: —

Let A B, *fig.* 17., be an illuminated object, placed before the window-shutter of a darkened room. Let *c* be a small hole made in the window-shutter, opposite the centre of the object. If a screen be held parallel to the window-shutter, and the object at some distance from the hole, an inverted picture of the object will be seen upon it, in which the form and colour of the object will be preserved; the magnitude, however, of such picture will vary according to the distance of the screen from the aperture. The less such distance, the less will be the magnitude of the picture.

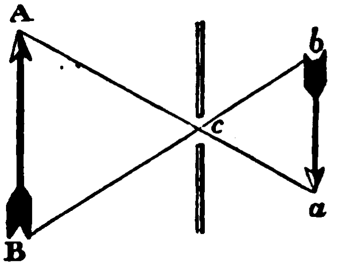

Fig. 17.

32. The smaller the aperture *c* is, by which the light is admitted,

the more distinct but the less luminous will be the picture. The effect of a small circular aperture in producing the image of a distant object is shown in *fig.* 18.

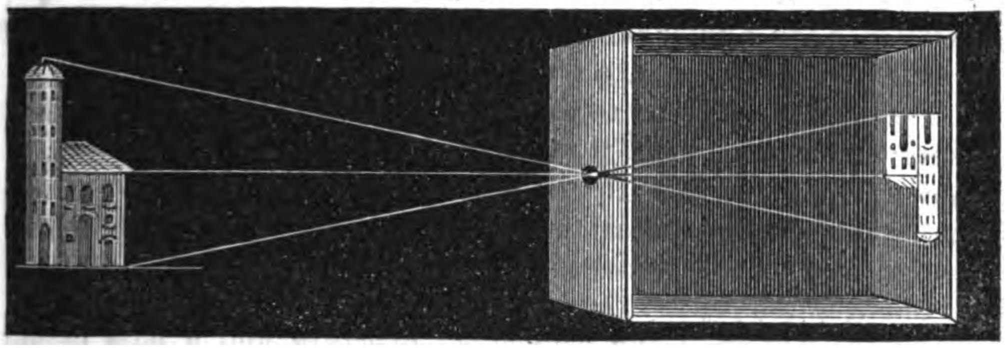

Fig. 18.

We have here supposed the shape of the aperture *c* to be circular, but the shape does not affect the production of the image, as may be proved experimentally by providing a movable cover for the aperture, upon which cards may be placed having in them holes of different forms. If a candle, as in *fig.* 19., be placed opposite these holes, the image of the candle will be equally distinct whatever be the shape of the hole.

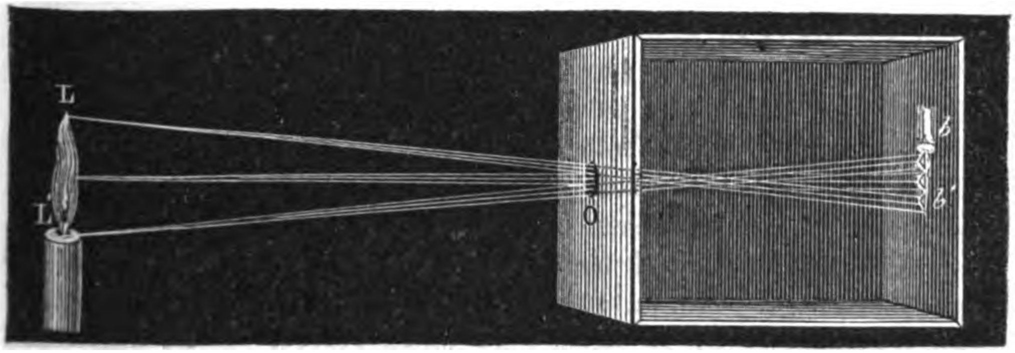

Fig. 19.

This effect is easily explained. According to what has been already stated, each point of the surface of the illuminated object A B (*fig.* 17.) is a focus of a pencil of rays of light having the colour peculiar to such point. Thus, each portion of the pencil of rays which radiates from the point B, and has for its base the area of the aperture *c*, will pass through the aperture, and will continue its rectilinear course until it arrives at the point *b* upon the screen, where it will produce an illuminated point corresponding in colour to the point B. In the same manner, the pencil diverging from A, and passing through the aperture *c*, will produce an illuminated point on the screen at *a*, corresponding to the point A.

Each intermediate point of the object will produce a corresponding illuminated point on the screen. It is evident, therefore, that a series of illuminated points corresponding in arrangement and colour to those of the object will be formed upon the screen between *a* and *b*, their position, however, being inverted, the points which are highest in the object being lowest in the picture.

These effects may be witnessed in an interesting manner in any room which is exposed to a public thoroughfare frequented by moving objects. Let the window-shutters be closed, and the interstices stopped, so as to exclude all light except that which enters through any small hole in the shutters, and if no hole be found in the shutters sufficiently small, a piece of paper or card may be pasted over any convenient aperture, and a hole of the required magnitude pierced in it. Coloured inverted images of all the objects passing before the window will thus be depicted on a screen conveniently placed. They will be exhibited on the opposite wall of the room; but unless the wall be white, the colours will not be distinctly perceptible. The smaller the hole admitting the light is, the more distinct but the less bright the pictures will be. As the hole is enlarged the brightness increases, but the distinctness diminishes. The want of distinctness arises from the spots of light on the screen, produced by each point of the object overlaying each other, so as to produce a confused effect.

33. Surfaces differ from each other in the proportion of light which they reflect and absorb. In general, the lighter the colour, other things being the same, the more light will be reflected and the less absorbed, and the darker the colour the less will be reflected and the more absorbed; but even the most intense black reflects some light. A surface of black velvet, or one blackened with lamp-black, are among the darkest known, yet each of these reflects a certain quantity of rays. That they do so we perceive by the fact that they are visible. The eye recognises such surfaces as differing from a dark aperture not occupied by any material surface, and it can only thus recognise the appearance of the material surface by the light which it reflects. The following experiment will render this more evident:—

34. **The deepest black reflects some light.**—Blacken the inside of a tube, and fasten upon the extremity remote from the eye a plate of glass. To the centre of this plate of glass attach a circular opaque disc, somewhat less in diameter than the tube, so that in looking through the tube a transparent ring will be visible, as represented in *fig.* 20. In the centre of this ring will appear an

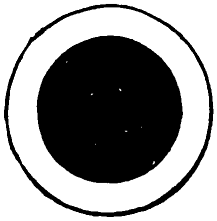

Fig. 20.

intensely dark circular space, being that occupied by the disc attached to the glass.

Now let a piece of black velvet be held opposite the end of the tube, so as to be visible through the transparent ring. If the velvet reflected no light, then the transparent ring would become as dark as the disc in the centre; but that will not be the case. The velvet will appear by contrast with the disc, not black, but of a greyish colour, proving that a certain portion of light is reflected, which in this case is rendered perceptible by the removal of the brighter objects from the eye.

35. Irregular reflection, as it has been so improperly called, is one of the properties of light which is most essential to the efficiency of vision.

Without irregular reflection, light must be either absorbed by the surfaces on which it falls, or it must be regularly reflected. If the light which proceeds from luminous objects, natural or artificial, were absorbed by the surfaces of objects not luminous, then the only visible objects in the universe would be the sun, the stars, and artificial lights, such as flames. These luminaries would, however, render nothing visible but themselves.

If the light radiating from luminous objects were only reflected regularly from the surface of non-luminous objects, these latter would still be invisible. They would have the effect of so many mirrors, in which the images of the luminous objects only could be seen. Thus, in the day-time, the image of the sun would be reflected from the surface of all objects around us, as if they were composed of looking-glass, but the objects themselves would be invisible. The moon would be as though it were a spherical mirror, in which the image of the sun only would be seen. A room in which artificial lights were placed would reflect these lights from the walls and other objects around as if they were specula, and all that would be visible would be the multiplied reflections of the artificial lights.

Irregular reflection, then, alone renders the forms and qualities of objects visible. It is not, however, merely by the first irregular reflection of light proceeding from luminaries that this is effected. Objects illuminated and reflecting irregularly the light from their surfaces, become themselves, so to speak, secondary luminaries, by which other objects not within the direct influence of any luminary are enlightened, and thus in their turn reflecting light irregularly from their surfaces, illuminate others, which again perform the same part to another series of objects. Thus light is reverberated from object to object through an infinite series of reflections, so as to render innumerable objects visible which are altogether removed from the direct influence of any natural or artificial source of light.

36. **Use of the atmosphere in diffusing light.** — The globe of the earth is surrounded with a mass of atmosphere extending forty or fifty miles above the surface. The mass of air which thus envelopes the hemisphere of the earth presented towards the sun is strongly illuminated by the solar light, and, like all other bodies, reflects irregularly this light. Each particle of air thus becomes a luminous centre, from which light radiates in every direction. In this manner the atmosphere diffuses in all directions the light of the sun by irregular reflection. Were it not for this, the sun's light could only penetrate those spaces which are directly accessible to his rays. Thus, the sun shining upon the window of an apartment would illuminate just so much of that apartment as would be exposed to his direct rays, the remainder being in darkness. But we find, on the contrary, that although that part of the room upon which the sun directly shines is more brilliantly illuminated than the surrounding parts, these latter are nevertheless strongly illuminated. All this light proceeds from the irregular reflection of the mass of atmosphere just mentioned.

37. But the solar light is further diffused by being again irregularly reflected from the surface of all the natural objects upon which it falls. The light thus irregularly reflected from the air, falling upon all natural objects, is again reciprocally reflected from one to another of these through an indefinite series of multiplied reflections, so as to produce that diffused and general illumination which is necessary for the purposes of vision.

Light and shade are relative terms, signifying only different degrees of illumination. There is no shade so dark into which some light does not penetrate.

It is the same with artificial lights. A lamp placed in a room illuminates directly all those objects accessible to its rays. These objects reflect irregularly the light incident upon them, and illuminate thus more faintly others which are removed from the direct influence of the lamp, and thus, these again reflecting the light, illuminate a third series still more faintly, and so on.

38. **Effect of the irregular reflection of lamp-shades.** — When it is desired to diffuse uniformly by reflection the light which radiates from a luminary, the object is often more effectually attained by means of an unpolished opaque reflector than by a polished one. White paper or card answers this purpose very effectually. Shades formed into conical surfaces placed over lamps are thus found to diffuse by reflection the light in particular directions, as in the case of billiard-tables or dinner-tables, where a uniformly diffused light is required. A polished reflector, in a like case, is found to diffuse light much more unequally.

In the case of white paper or card, each point becomes a centre of radiation, and a general and uniform illumination is the consequence. The light obtained by reflection in such cases is always augmented by rendering the reflector perfectly opaque; for if it be in any degree transparent, as is sometimes the case with paper shades put over lamps, the light which passes through them is necessarily subtracted from that which is reflected.

REFLECTION FROM PERFECTLY POLISHED SURFACES.

39. **Regular reflection.** — By what has been just explained, it appears that light reflected from rough and unpolished surfaces radiates from all the parts composing them, as from so many foci of divergent pencils. If, however, the surface were absolutely smooth and perfectly polished, then totally different phenomena would ensue, which have been denominated *regular reflection.*

40. Surfaces which possess this reflecting power in the highest degree are called *mirrors* or *specula.* The most perfect specula are those composed of the metals, the best being produced by various alloys of copper, silver, and zinc. If a glass plate be blackened on one side, the surface of the other will form, for certain purposes, a good reflector.

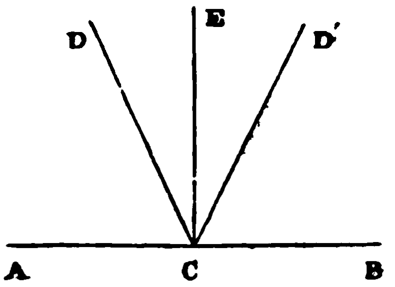

Fig. 21.

41. To explain the law of regular reflection, let C (*fig.* 21.) be a point upon a reflecting surface A B, upon which a ray of light D C is incident. Draw the line C E perpendicular to the reflecting surface at C; the angle formed by this perpendicular, and the incident ray D C, is called the *angle of incidence.*

From the point C, draw a line C D′ in the plane of the angle of incidence D C E, and forming with the perpendicular C E an angle D′ C E equal to the angle of incidence, but lying on the other side of the perpendicular. This line C D′ will be the direction in which the ray will be reflected from the point C. The angle D′ C E is called the *angle of reflection.*

The plane of the angles of incidence and reflection which passes through the two rays C D and C D′, and through the perpendicular C E, and which is therefore at right angles to the reflecting surface, is called the *plane of reflection.*

This law of regular reflection from perfectly polished surfaces, which is of great importance in the theory of light and vision, is expressed as follows: —

When light is reflected from a perfectly polished surface, the angle of incidence is equal to the angle of reflection, in the same plane with it, and on the opposite side of the perpendicular to the reflecting surface.

From this law it follows that if a ray of light fall perpendicularly on a reflecting surface, it will be reflected back perpendicularly, and will return upon its path; for in this case, the angle of incidence and the angle of reflection being both nothing, the reflected and incident rays must both coincide with the perpendicular. If the point c be upon a concave or convex surface, the same conditions will prevail; the line c E, which is perpendicular to the surface, being then what is called in geometry the normal.

42. This law of reflection may be experimentally verified as follows: —

Let *c d c′* (*fig.* 22.) be a graduated semicircle, placed with its diameter *c c′* horizontal. Let a plumb-line *b d* be suspended from its centre *b*, and let the graduated arc be so adjusted that the plumb-line shall intersect it at the zero point of the division, the divisions being numbered from that point in each direction towards *c* and *c′*. Let a small reflector (a piece of looking-glass will answer the purpose) be placed upon the horizontal diameter at the centre with its reflecting surface downwards, and let any convenient and well-defined object be placed upon the graduated arc at any point, such as *a*, between *d* and *c*. Now, if the point *a′* be taken upon the arc *d c′* at a distance from *d* equal to *d a*, the eye placed at *a′* and directed to *b* will perceive the object *a* as if it were placed in the direction *a′ b*. It follows, therefore, that the light issuing from the point of the object *a* in the direction *a b* is reflected to the eye in the direction *b a′*. In this case the angle *a b d* is the angle of incidence, and the angle *d b a′* is the angle of reflection; and whatever position may be given to the object *a*, it will be found that, in order to see it in the reflector *b*, the eye must be placed upon the arc *d c′* at a distance from *d* equal to the distance at which the object is placed from *d* upon the arc *d c*.

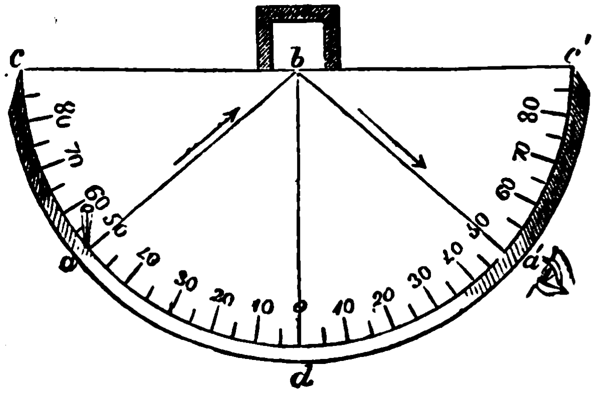

Fig. 22.

The same principle may also be experimentally illustrated as follows: —

If a ray of sun light admitted into a dark room through a small hole in a window-shutter strike upon the surface of a mirror, it will be reflected from it, and both the incident and reflected rays will be rendered visible by the particles of dust floating in the room. By comparing the direction of these two visible rays with the direction of the plane of the mirror and the position of the point of incidence, it will be found that the law of reflection which has been announced is verified.

43. **Plane reflectors.** — If parallel rays be incident upon a polished plane reflecting surface, they will be reflected parallel; for since they are parallel, they will make equal angles with the perpendiculars to the surface at their points of incidence, and the planes of these angles will also be parallel. The reflected rays will therefore also make equal angles with the perpendiculars, and the planes of reflection will be parallel; consequently, the reflected rays will be parallel.

This may also be experimentally verified by admitting rays of solar light into a dark room through two small apertures. Such rays will always be parallel; and if they are received upon a plane mirror their reflections will be found to be parallel, the rays and the reflections being rendered visible, as already explained.

44. If a pencil of divergent rays fall upon a plane mirror, the reflected rays will also be divergent, and their focus will be a point behind the mirror similarly placed, and at the same distance, as the focus of incident rays is before it. To demonstrate this, let D D (*fig.* 23.) be the reflecting surface. Let F be the focus of the incident pencil from which the rays F A, F B, F C, &c. diverge, and let F A be perpendicular to the reflecting surface A B. If we take A F′ on the continuation of F A equal to A F, and draw the line F′ B B′ and F′ C C′, then it can easily be perceived that the lines B B′ and C C′ make angles with the reflecting surface, and therefore with the perpendicular to it, equal to the angles which the incident rays F B and F C make with it respectively; for since A F is equal to A F′, F B will be equal to F′ B, and F C will be equal to F′ C; consequently the angles B F A and B F′ A will be equal, as will also the angles C F A and C F′ A. But the angles B F A and C F A are the angles of incidence of the two rays F B and F C; and since the angles B F′ A and C F′ A are respectively equal to them,

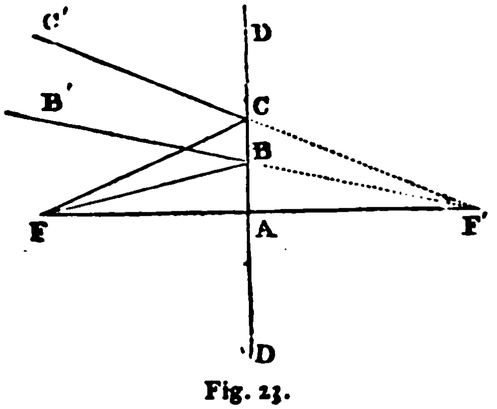

Fig. 23.

and lie on opposite sides of the perpendicular, they will be the angles of reflection; consequently the ray F B will be reflected in the direction B B′, and the ray F C in the direction C C′. These two rays, therefore, will be reflected from the points B and C as if they had originally radiated from F′ as a focus; and in the same way it may be shown that the other rays of a pencil diverging from F will be reflected from the mirror as if they had diverged from F′. But F′ is the point on the other side of the mirror which is placed similarly and at the same distance from the mirror as the point F is in front of it.

45. **Image formed by a plane reflector.**—It follows, from what has been just explained, that an object placed before a plane reflector will have an image at the same distance behind the reflector as the object is before it; for the rays which diverge from each point of the object will, after reflection, according to what has been shown, diverge from a point holding a corresponding position behind the reflector, and if received after reflection by the eye of an observer will produce the same effect as if they had actually diverged from such point. All the rays, therefore, proceeding from the object will, after reflection, follow those directions which they would follow had they proceeded from a series of points on the surface of a similar object placed behind the reflector at the same distance as the object itself is before it, and consequently they will produce the same effect on the organs of vision as would be produced by a similar object placed as far behind the mirror as the object itself is before it.

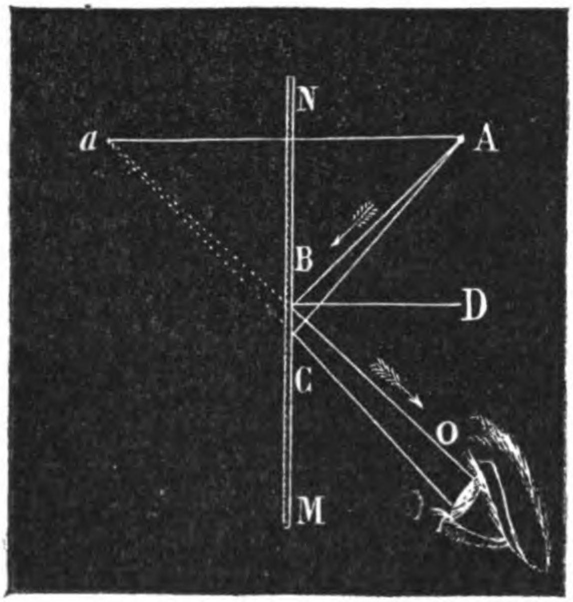

Fig. 24.

Let A (*fig.* 24.) be any point of a visible object placed before a plane reflector M N. Let A B and A C be two rays diverging from it, and reflected from B and C to an eye at O. After reflection, they will proceed as if they had issued from a point, *a*, as far behind the reflector as the point A is before it; that is to say, the distance A N will be equal to *a* N.

It is easy to verify this, by taking into account the law of reflection already explained. If B D be at right angles to M N, the angle D B O will be equal to B *a* N, and also to D B A, and consequently to B A N, from whence it follows that B A is equal to B *a*, and A N to *a* N; and since the same will be true of all rays which issue from A towards the reflector, it follows that, after reflection, all such rays will enter the eye, O, as if they had diverged from *a*.

The eye O will therefore see the point A in the reflector as if it were at *a*.

46. But since the same will be true of each point in an object, A B (*fig.* 25.), placed before the reflector, it follows that the rays which proceed from the several points of the object will, after reflection, enter the eye, as if they came from corresponding points of a similar object *a b*, placed just as far *behind* the reflector as the object itself, A B, is *before* it.

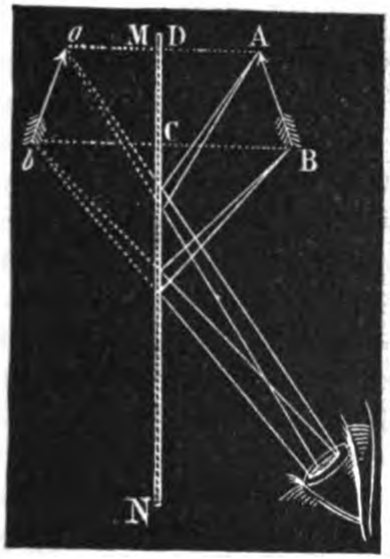

Fig. 25.

It is evident that in this case the image *a b* is not only similar to the object but precisely equal to it. Its position relatively to the reflector is similar to that of the object, but in an absolute sense it is different, as will be evident from observing that while the arrow, A B, points to the left, its image, *a b*, points to the right.

The position of the different parts of the image formed in a plane reflector will be exactly determined by supposing perpendiculars drawn from every point on the object to the reflector, and these perpendiculars to be continued beyond the reflector to distances equal to those of the points from which they are drawn before it. The extremities of the perpendiculars so continued will then determine the corresponding points of the image.

It follows from this, that the images of objects in a plane reflector appear *erect*; that is to say, the top of the image corresponds with the top of the object, and the bottom of the image with the bottom of the object. But considered laterally with regard to the object itself, they will be *inverted*; that is to say, the left will become the right, and the right the left. This will be easily understood by considering that if a person stand with his face to a plane reflector, in a vertical position, his image will be presented with the face towards him, and the image of his right hand will be on the right side of his image as he views it, but will be on the left side of the image itself, and the same will apply to every other part of the image in reference to the object. There is, therefore, *lateral inversion*.

If *a b c d* (*fig.* 26.) be an object placed before a plane reflector, the manner in which its image is rendered visible will be understood by observing the course pursued by the rays issuing from each part of the object, and reflected to the eye. Thus, a ray proceeding from the foot *d* will be reflected to the eye *a*, as if it came from *d′*; one proceeding from the leg *c* will be reflected to the eye as if it came from *c′*; and so on.

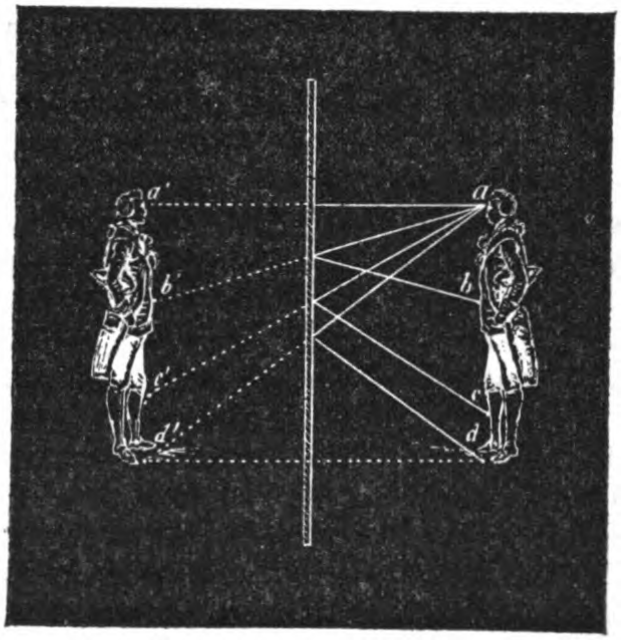

Fig. 26.

47. The effect of the lateral inversion produced by a plane reflector is rendered strikingly manifest by holding before it a printed book. On the image of the book all the letters will be reversed.

It follows also, from what has been explained, that if an object be not parallel to a reflector, but forms an angle with it, the image will form a like angle with it, and will form double that angle with the direction of the object.

Let A B (*fig.* 27.) be a plane reflector, before which an object, C D, is placed. From C draw the perpendicular C O, and continue it from O to C′, so that O C′ shall be equal to O C. In like manner, draw the perpendicular D P, and continue it so that P D′ shall be equal to P D. Then the image of C will be at C′, and the image of D at D′, and the image of all the intervening points between C and D will be at points

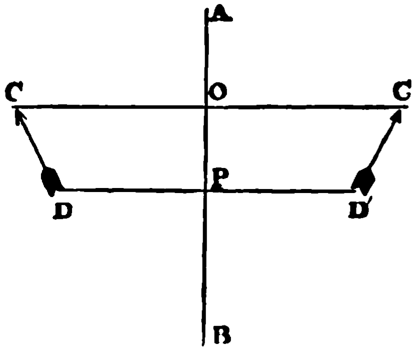

Fig. 27.

intermediate between c′ and d′, so that c′ d′ shall be inclined to the reflector at the same angle as c d is inclined to it, and the object and the image will be inclined to each other at twice the angle at which either is inclined to the reflector. Hence, if an object in a horizontal position be reflected by a reflector forming an angle of 45° with the horizon, its image will be in a vertical position; and if the object being in a vertical position be reflected by such a mirror, its image will be in a horizontal position.

If a reflector be placed at an angle of 45° with a wall, the image of the wall will be at right angles with the wall itself. If a reflector be horizontal, the image of any vertical object seen in it will be inverted. Examples of this are rendered familiar by the effect of the calm surface of water. The country on the bank of a calm river or lake is seen inverted on its surface.

48. **Series of images formed by two plane reflectors.** — If an object be placed between two parallel plane reflectors, a series of images will be produced lying on the straight line drawn through the object perpendicular to the reflector. This effect is seen in rooms where mirrors are placed on opposite and parallel walls, with a lustre or other object suspended between them. An interminable range of lustres is seen in each mirror, which lose themselves in the distance and by reason of their faintness. This increased faintness by multiplied reflection arises from the loss of light caused in each successive reflection, and also from the increased apparent distance of the image.

Let a b and c d (*fig.* 28.) be two parallel reflectors; let o be an object placed midway between them. An image of o will be formed at o′ as far behind c d as o is before it, and another image will be formed at *o′* as far behind a b as o is before it. The image o′, becoming an object to the mirror a b, will form in it another image *o″* as far behind a b as o′ is before it; and in like manner the image *o′*, becoming an object to the mirror c d, will form an image o″ as far behind c d as *o′* is before it. The images o″ and *o″* will again become objects to the mirrors a b and c d respectively; and two other images will be formed at equal distances beyond these latter. In the same way we shall have, by each pair of images

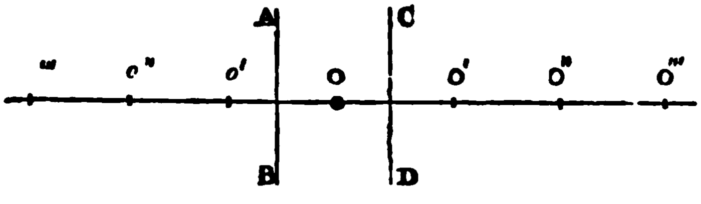

Fig. 28.

becoming objects to the respective mirrors, an indefinite series of equidistant images.

The distance between each successive pair of images will be equal to the distance of the object o from either of the images o′ or *o*′, and consequently to the distance between the mirrors.

49. **Images repeated by inclined reflectors.** — A variety of interesting optical phenomena are produced by the multiplied reflection of plane mirrors inclined to each other at different angles. As all these phenomena may be explained upon the same principle, it will suffice here to give a single example.

Let A B, A C, *fig.* 29., be two reflectors, inclined to each other at a right angle, and let o be an object placed at a point between them, equally distant from each. From o draw O M and O N perpendicular to A C and A B, and produce O M to o′, so that M o′ will be equal to M o; and produce O N to o″, so that N o″ shall be equal to N o. Two images of the point o will be formed at o′ and o″. The image o′ becoming an object to the mirror A B will have an image at o‴ just as far behind A B as o′ is before it; and, in like manner, the image o″ becoming an object to the reflector A C, will have an image just as far behind A C as o″ is before it; but, in the present case, this latter image of o″ in the reflector A C will coincide with the image of o′ in the reflector A B, and will appear at o‴. Thus, the mirrors will present three images of the object o, which are placed at the angles of a square, of which the point A is the centre.

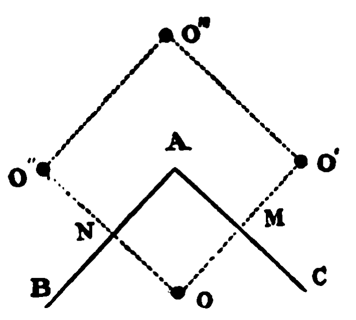

Fig. 29.

In the same manner, if the reflectors A B and A C be placed at an angle which is the eighth part of 360°, there will be formed seven images of the point o, which, with the point o, will be placed at the eight angles of a regular octagon of which the point A, where the mirrors meet, will be the centre; and like results will be found by giving the mirrors other inclinations.

50. **Formation of images by reflecting surfaces in general.** — In order that a reflector should produce a distinct image of an object placed before it, it is necessary that the rays diverging from each point of the object should, after reflection, diverge from, or converge to, some common point.

Thus, the surface of the object may be considered as an assemblage of foci of an infinite number of pencils of incident rays. These pencils will, by reflection, be converted into other pencils, having other foci, the assemblage of which will determine the form and magnitude of the image of the object produced by the reflector. In the case of a plane reflector, it has been shown that the assemblage of these foci corresponds in form and magnitude to the object, and therefore the image is equal, and in all respects similar to the object; but this does not always happen.

51. The pencils of incident rays may be converted by reflection into pencils of reflected rays having different foci, but the assemblage of these foci may not correspond with the points forming the surface of the object. They may be similar to it in form, but greater in magnitude, in which case the reflector is said to magnify the object; or they may be similar to it in form and less in magnitude, in which case the reflector is said to diminish the object. In fine, they may assume such a form as to present the object in altered proportions. Thus, while the proportion of the vertical dimensions is preserved, that of the horizontal dimensions may be increased or diminished, or *vice versâ;* or either of these dimensions may be differently increased at various points of the image. In such case, the reflector is said to present a distorted image.

52. Since to produce a distinct image of any point in an object, it is necessary that the rays diverging from that point should be reflected, so as to diverge from some other point, if after reflection they have no common point of intersection, the point of the object from which they originally diverged can have no distinct image. In this case the effect of the reflection will be to produce upon the vision a confused impression of the colour of the object, without any distinct form.

53. In order, therefore, that a polished surface should reflect the rays which diverging from any point are incident upon it exactly to or from another point, it is necessary that the surface should have that property in virtue of which lines drawn from the two points in question to any one point on the surface shall make equal angles with the surface. No surface possesses this property except one whose section made by a plane passing through the two points is an ellipse, the two points being its foci. It follows, therefore, that if a pencil of light have its focus at one of the foci of an ellipse, the rays which diverging from such focus strike upon the ellipse, or upon any surface with which the ellipse would coincide, will be reflected to the other focus.

54. **Elliptic reflector.**—To render this more clear, let A C B D, *fig.* 30., be an ellipse whose foci are F and F′. Then, according to what has been explained, if two lines be drawn from F and F′ to any one point, such as P, in the ellipse, they will make equal angles with the ellipse; and, consequently, if F P be a ray of light forming part of a pencil of rays whose focus is F, it will be re-

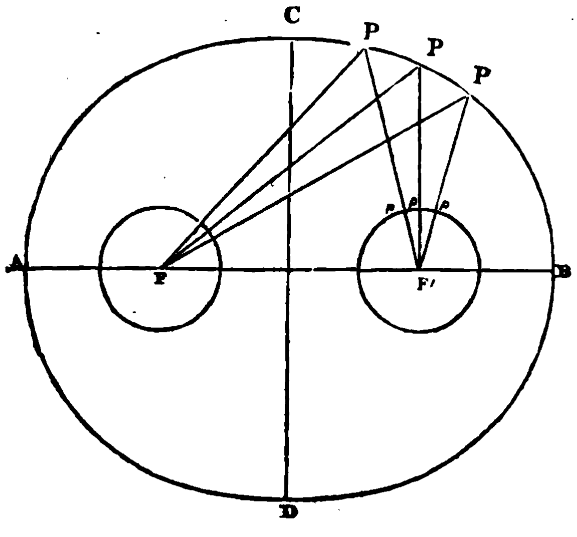

Fig. 30.

flected along the line P F′ to the other focus.

Now if we suppose a reflecting surface so formed, that the ellipse by turning round the line A B as an axis will everywhere coincide with it, this surface is called an *ellipsoid;* and if it were a polished and reflecting surface, it would be called an *elliptic reflector.*

It is evident that it is not necessary that such a surface should form a complete ellipsoid. Any portion of it upon which a pencil of rays passing from one of the foci would fall, would reflect such pencil so as to make it converge to the other focus. In this case the pencil proceeding from the focus in which the luminous point is placed, would be a diverging pencil, and that which is reflected to the other focus would be a converging pencil.

55. **Parabolic reflectors.**—A parabola has a property in virtue of which a line drawn from any point in it, such as P, *fig.* 31., to a point F called its focus, and another, P M, parallel to its axis, make equal angles with the curve. It follows from this, that if the parabola possessed the power of reflecting light, rays diverging from its focus F would be reflected parallel to its axis V M; and, on the other hand, if rays directed along lines parallel to its axis were incident on the parabola, they would be reflected in the form of a pencil converging to its focus.

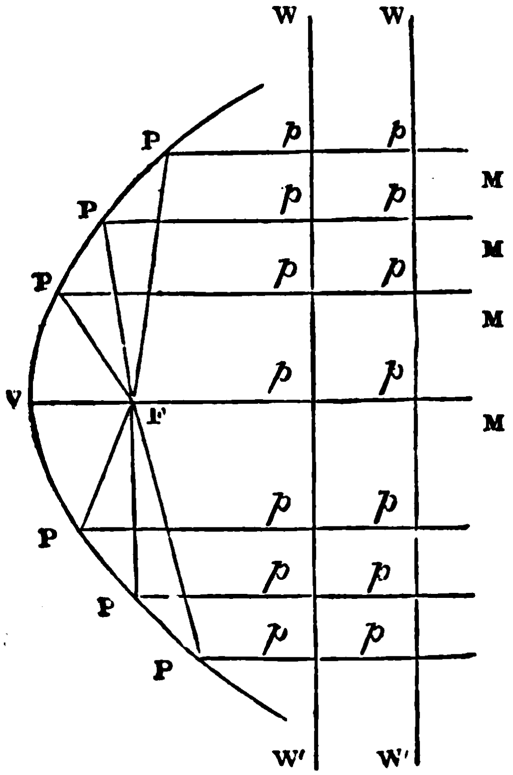

Fig. 31.

If we suppose the parabola to revolve round its axis V M, a surface with which it would everywhere coincide as it revolves is called a *paraboloid;* and if such a surface were polished so as to reflect light regularly, it would form a parabolic reflector. It follows, therefore, that if a luminous point be placed in the focus of such a reflector, its rays after reflection will be parallel to the axis; and, on the other hand, if rays strike upon the reflector in directions parallel to its axis, they will be reflected to its focus.

56. These remarkable properties of elliptic and parabolic reflectors may be easily verified by experiment. Let A B C, *fig.* 32., be the section of an elliptic reflector made by a plane passing through its focus F, the other focus being at F′. Let a luminous point, such as a small flame, or, still better, the light produced by two charcoal points when a galvanic current passes through them, be placed at the focus F.

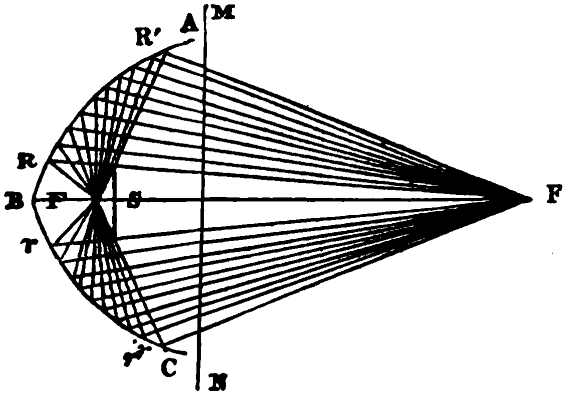

Fig. 32.

Let straight lines be imagined to be drawn from F′ through the extremities of a screen S, meeting the reflector at R and *r*, and from the luminous point F draw the lines F R and F *r*. It is clear from what has been stated that a ray of light passing from F to R will be reflected from R to F′; and one passing from F to *r* will be reflected from *r* to F′, both grazing the edge of the screen S; and the same will be true for all rays passing from F which are incident upon a circle traced on the reflector whose diameter would be a line joining R and *r*.

The rays proceeding from F, and incident between the points R and *r*, will, after reflection, strike upon the screen S, and will thus be prevented from proceeding towards the point F′. From the point F draw the lines F R′ and F *r*′ passing the extremities of the screen S. It is clear that the rays passing from F between the lines F R′ and F *r*′ will be intercepted by the screen.

Thus it follows that all the rays which strike upon the reflector, and which are not intercepted by the screen S, are included on the one side by the lines F R and F R′, and on the other by the lines F *r* and F *r*′. Now, according to what has been explained, all the rays incident upon the surface of the reflector would, after reflection, converge to the point F′, as represented in the figure. To verify this fact, let a white screen M N be placed between F′ and S, at right angles to the line F′ S. The reflected light will appear upon this screen when held near to S as an illuminated disc with a small circular dark spot in its centre, this dark spot corresponding to the space from which the light both direct and reflected is excluded by the small screen S. If the screen M N be now gradually moved towards F′, being kept perpendicular to the line S F′, the illuminated disc will gradually diminish in diameter, as will also the dark circular spot in its centre, and this diminution will continue until the screen arrives at the point F′, when the illuminated disc will be reduced to a small light spot, and the dark spot in its centre will disappear.

This experiment may be further varied by placing the screen M N as near the reflector as possible, and piercing several holes in it within the area of the illuminated disc. The rays of light passing through these holes will severally converge to the point F′, as may be shown by holding another screen beyond M N, by means of which the course of the rays may be traced, since their light will produce light spots upon this screen. As it is moved towards F′, these light spots will gradually approach each other, and when it arrives at F they will coalesce and form a single spot.

57. The reflecting property of a parabolic reflector may be experimentally exhibited by a like expedient. Let A B C, *fig.* 33., represent a section of the reflector, the focus being F. Let a luminous point be placed at F, and a small circular screen S, as before, be placed perpendicular to the axis, and near the point F. It may be shown, as in the case of the elliptic reflector, that the rays F R′ and F *r*′, which grazed the screen, will be reflected in the direction R′ Y′ and *r*′ *y*′, parallel to the axis B X; and, in like manner, that the rays F R and F *r*, which, after reflection, graze the screen, will also be reflected in the direction R Y and *r* *y*, parallel to the axis.

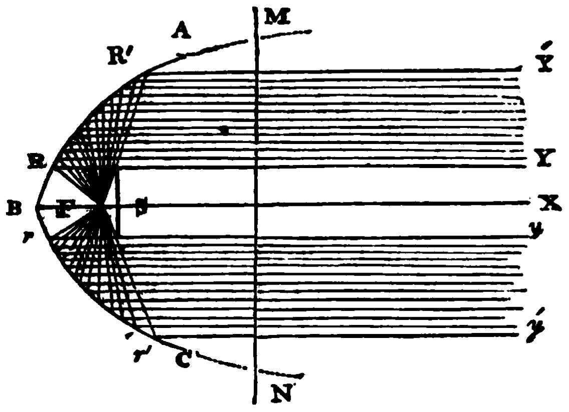

Fig. 33.

Hence it follows that the reflected light will be excluded from a cylindrical space, of which the screen S is the circular base, and whose axis coincides with the axis B X of the reflector.

It also appears that no light diverging from the focus F will strike the reflector beyond the points R′ and *r*′. The light reflected will therefore be included between two cylindrical surfaces, having the axis of the parabola as their common axis, the sides of the exterior cylinder being R′ Y′ and *r′ y′*, and those of the interior cylinder being R Y and *r y*.

It is easy to verify these phenomena. Let a white screen M N be held as before at right angles to the axis B X, an illuminated disc will appear upon it, whose diameter will be equal to the line R′ *r′*, having a small dark spot in the centre equal in magnitude to the screen S. If the screen M N be moved towards or from the screen S, this illuminated disc will continue of the same magnitude, having a dark spot in the centre constantly of the same magnitude also. Thus it appears that the reflected rays must follow the course already described.

The experiment may be further varied, as in the case of the ellipse, by piercing several holes in the screen M N, through which distinct rays shall pass. These rays, being received upon another screen behind M N, will produce upon it luminous spots, and if then either screen be moved towards or from M N, these spots will maintain always the same relative position.

If, in the case of the elliptic reflector, the luminous point be placed at F′, *fig.* 32., instead of F, then the effects will take place in an inverse order, the incident rays being in this case what the reflected rays were in the former, and *vice versâ;* and the phenomena may be verified by a like expedient. If a small circular screen be held between S and B at right angles to the axis, it will be found that the rays reflected from the elliptic surface will be inclosed between two conical surfaces, one of which is bounded by F R′ and F *r′*, and the other by F R and F *r*. The light will be excluded from the cone whose base is the screen S, and whose vertex is at F; and also from the cone whose base is R *r*, and whose vertex is also at F.

In the same manner, all the effects will be inverted if a cylinder of rays parallel to the axis be directed upon a parabolic reflector. In this case, the reflected rays will be included between the conical surface bounded by the lines F R′ and F *r′*, *fig.* 33., and the conical surface bounded by the lines F R and F *r*.

This may be in like manner experimentally verified by means of a white screen moved between the screen S and the vertex B of the reflector.

58. **Burning reflectors.** — In consequence of this property, parabolic reflectors are well adapted for collecting the rays of the sun or moon into a focus. Owing to the enormous distance of these objects, compared with any magnitudes which can be subject to experiment, all pencils proceeding from them may be considered as parallel. If, then, a parabolic reflector be placed so that its axis shall be directed towards the sun, the rays of the sun reflected by it will be collected in its focus; and as their heating power will

then be proportionally augmented, the apparatus may be used as a burning reflector.

59. **Experiment with two parabolic reflectors.** — If two parabolic reflectors be placed at any distance asunder, their axes coinciding, the rays proceeding from a luminous point placed in the focus of one will, after two reflections, be collected into the focus of the other.

Thus, if A B and A′ B′, *fig.* 34., be the two parabolic reflectors,

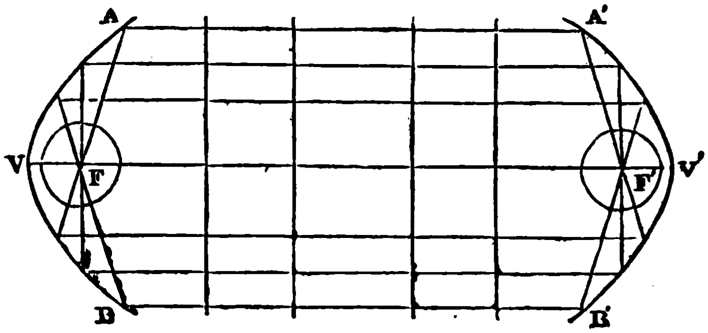

Fig. 34.

the light proceeding from a luminous point at F will be reflected by the surface A B in lines parallel to A′ B′, and striking upon the reflector will converge to the focus F′.

This is precisely similar to and explicable on the same principles as the phenomena of echo; all that has been explained above in reference to elliptic reflectors is also analogous to the phenomena of echo, which are explained in our course of "Acoustics." Thus the reflection of light is in all respects analogous to the reflection of sound, and subject to the same laws.

60. If, in the preceding experiments, the luminous point be moved from the position of the focus F, and be placed either nearer to or further from the reflector, or above or below the focus, the reflected rays will no longer converge to a common point after reflection by an elliptic surface, nor will they proceed in parallel directions after reflection by a parabolic surface. These effects may be verified experimentally by the same expedients as before.

If, when the luminous point is placed before the reflector out of the focus F, the screen M N be moved as before, the reflected rays will produce upon it as before an illuminated disc; but this disc will not be reduced to a luminous point by moving the screen from the reflector; it will diminish in magnitude to a certain limit, and then increase, but will not in any case be reduced to a point.

In the same manner with the parabolic reflector, when the light is placed out of the focus, the illuminated disc produced upon the screen will not continue to be of the same magnitude, but will either increase or diminish, according as the luminous point is placed within or beyond the focus. In the latter case, however, although the illuminated disc will diminish, it will not be reduced to a point, but after being reduced to a certain magnitude, it will again increase, and in all these cases the disc will be much more regular in its outline than in the former case.

It appears, therefore, that an elliptical reflector will only convert rays diverging from a determinate point into rays converging to another determinate point, when the former of these points is at one of the foci; and a parabolic reflector will only convert diverging rays into parallel rays when these rays diverge from the focus, and will only convert parallel rays into rays converging to a determinate point when these parallel rays are parallel to the axis.

61. **Spherical reflectors.** — The form of reflecting surface, however, which is most easy of construction, and most convenient in practice, and consequently which is most generally used, is the *spherical reflector*. The spherical reflector is a surface which may be conceived to be formed by the arc of a circle less in magnitude than a semicircle revolving round that diameter which passes through its middle point.

Thus, let us suppose A B C, (*fig.* 35.), to be such an arc, B being its middle point, and O its centre. Taking the line B O X as an axis of revolution, let the arc be imagined to rotate round it. Now let a surface be conceived, which would be everywhere in exact contact with the arc as it revolves. Such a surface is that of a spherical reflector. If the concave side of it be the polished side, it is called a *concave reflector*, the solidity and thickness being then on the convex side; but if the solidity be included within the concavity, and the convex side be polished, then the reflector is said to be *convex*.

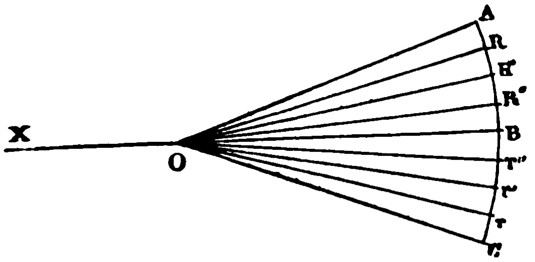

Fig. 35.

These two classes of spherical reflectors, *concave* and *convex*, have distinct properties, which will be explained in succession.

The point B, which is the middle point of the generating arc, is called the *vertex* of the reflector; and the point O, the centre of the generating arc, is called its *centre*. The length A C of the generating arc itself, expressed in degrees, is called the *opening of the reflector*. Consequently, the angle which the axis O B makes with the radius O A drawn to the edge of the reflector is half the opening. The right line B O X, drawn through the vertex and the centre of the reflector, is called the *axis of the reflector*.

Since all radii of a circle are at right angles to the circumference at the point where they meet it, it follows also that the radii of a spherical surface are at right angles to such surface. Hence it follows that all radii of a spherical reflector, such as O B, O B′, O B″, &c., are respectively at right angles to the surface of the reflector.

These definitions and consequences are equally applicable to concave and convex reflectors.

When a pencil of rays proceeding from any luminous point or illuminated object is incident upon a spherical reflector, that ray of the pencil which passes through the centre O of the reflector is called the *axis of the pencil.* Thus, if a pencil of rays diverging from the point I (*figs.* 36, 37.), be incident upon the reflector A B C, the axis of that pencil will in such case be the line I O passing through the centre O of the reflector, and meeting the surface.

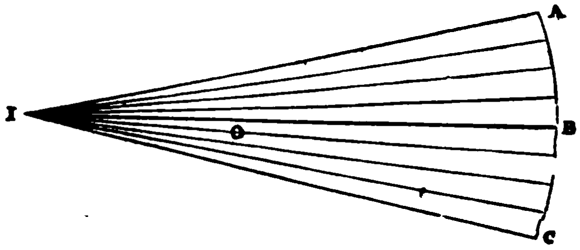

Fig. 36.

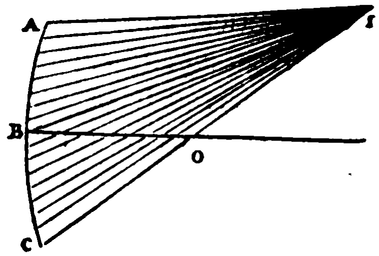

Fig. 37.

In the case represented in *fig.* 36., the axis of the pencil coincides with the axis of the reflector; but in the case represented in *fig.* 37., it is inclined to it at the angle B O C. A pencil, such as that represented in *fig.* 36. is called the *principal pencil*, and the line I B the principal axis. The pencil represented in *fig.* 37. is called a *secondary pencil*, and the axis I O a secondary axis. It is clear, from mere inspection of the diagram, that the axis of the principal pencil is the axis of the reflector. But in the case of the secondary pencil, represented in *fig.* 37., the axis I O C of the pencil is not in the centre of the rays which strike the reflector, there being more on the side B A than on the side B C.

The axis of a pencil of parallel rays is defined in the same manner; a principal pencil of parallel rays being one whose direction is parallel to, and whose axis coincides with, the axis of the reflector, and a secondary pencil of parallel rays being one whose rays and axis are inclined to the axis of the reflector.

A principal pencil of parallel rays is represented in *fig.* 38., B O X being its axis; and a secondary pencil of parallel rays is represented in *fig.* 39., X O B′ being its axis.

converge to a common point, still nearer to the middle point F of the axis O B, but still between F and B; in a word, the nearer such rays are to the axis B X, the nearer will be their common point of convergence after reflection to the middle point F; but, however near they may be to B X, they cannot converge to any point beyond F in the direction of the centre O.

It is evident, therefore, from these results, that parallel rays incident upon a spherical surface do not after reflection converge to any common point, since each cylindrical surface formed by such rays converges to a different point upon the axis; nevertheless, it appears that all these points of convergence are included within a small space P F upon the axis, provided that the reflector have not great breadth; and it is found that if the reflector do not extend to more than about 5° or 6° on each side of its vertex, all the parallel rays reflected from it will converge so nearly to the middle point F of the radius O B passing through its vertex, that, for practical purposes, the reflector may be considered as possessing the properties of a parabola already explained, and the reflected rays may be considered as virtually converging to a common point. This common point will be F, the middle point of the radius O B, which forms the axis of the reflector, parallel to the incident rays.

If a secondary pencil of parallel rays be incident on the reflector, as represented in *fig.* 41., the focus to which its rays will be reflected will be the middle point F of the radius O B′, which forms the secondary axis.

All the reasoning which has been applied to the principal pencil (*fig.* 40.) will be equally applicable in this case.

If a secondary pencil be inclined to the axis O B, at an angle greater than half the opening of the reflector, its axis will not meet the reflecting surface. This case is represented in *fig.* 42., where the line O F B′, drawn through the centre, parallel to the rays of

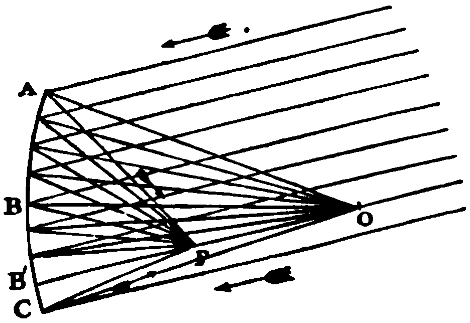

Fig. 41.

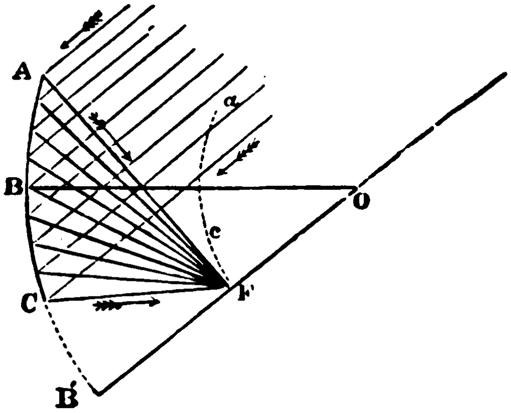

Fig. 42.

the pencil, passes below the limit C of the reflector. In such a case, nevertheless, the focus of the reflected rays is determined in the same manner as it would be if the reflector extended to B, and, accordingly, the rays reflected from A C will converge to a focus at F, the middle point of O B′.

63. **Principal focus of spherical reflector.**—If, therefore, any number of pencils of parallel rays, principal and secondary, are incident upon the same reflector, their several foci will lie at the middle point of the radii of the reflector which coincide respectively with their several axes; and if an infinite number of such pencils fall at the same time on their reflector, their foci will form a circular arc *a c* (*fig.* 42.), whose centre is the centre of the reflector O, and whose radius is O F, one half the radius of the reflector

64. All these effects may be experimentally verified by means of screens, in a manner similar in all respects to that which has been already explained in the case of a parabolic reflector. Thus it can be shown, that if the opening of a reflector be much greater than 20°, parallel rays will not be reflected converging to a common point; and, on the other hand, if a luminous point be placed at F (*fig.* 41.), the reflected rays will not be parallel; but if the opening do not exceed 20° or thereabouts, parallel rays will be sensibly convergent to the point F after reflection, and rays diverging from F will be reflected in directions sensibly parallel.

The focus to which parallel rays converge after reflection is called the principal focus of the reflector.

It follows, therefore, from what has been stated, that the principal focus of a spherical reflector is the middle point of that radius which is parallel to the incident rays; and the principal foci for secondary pencils of parallel rays lie in a spherical surface A C (*fig.* 42.), whose centre is the centre of the reflector, and whose radius is half the radius of the reflector.

65. **Aberration of sphericity.**—When the opening of a spherical reflector exceeds the limit already stated of about 20°, parallel rays, falling on that part of its surface which is more than 10° from its vertex, will be reflected sensibly distant from the principal focus, and consequently the entire pencil of rays whose base is the reflector will not have a common point of convergence. Those which are incident upon the reflector within a distance of 10° from its vertex will converge sensibly to the principal focus; but those beyond that limit will converge to points more or less distant from the principal focus, according as these points of incidence, more or less, exceed a distance of 10° from the vertex of the reflector.

This departure from correct convergence, produced by the too great magnitude of the reflecting surface, is called the *aberration of sphericity*.

To convey a more exact idea of the form and curvature of a spherical reflector which has the effect of effacing spherical aberration, such a reflector is represented in *fig.* 43., where A C is an arc 20° in length, representing the vertical section of the reflector, B being its vertex, O its centre, and F its principal focus. Rays falling on A C parallel to O B would be reflected sensibly to the point F; but if the reflector were greater in the opening, as, for example, if it extended to A′ and C′, being 20° on each side of the vertex B, then the parallel rays incident at its extreme points A′ and C′ would be reflected to *f*, a point between F and B. In such cases, the space *f* F would be that within which all the rays incident between A and A′, and between C and C′, would be collected. This space *f* F would then be the extreme limit of the aberration of sphericity due to a reflector 40° in magnitude.

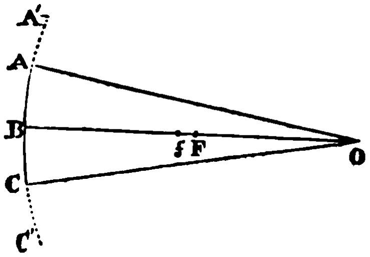

Fig. 43.

The spherical aberration of a secondary pencil will be greater than that of a principal pencil; for in the case of the secondary pencil represented in *fig.* 42., the axis of which is in the direction of O B′, the aberration will be the same as if the opening of the reflector were twice the arc A B′; and in proportion as the angle formed by the axis of the secondary pencil O B′ with the axis of the reflector O B is increased, this cause of aberration will be also increased. Thus, in the secondary pencil represented in *fig.* 42., the aberration would be the same as if the opening of the reflector were twice the angle A O B′.

In fine, the aberration attending any secondary pencil will always be the same as that which would be produced with a principal pencil by a reflector whose opening would be equal to the opening of the proposed reflector, added to twice the angle formed by the axis of the reflector and the axis of the secondary pencil. Thus, in the case represented in *fig.* 42., the aberration of the secondary pencil is the same as would be produced upon a principal pencil by a reflector having an opening equal to twice A B′.

66. **Case of convex reflectors.**— In what precedes, the case of concave reflectors only has been contemplated. The same conclusions, however, will be applicable, with but little qualification, to the case of convex reflectors.

Let such a reflector be represented by A C (*fig.* 44.), a pencil of rays parallel to the axis B X being incident upon it. The

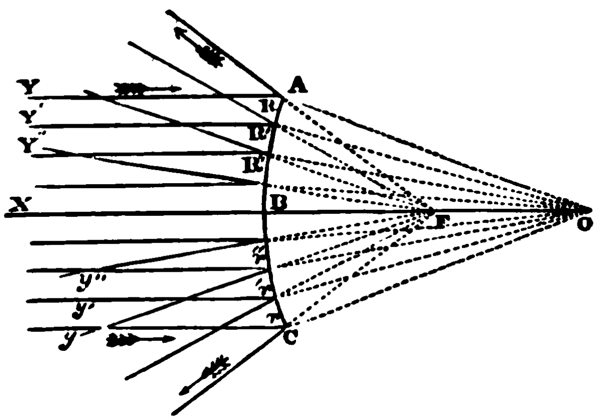

Fig. 44.

extreme rays R Y and *r y*, equidistant from B X, will be reflected from R and *r*, as if they had diverged from F, the middle point of O B, provided R and *r* be not more distant than 10° from B. In the same manner, the rays R′ Y′ and *r′ y′*, and also the rays R″ Y″ and *r″ y″*, and, in a word, all rays between the extreme rays and the axis, will be reflected as if they had diverged from F. This point F, being the middle point of the radius O B, is therefore, as in the case of the concave reflector, the principal focus.

A difference is presented here in the two cases, which suggests a distinction to which we shall often have occasion to refer in other instances. In the case of the concave reflector represented in *fig.* 40., the principal focus is a point to which the reflected rays do actually converge, and where, as has been shown, the light is concentrated. In the case, however, of the convex reflector represented in *fig.* 44., the rays diverging from the surface diverge as if they had originally been united at F. This point F is, therefore, in such case, not a point, as in the case of a concave reflector, where the rays do actually coalesce, but a point where they would coalesce if they had been continued backwards from the points on the surface of the reflector.

67. **Foci real and imaginary.** — A focus like the former, where the rays do actually converge, is called a real focus, and sometimes a physical focus; whereas a focus like the latter, in which the rays do not actually converge, but which merely forms the point of convergence of their directions, is called an imaginary focus. In the case already explained of plane reflectors, the focus of reflection of a divergent pencil is an imaginary focus; and, on the other hand, the focus of a convergent pencil is a real or physical focus.

68. **Images formed by concave reflectors.**—If an object be placed before a concave reflector at so great a distance from it that all pencils of rays passing from such object would be considered as parallel, an image of such object will be formed at the principal focus of the reflector; that is to say, midway between its centre and its surface.

Let A C (*fig.* 45.) be such a reflector, B being its vertex, O its centre, and F the principal focus. Let L M be an object, placed at so great a distance from the reflector, that the divergence of a pencil of rays passing from any point upon it, and having the reflector as their base, shall be so small that the rays may be considered as practically parallel.

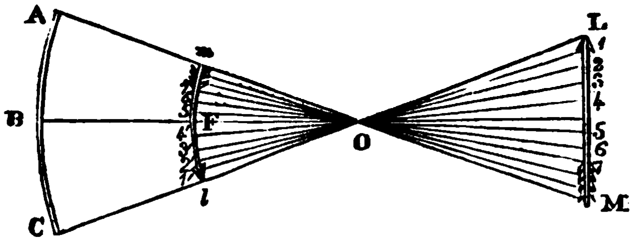

Fig. 45.

Let L O *l* be the axis of the secondary pencil passing from L, and M O *m* the axis of the secondary pencil passing from M, *l* and *m* being respectively the middle points of the radii, and therefore the foci to which the pencils proceeding from L and M respectively are collected after reflection. Images, therefore, of the points L and M respectively will be produced at *l* and *m*. In the same manner, the pencils proceeding from the several points marked 1, 2, 3, 4, 5, &c., will converge, after reflection, to the corresponding points marked 1′, 2′, 3′, 4′, 5′, &c., which are the middle points of the several radii which are in the direction of the axes of the several pencils. At these points, therefore, images will be formed of the corresponding points in the object, and the assemblage of these images will form a complete image of the object in an inverted position, midway between the centre O and the surface A B C of the reflector.

It is evident that the points forming the image *m l* will lie in a spherical surface, whose centre is O, and whose radius is half the radius of the reflector. If, therefore, the object be a straight line, its image will be the arc of a circle; and if the object be a plane surface, its image will be a spherical surface.

In the case represented in *fig.* 45., the centre point of the object is placed in the direction of the axis of the reflector, and the centre point of the image lies consequently also in the axis, and the image is at right angles to the axis of the reflector and is bisected by it.

It will be explained hereafter that the apparent visual magnitude of an object is determined by the angle formed by two straight lines drawn from the eye to the extremities of the object. Thus, if the eye were placed at O, the centre of the reflector, the angle L O M would be the apparent magnitude of the object. The full import and propriety of this term will be explained more fully hereafter, but for the present it will be convenient to use it in the sense just explained.

It is evident, then, that the apparent magnitude of the object L M, as viewed from the centre of the reflector O, is the same as the apparent magnitude of its image *l m* viewed from the same point, since the lines drawn from the limits of the object and the image intersect each other at the point O.

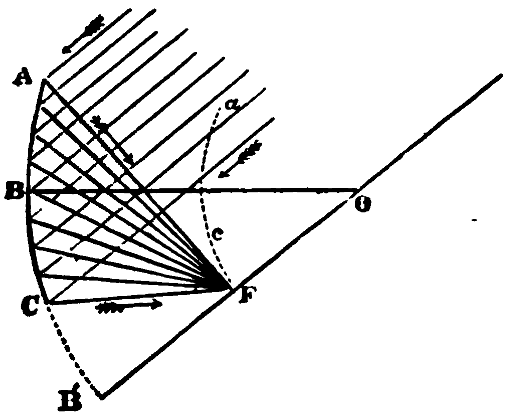

Fig. 46.

It is evident also that the linear magnitude of the image will be just so much less than the linear magnitude of the object, as one half the radius of the reflector O F is less than the distance of the object from the centre O.

69. The case in which the axis of the reflector is not directed to the centre of the object is represented in *fig.* 46.

In this case the image of the object is produced between the axes of the secondary pencil, proceeding from the extremities of the object, and at the middle points of the radii which coincide with the axes.

In the case of a convex reflector, let L M (*fig.* 47.) be the object,

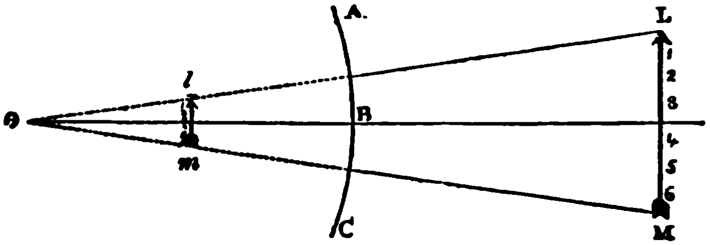

Fig. 47.

placed, as before, at such a distance that each pencil of rays proceeding from a point in the object to the reflector may be considered as parallel. Let L O and M O be the axes of the pencils proceeding from the extreme points of the object. After reflection, the rays of these pencils will diverge as if they had proceeded from the points *l m* respectively, which are the middle points of the radius of the reflector; and therefore, if such rays were received by the eye of the observer, they would produce the same effect on vision as if they had proceeded from the points *l m*, and consequently the points L M of the object would appear as if they were placed at *l m*. In the same manner, it may be shown that the intermediate points 1, 2, 3, 4, 5 of the object will appear as if they were at the intermediate points 1′, 2′, 3′, 4′, 5′ of the radii, which are in the direction of their respective pencils. Thus, an eye directed to the reflector, receiving the rays of the reflected pencils, will see the object as if it were on a spherical surface, of which the centre is o, and of which the radius is one half the radius of the reflector. The image *l m* in this case, though not formed by the real intersection of the rays of light, is not the less present to vision. The eye receives the rays exactly as it would receive them if they had actually diverged from the points *l*, 1′, 2′, 3′, 4′, 5′, *m*, and consequently the effect on vision is the same as if a real image of the object were placed at *l m*. It is evident from the figure that in this case the image is erect, and not inverted, as in the case of the concave reflector.

All that is said, however, of the relative magnitudes of the image and object in the case of the concave reflector, will be equally applicable here. Thus, to an eye placed at o, the apparent magnitude of the object L M, and of its image *l m*, will be the same; and the real linear magnitude of the image will be just so much less than that of the object, as one half the radius of the reflector is less than the distance of the object.

70. The phenomena which have been just explained in the case of the reflection of very distant objects produced by concave and convex reflectors, may easily be verified experimentally.

If a concave reflector be directed towards the sun or moon, an image of either of those objects will be found at its principal foci, and such image may be rendered apparent by holding at its principal focus, and at right angles to the radius directed to the object, a small semi-transparent screen, which may be formed of ground glass or oiled paper. A small image will be seen upon the screen, the diameter of which will bear the same proportion to the *real* diameter of the sun or moon, as half the radius of the reflector bears to the distance of one or other of these objects.

The effects of a convex reflector can be still more easily made manifest. When a convex reflector is presented to any distant objects, their images will be seen in it, and will appear as if they were behind the reflector. They will be less in magnitude than the objects in the proportion in which half the radius of the reflector is less than the distance of the objects, and they will appear as if they were painted on a spherical surface, having its centre at the centre of the reflector, and having half the reflector for its radius.

71. Before proceeding to explain the effects produced by spherical reflectors on diverging and converging pencils, it will be convenient here briefly to state some principles derived from geometry, to which it will be necessary frequently to recur in explanation of the effects produced on pencils of rays by spherical surfaces on which they are incident, whether these surfaces belong to opaque bodies or transparent media.

The magnitude of angles is easily explained by stating the degrees and parts of degrees of which they consist. It may also be often more conveniently expressed by stating the ratio which the arc which bounds them bears to the radius. Thus an angle B A C will be perfectly defined if the ratio which the arc B C bears to its radius A B be stated. Any other angle, such as *b a c*, the arc of which *b c* bears the same ratio to the radius *b a*, will necessarily have the same magnitude. This principle may be rendered still clearer, if, with A as a centre, several arcs, such as B′ C′, B″ C″, B‴ C‴, &c. be drawn subtending some angle A. It is demonstrated in geometry, and made evident from the figure, that the arcs B′C′, B″ C″, B‴ C‴, bear respectively the same ratio to their radii A B′, A B″, A B‴, as the arc B C bears to its radius A B.

On this principle, the magnitude of an angle may, with great convenience, be expressed by a fraction, whose numerator is its arc, and whose denominator is its radius. Thus the angle A, *fig.* 48., may be expressed by $\frac{BC}{BA}$, or $\frac{B'C'}{B'A}$, or $\frac{B''C''}{B''A}$, &c.

If the angles be very small, perpendiculars drawn from the extremity of either side including them upon the other, may be considered as equal to the arc. Thus, in *fig.* 48., the perpendiculars B *m* and C *m′* may be regarded as equal to the arc B C, provided the angle A do not exceed a few degrees.

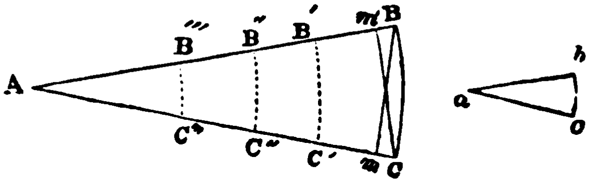

Fig. 48.

In the case of such angles, therefore, their magnitude may be easily expressed by a fraction whose numerator is the perpendicular, and whose denominator is the radius.

Thus the angle A, being small, will be expressed by $\frac{Bm}{BA}$ or by $\frac{Cm'}{CA}$.

REFLECTION OF DIVERGENT AND CONVERGENT RAYS BY SPHERICAL SURFACES.

72. **Concave reflectors.**—Let I, *fig.* 49., be the focus of a diverging pencil of rays, incident upon a concave reflector A B C, the point I being supposed to be upon the axis of the reflector. Draw I A and I C, representing the extreme rays of the pencil,

Draw O A and O C, the radii, to the points of incidence. The angles O A I and O C I will then be the angles of incidence; and these will evidently be equal, because the three sides of the two triangles are respectively equal.

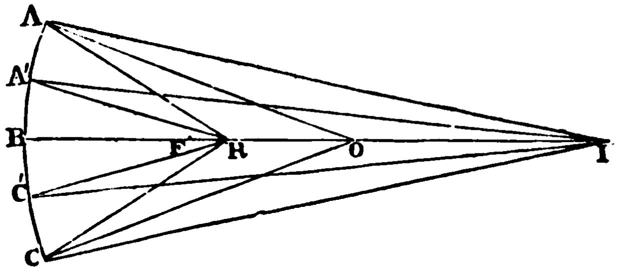

Fig. 49.

To find the direction of the reflected rays, it will be only necessary to draw from A and C lines which make with A O and C O angles equal to the angles of incidence.

Let these lines be A R and C R. The two rays I A and I C will therefore be reflected converging, and will meet at R.

By the principles of geometry*, the angle O A R of reflection is equal to the difference between the angles A R B and A O B, and the angle O A I of incidence is equal to the difference between the angles A O B and A I B.

Now, let f express the distance I B of the focus of incident rays from the vertex, and f' the distance R B of the focus of reflected rays from the same point, and let r express the radius O B of the surface. We shall then have, according to what has been explained,—

$$\text{OAI} = \frac{\text{AB}}{r} - \frac{\text{AB}}{f},$$

$$\text{OAR} = \frac{\text{AB}}{f'} - \frac{\text{AB}}{r}.$$

But since the two angles are equal, we shall have

$$\frac{\text{AB}}{r} - \frac{\text{AB}}{f} = \frac{\text{AB}}{f'} - \frac{\text{AB}}{r}.$$

Omitting the common numerator A B, we shall then have

$$\frac{1}{r} - \frac{1}{f} = \frac{1}{f'} - \frac{1}{r};$$

and consequently we shall have

$$\frac{1}{f} + \frac{1}{f'} = \frac{2}{r} \dots (\text{A}).$$

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* Euclid, Book 1. Prop. 32.

The same formula is applicable to rays incident at every point between A or C and the vertex B; so that rays reflected from all such points will converge to a common point on the axis, whose distance from B will be determined by the value of f', found by the preceding formula.

The formula (A) is of the utmost importance, and may be both understood and remembered with the greatest facility.

It may be expressed in common language as follows:—

If the fractions, whose numerator is 1, and whose denominators are the numbers expressing the distances of the foci of incident and reflected rays from the vertex, be added together, their sum will be equal to a fraction, whose numerator is 2, and whose denominator is the radius of the reflecting surface.

73. By this formula (A) the position of the focus of reflected rays can always be found when that of the incident rays is known. We have only to subtract the fraction whose numerator is 1, and whose denominator is the distance of the focus of incident rays from the vertex; that is to say, the fraction $\frac{1}{f}$ from the fraction whose numerator is 2, and whose denominator is the radius, and the remainder will be equal to a fraction whose numerator is 1, and whose denominator is the distance of the focus of reflected rays from the vertex. It is evident that by a like process the focus of incident rays can be found whenever the focus of reflected rays is known.

Since the two fractions $\frac{1}{f}$ and $\frac{1}{f'}$ added together always produce the same sum, it follows that whatever circumstances increase one, must diminish the other; and hence it follows that the more distant the focus of incident rays I is from the reflector, the nearer the focus of reflected rays R will be to it, and *vice versâ;* because as I B increases, R B must diminish, and *vice versâ,* as has been just explained.

If the focus I were removed to an infinite distance, then the fraction $\frac{1}{f}$ would be infinitely small, and consequently the other fraction $\frac{1}{f'}$, would be equal to $\frac{2}{r}$, and consequently f' would be equal to $\frac{1}{2}r$; that is to say, the focus of reflected rays would then be coincident with the principal focus, which is only what might have been anticipated; because, if the focus of incident rays I be removed to an infinite distance, the pencil of incident rays having the reflector for its base must be parallel.

But in order to produce this effect, it is not necessary that the focus of the pencil of incident rays should be either infinitely or even very considerably distant. Let us suppose that the distance I B, which is here expressed by f, is only one hundred times the length of the radius of the reflector, and let half the radius, or the distance of the principal focus from the vertex, be expressed by F. Then we shall have

$$f = 200\,\mathrm{F}.$$

Consequently we shall have

$$\frac{1}{f} + \frac{1}{200\,\mathrm{F}} = \frac{1}{\mathrm{F}};$$

and therefore

$$\frac{1}{f} = \frac{1}{\mathrm{F}} - \frac{1}{200\,\mathrm{F}} = \frac{199}{200\,\mathrm{F}},$$

and therefore

$$f = \frac{200\,\mathrm{F}}{199} = \mathrm{F} + \frac{1}{199} \times \mathrm{F};$$

that is to say, the distance of the focus of reflected rays from the vertex will exceed the distance of the principal focus by the 199th part of half the radius, or nearly the 400th part of the radius of the reflector, an insignificant quantity.

It follows, therefore, that whenever the distance of an object from the reflector is not less than 100 times its radius, all pencils proceeding from it may be regarded as parallel, and therefore as coincident with the principal focus of the reflector.

It follows also from the preceding formula, that when the focus of incident rays is beyond the centre, the conjugate focus of reflected rays will be between the centre and the principal focus; and that when the focus of incident rays is between the centre and the principal focus, the conjugate focus of reflected rays will be beyond the centre.

In the preceding cases we have supposed the focus of incident rays to be situate at some point beyond the principal focus of the reflector.

Let us now consider the case in which the focus of incident rays I, *fig.* 50., is placed between the principal focus F and the vertex.

Let I A and I C, as before, be the two extreme rays of the pencil, and draw the radii A O and C O. To find the direction of the reflected rays, it is only necessary to draw from A and C two lines, which shall make with O A and O C angles equal to those which A I and C I make with it. Let this direction be A X and C X′. It follows, therefore, that in this case the reflected rays will diverge instead of converging, as in the former case, and that their point of divergence will be at R, upon the axis behind the reflector; consequently the focus will be an imaginary focus.

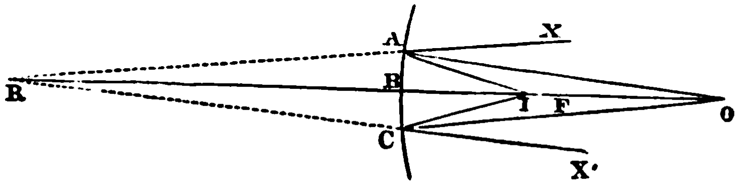

Fig. 50.

By geometrical principles already referred to *, the angle of incidence I A O is equal to the difference between the angles A I B and A O B, and the angle of reflection X A O is equal to the sum of the angles A R B and A O B; and since the angles formed by O A, I A, and R A with the axis O R, are so small as to come within the scope of the principles already expressed, we shall have

$$\text{IAO} = \frac{\text{AB}}{f} - \frac{\text{AB}}{r},$$

$$\text{XAO} = \frac{\text{AB}}{f'} + \frac{\text{AB}}{r},$$

where f and f' express, as in the former case, the distances of the foci of incidence and reflection respectively from the vertex B.

We shall therefore have

$$\frac{\text{AB}}{f} - \frac{\text{AB}}{r} = \frac{\text{AB}}{f'} + \frac{\text{AB}}{r};$$

and omitting the common numerator A B, we shall have

$$\frac{1}{f} - \frac{1}{f'} = \frac{2}{r} \ldots (\text{B}),$$

a formula which is identical with formula (A), p. 48., only that $\frac{1}{f'}$ in it is negative, which indicates that the focus of reflected rays is imaginary and behind the reflector.

In the formula (B) it is not the sum of the two fractions $\frac{1}{f'}$ and $\frac{1}{f}$, but their difference, which is equal to $\frac{2}{r}$.

Analogous results, however, follow from this formula, which may be expressed in common language as follows: —

When the focus of rays incident upon a concave reflector is

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* Euclid, Book 1. Prop. 32.

placed between its principal focus and the vertex, the difference between the fraction whose numerator is 1 and whose denominator is the distance of the focus of incident rays from the vertex, and the fraction whose numerator is 1 and whose denominator is the distance of reflected rays from the vertex, will be equal to the fraction whose numerator is 2 and whose denominator is the radius of the reflecting surface.

Since the difference between these two fractions is always the same, however they may separately vary, it follows that when one increases, the other must increase to the same extent. Hence it follows that f and f' increase and diminish together; and therefore it also follows that as the focus of incident rays I approaches the vertex B, the focus of reflected rays R must also approach it; and as the focus of incident rays I recedes from the vertex, the focus of reflected rays R must also recede from it.

When the focus of incident rays I arrives at the principal focus F, the focus of reflected rays R recedes to an infinite distance.

74. **Case of converging incident rays.** — If rays fall on the reflector converging to a point R behind it, they will be reflected converging to the point I. In this case, the focus of incident rays, being behind the reflector, will be imaginary, and the focus of reflected rays, being before it, will be real. The relative positions of the two foci, however, will be determined in the same manner exactly as if I were the focus of incidence, and R the focus of reflection. It may be useful to observe in general, that the conjugate foci are in all cases interchangeable.

If the focus of incidence become the focus of reflection, the focus of reflection will become the focus of incidence, and *vice versâ.*

75. **Convex reflectors.** — The effects attending diverging or converging rays incident upon convex reflectors, are in all respects analogous to those which have been just established respecting concave reflectors.

It is only necessary to observe that converging rays upon a convex reflector are analogous to diverging rays upon a concave reflector; and diverging rays upon a convex reflector are analogous to converging rays upon a concave reflector.

Thus, if A C, (*fig.* 49.) instead of representing a concave, represent a convex reflector, and a pencil of rays be supposed to be incident upon it, which if not intercepted would converge upon the point I, those rays after reflection will diverge from the point R. The conjugate foci will be in this case precisely similar, and determined by similar conditions as in the former case, except that the incident rays are convergent, while the reflected rays are divergent, the contrary being the case in a concave reflector.

In like manner, if the reflector A C, (*fig.* 50.), be a convex instead of a concave reflector, a pencil of rays incident upon it, which if not intercepted would converge to I, will be reflected converging to R. In this case, the focus of incident rays will be imaginary, and the focus of reflected rays real, contrary to what was shown for a concave reflector; but the relative position of the two foci will be determined as before.

76. **Images of near objects formed by spherical reflectors.** — The manner has been explained in which images are formed by spherical reflectors of objects whose distance is so great that the pencils of rays proceeding from them may be considered as consisting of parallel rays. It is in this and like cases important that the student should not confound the directions of the pencils themselves with the directions of the rays which form them. Thus, the pencils of rays proceeding from points upon the surface of the sun or moon are pencils of parallel rays, because the distance of the foci of such pencils from the observer is incomparably great compared with any surface which can form the base of the pencil. Thus, the surface of the largest reflector is as nothing compared with the distance of any point in the sun; and consequently, the rays which form a pencil, whose vertex is a point in the sun, and whose base is the surface of such a reflector, may be practically considered as parallel; but this parallelism must not be applied to the direction of the pencils themselves which proceed from different points in the sun. The directions of these pencils, or, to speak strictly, those of their respective axes, are not parallel, the axes of the extreme pencils forming an angle with each other equal to the apparent diameter of the sun; and the same observations would be applicable to any other object whose distance is so great that a pencil of rays proceeding from it may be regarded as parallel.

These observations being premised, we shall now explain the manner in which images are formed by spherical reflectors of objects which are not so distant that the rays of the pencils proceeding from points in them can be regarded as parallel.

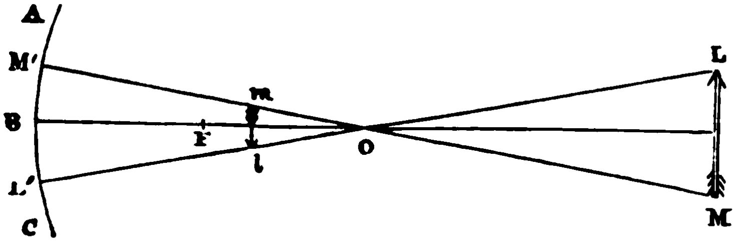

Fig. 51

Let A B C, *fig.* 51., be a concave reflector, whose centre is O, and whose vertex is B. Let L M be an object, whose form we shall for the present assume to be that of an arc of a circle whose centre is O. Let L L′ and M M′ be the axes of the extreme secondary pencils proceeding from this object, and let *l* and *m* be the foci of reflection conjugate to the points L and M. An image of the point L will be formed at *l*, and an image of the point M will be formed at *m*, and images of all the intermediate points between L and M will be formed at intermediate points of an arc drawn from *l* to *m*, having O as a centre.

Since the lowest point of the image corresponds to the highest point of the object, and *vice versâ*, the image will in this case be inverted with respect to the object, and the linear magnitude of the image will bear to that of the object the same proportion as O *l* bears to O L. These results follow in the same manner as in the case of the images of distant objects already explained.

The distance O *l* is determined when O L is known by the formulæ (A) and (B), p. 48. and p. 51.; that is to say, the position and magnitude of the image will be determined when the position and magnitude of the object are known. In this case, the object L M has been supposed to have the form of a circular arc, and its image to have a similar form. If the object form part of a spherical surface whose centre is O, the image would have a like form; but if the object were a straight line or flat surface, then the image would be more or less curved, and would consequently be distorted. But as, in general, the angle O, under which the object or image would be seen from the centre, is small, this curvature may be disregarded, and we may assume that the image will be similar to the object.

77. **Spherical aberration of reflectors.** — The pencils of rays proceeding from or to the incident focus will be reflected to a common point, only on the condition that the opening of the reflector is limited, as was explained in the case of parallel rays. If it be not so limited, then the extreme rays of the pencil will converge to points sensibly different from those which are within such limit of distance of the vertex already defined, and hence will arise a spherical aberration.

If even the reflector be sufficiently limited in its opening, a sensible spherical aberration will arise from the secondary pencils which proceed from the borders of the object, and are inclined at the greatest angles to the axis of the reflector, for in this case the angle of divergence of such pencils will, as has been already explained, exceed that limit which would efface the spherical aberration. Hence it arises that images produced by spherical reflectors when the objects are too great, are indistinct towards the borders, the pencils which form each part of the image not being brought to the same focus, and consequently producing a confused effect.

78. In what precedes, the position of the object before a concave reflector has been considered as being either beyond the centre or between the centre and the principal focus F. Let us now consider the position of the object to be at L M (*fig*. 52.), between the principal focus F and the reflector. In this case the image *l m* will be behind the reflector at the points which form the foci conjugate to the several points of the object L M.

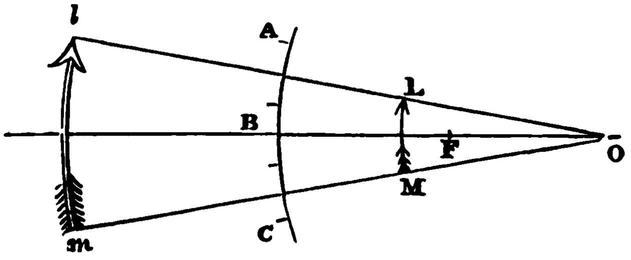

Fig. 52.

The image will in this case evidently be erect with respect to the object, and will be greater in magnitude than the object in the proportion of O *l* to O L.

If the reflector be convex, the object L M (*fig*. 53.) will have its image at the points *l m*, which are the foci conjugate to the points at L M, and those points will, according to what has been already explained, lie between the reflector and the principal focus F.

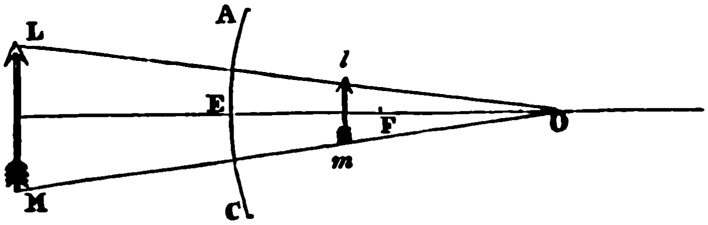

Fig. 53.

The rays proceeding from the several points of the object L M will, after reflection, diverge as if they had proceeded from the corresponding points of *l m*, and will produce upon the vision the same effects as if an object had been actually placed at *l m*.

The image in this case, therefore, will be erect, and it will be less than the object in the proportion of O *l* to O L. In this manner is explained the effect familiar to every one, that convex reflectors exhibit a diminished picture of the object placed before them.

All the preceding observations on the effect of spherical aberration, and the indistinctness incident to the borders of the image, will be equally applicable in the present case.

79. In the preceding example, the object has been supposed to be placed so that its centre coincides with the axis of the reflector. The image, however, is determined on like principles, whatever other position it may have.

Thus, let L M (*fig.* 54.) be the object, A B C being the reflector, O its centre, and F its principal focus. From the extremities of the object draw lines L O and M O through the centre O of the reflector to meet the continuation of the section of the reflector at P and Q. Let *l* be the focus conjugate to L, and *m* the focus conjugate to M, determined according to the principles and formulæ already established. Images, therefore, of the points L and M will be formed at *l m*, and images of all the intermediate points of the object will in like manner be formed between *l* and *m*, so that an inverted image of the object will be formed at *l m*.

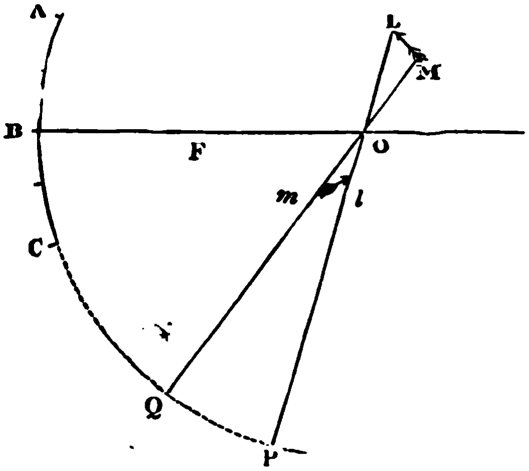

Fig. 54.

In like manner, if the object be placed at *l m*, its image will be formed at L M.

80. All the preceding results may be verified experimentally by means analogous to those already explained. Thus, if the flame of a candle be placed at L M (*fig.* 51.), outside the centre of a concave reflector, and a small semi-transparent screen, such as a piece of ground glass or oiled paper, be held at *l m*, an inverted image of the candle will be seen upon it; and, on the other hand, if the candle be placed at *l m*, and the screen held at L M, the image will be again seen. If any object, such as one's hand, be presented between the principal focus F and a concave reflector, as at L M (*fig.* 52.), a magnified image of the hand will be seen at *l m*.

Amusing optical deceptions are often exhibited with concave reflectors founded on this principle. Thus, a hand presenting a dagger is held between O and F (*fig.* 51.), when immediately a magnified image of the hand and dagger is presented outwards at L M. If a candle be held at L M (*fig.* 54.), opposite the upper edge of a concave reflector, an inverted image of the candle may be exhibited on a screen at *l m*, opposite the lower edge.

81. **Cylindrical and conical reflectors.** — A cylindrical surface is circular in one direction and rectilinear in the other, these directions being at right angles to each other. A sheet of paper, or a plate of metal bent into the form of a circle, will be a cylindrical surface. It may be polished either on the concave or convex side, thus presenting the varieties of a concave or convex cylindrical reflector.

If a cylindrical reflector be placed vertically before an object, its effects upon the vertical dimensions will be the same as those of a plane reflector, and its effects upon the horizontal dimensions the same as those of a spherical reflector. An image, therefore, will be presented, which will be identical in form with the object in all its vertical dimensions, but enlarged, diminished, or reversed in its horizontal dimensions in the same manner as it would be in a spherical reflector.

If a cylindrical reflector be placed with its axis horizontal before a vertical object, it will have the same effect as a plane reflector on the horizontal dimensions, and as a spherical reflector on the vertical dimensions. The horizontal dimensions, therefore, will be preserved in the image, while the vertical dimensions will be enlarged, diminished, or reversed, in the same manner as would be the case with a spherical reflector.

A conical reflector, whether concave or convex, is circular in all sections made at right angles to its axis, and rectilinear in all sections made by planes through its axis. It will, therefore, if placed with its axis vertical, have the effect of an inclined plane reflector on the vertical dimensions of an object, and will have the effect of a spherical reflector on the horizontal dimensions; but each horizontal section will be differently magnified or diminished, according to the position of each section with reference to the axis of the cone, since the circular section of the cone will diminish in approaching the axis, and increase in receding from it. An infinite variety of amusing deceptions are thus produced.

REFLECTION FROM IMPERFECTLY POLISHED SURFACES.

82. If the surface of an opaque body were perfectly polished, and capable of reflecting regularly all the light incident upon it, such surface would itself be invisible. The images of all objects placed before it would appear in the position and with the form and magnitude determined in the preceding paragraphs; and an observer receiving the reflected light would perceive nothing but such images. Thus, a plane reflector of that kind placed vertically against the wall of a room, would appear to the eye merely as an opening leading into another room, precisely similar and similarly

furnished and illuminated; and an observer would only be prevented from attempting to walk through such an opening by encountering his own image as he would approach it.

83. But such a reflector as this has no practical existence, for there is no surface natural or artificial possessing the power of reflecting regularly all the light incident upon it. The absence of complete polish is one of the principal causes of this.

84. The consequence is, that even the most polished surfaces reflect irregularly a certain portion of the light incident upon them; that is to say, the material points, the assemblage of which forms such surfaces, becoming separately illuminated, form so many radiant points, from which pencils of light diverge, and render such surfaces visible exactly in the same manner, though much more faintly than is the case with unpolished surfaces. The quantity of light which is thus irregularly reflected, and which therefore renders the reflecting surface itself more or less visible, diminishes in the same proportion as the perfection of the polish of the surface increases.

The most perfectly polished surfaces, which serve as reflectors, are certain alloys of metal known as speculum metal. These are used generally for the metallic specula of telescopes, microscopes, and other optical instruments.

85. When light falls, therefore, on any imperfectly polished and opaque surface, it is disposed of in three ways: — 1°. A part is regularly reflected, and forms the optical image of the object from which it proceeded. 2°. A part is irregularly reflected, and renders the surface of the reflector perceivable. 3°. A part is absorbed by the surface, and, consequently, not reflected. The smaller the proportion of the light subject to the two last mentioned effects, the more perfect will be the reflector.

The quantity of light regularly reflected by a given surface also varies with the angle of incidence. When the angle of incidence is nothing, and consequently the light falls perpendicularly on such a surface, a less proportion of it is regularly reflected, and a greater proportion irregularly reflected and absorbed, than when the angle of incidence has some magnitude, and, consequently, the light falls more or less obliquely; and in general, as the angle of incidence increases, the quantity of light reflected regularly is augmented, and, consequently, the quantities reflected irregularly and absorbed are diminished.

The following is given by Bouguer as the proportion of the light regularly reflected from different reflecting surfaces, at different angles of incidence: —

86 *Table showing the Proportion of Light incident on reflecting Surfaces which are regularly reflected at different Angles of Incidence.*

Species of reflecting Surface.	Angle of Incidence.	Number of Rays incident.	No. of Rays regularly reflected.	No. of Rays irregularly reflected and absorbed.	Species of reflecting Surface.	Angle of Incidence.	Number of Rays incident.	No. of Rays regularly reflected.	No. of Rays irregularly reflected and absorbed.
Water -	89° 30′	1000	721	279	Black marble, polished -	80° 45′	1000	600	400
	75	1000	211	789		75	1000	156	844
	60	1000	65	935		60	1000	51	949
	30 to 0	1000	18	982		30 to 0	1000	23	979
Glass -	85	1000	543	457	Metallic reflectors	Great angles	1000	700	300
	75	1000	300	700					
	60	1000	112	888		Small angles	1000	600	400
	30 to 0	1000	25	975					

In the preceding table, the light is understood to pass from air to the several media indicated in the first column. The law by which the quantity of light regularly reflected varies according to the density or other physical qualities of the media has not been ascertained.

It is, however, certain that it depends upon the qualities of the medium from which the light passes, as well as those of the medium into which it passes.

87. The angle of incidence has often so much effect upon the quantity of light regularly reflected, that it will sometimes happen that a surface which reflects no light regularly when the angle of incidence is nothing, reflects a considerable quantity when such angle has much magnitude. Thus, a surface of unpolished glass produces no image of an object by reflection when the rays fall on it nearly perpendicularly; but if the flame of a candle be held in such a position that the rays fall upon the surface at a very small angle, a distinct image of it will be seen. Similar phenomena will be observed with surfaces of wood, of common woven stuff, and of paper blackened by smoke.

88. When light is incident upon the surface of a transparent body, such as glass or water, it is disposed of as follows:— 1°. A part is regularly reflected, and produces an optical image of the object from which the light proceeds. 2°. A part is irregularly reflected, and renders the surface visible. 3°. A part is absorbed, and, consequently, neither reflected nor transmitted. 4°. A part is transmitted through the transparent medium.

If light be incident upon the surface of a transparent medium bounded by parallel surfaces, such as a flat plate of glass, all the circumstances above mentioned will take place both at its entrance at the one surface and its escape from the other. Light will be reflected regularly and irregularly at both surfaces; light will be absorbed at both, and light will be transmitted from both. The quantity of light, therefore, transmitted in such a case from the second surface will be less than the quantity of light incident upon the first surface by the sum of all the light regularly and irregularly reflected from the first surface, and all the light regularly and irregularly reflected from the second surface, and all the light absorbed at both surfaces in its transit through the medium.

This will explain a phenomenon which is familiar to every eye. A spectator stationed on the banks of a river or lake, as at S (*fig.* 55.), will see the opposite bank, and objects such as O upon it, reflected in the surface of the water, and will see in the same way distant boats or vessels, such as B, reflected, the images being inverted according to what has been already explained (45.). But he will not see any reflection of a near object, such as A. In the case of distant objects, such as O and B, the rays O R, B R, which proceed from them, striking the surface of the water very obliquely, the part of the light which is reflected in the direction R S is so considerable as to make a very sensible impression on the eye, although it is far from being as strong as a more complete reflection would produce, as is proved by the fact of which every one is conscious, that the images of objects thus reflected in water are far less intense and vivid than images would be reflected from the surface of looking-glass.

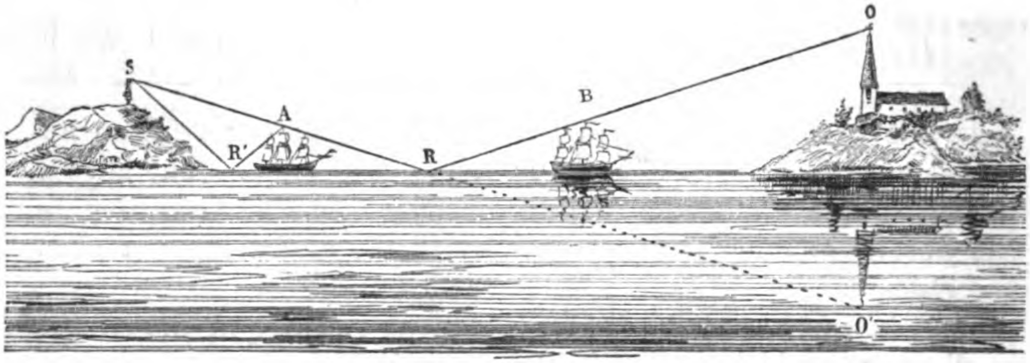

Fig. 55.

As for objects, such as A, placed near the spectator, they are not seen reflected, because the rays A R′, which proceed from them, strike the water with but little obliquity, and, consequently, the part of their light which is reflected in the direction R′ S towards the spectator is not sufficiently considerable to produce a sensible impression on the eye.

For this reason, also, a person on board a vessel may see plainly enough the banks or shores reflected in the water; but if he lean over the bulwark, and look down, he cannot see his own image.

If, however, the observer present himself nearer to the surface of the water, as, for example, at two or three feet distance, the reflection, though still very faint, will in some cases be sensible, so that he will perceive his own image, though faintly. The impression will be stronger if means be taken to exclude from the eye the surrounding light, so that the only rays acting upon the eye shall be those proceeding from the surface of the water.

89. Even when the transparent medium consists of the same substances, these effects take place if the substance composing it varies in density. The successive strata of the atmosphere present an example of this.

In ascending in the atmosphere the succeeding strata of air gradually diminish in density.* The light, therefore, of the sun and other celestial bodies in passing through the atmosphere is transmitted through a succession of strata of increasing density, and is subject consequently to all the effects just explained. Light is gradually absorbed and reflected by the successive strata of air through which it passes, and consequently the direct solar light which arrives at the surface of the earth is less in quantity considerably than the light originally incident upon the superior surface of the atmosphere. A portion, however, of the light irregularly reflected from the successive strata of the atmosphere arrives at the earth from these strata, in the same manner as light is received from the surface of any opaque illuminated body. A part of the light which enters the air is absolutely absorbed by it, and a certain depth might be assigned to the atmosphere, which would completely intercept the solar light. It is calculated that seven feet thickness of water is sufficient to intercept one half of the light transmitted through it.

90. **Blackened glass reflectors.** — A reflecting surface convenient for certain optical purposes is produced by blackening one side of a plate of glass. By this means the light transmitted through the plate is absorbed by the blackened surface on the other side, and light is prevented from being transmitted from the opposite side by the opaque coating; consequently, the only light regularly reflected in this case will be that which is reflected from the superior surface.

91. **Common looking-glass.** — The effects of a common looking-glass are produced by the reflection of the metallic surface attached to the back of the glass, and not by the glass itself. The effect may be explained as follows: — A portion of the light s o, (*fig.* 56.), incident upon the anterior surface is regularly reflected, and another portion irregularly. The former produces an image of the object placed before the glass visible in it; the other renders the surface of the glass itself visible. Another and much greater

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* See "Course of Pneumatics," Chap. II.

portion, o o′, however, of the light incident upon the anterior surface penetrates the plate, and arrives at the posterior surface M′ M′. This surface, coated with an amalgam produced by the combination of tinfoil and quicksilver, has an intense metallic lustre, and possesses therefore strong reflecting power. The chief part of the light, therefore, which passes through the plate of glass is regularly reflected by this metallic surface, and returning to the eye E, produces a strong image of the objects placed before the glass. There are, therefore, strictly speaking, two such images formed: first, a faint one by the light reflected regularly from the anterior surface; and, secondly, a vivid one by the light reflected regularly from the metallic surface. One of these images will be before the other, at a distance equal to the thickness of the glass.

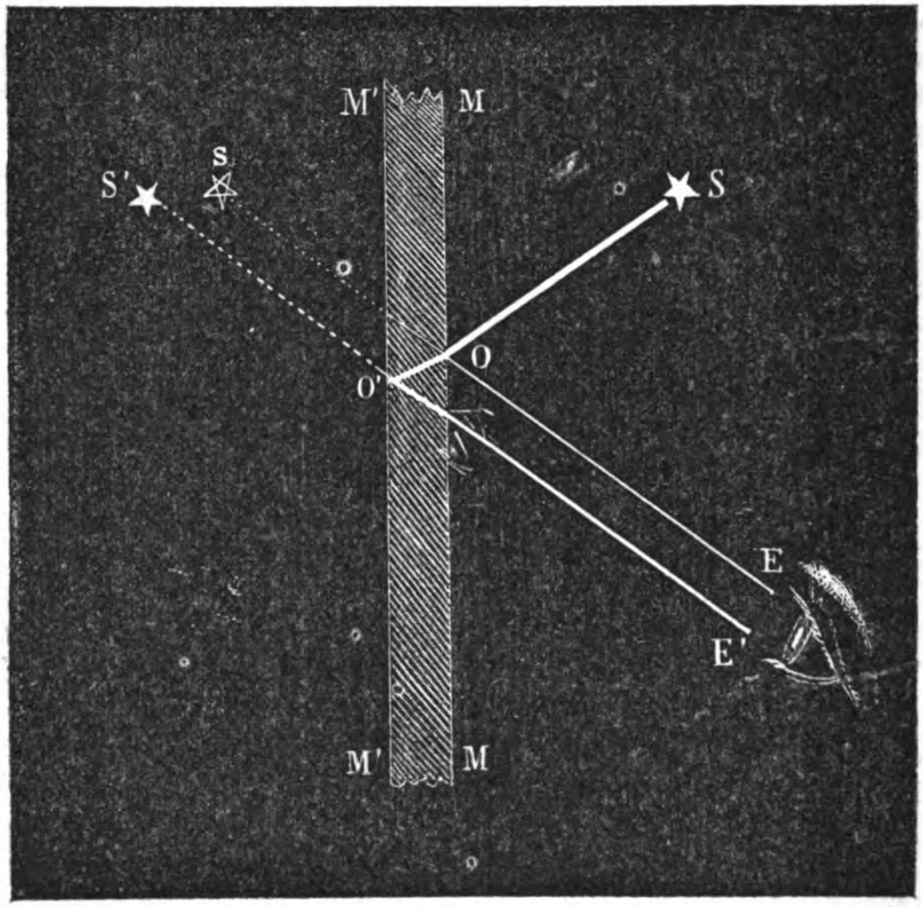

Fig. 56.

In good mirrors which are well silvered, the superior brilliancy of the image produced by the metallic surface will render the faint image produced by the anterior surface of the glass invisible; but in glasses badly silvered, the two images may be easily seen.

CHAP. IV.

REFRACTION AT PLANE SURFACES.

92. When a ray of light, after passing through a transparent medium, enters another of a different density, or possessing other physical properties, it will change its direction at the point which separates the two media, and consequently the direction it follows in the second medium will form a certain angle with that which it has followed in the first medium. The ray is as it were broken at the common surface of the two media, which has caused this phenomenon to be called *refraction*.

That such a deflection really takes place may be rendered visibly evident by the following experiment: — Let a coin or any similar object c, (*fig.* 57.), be placed upon the bottom of a vessel and near its side, and let an observer place his eye at E in such a position that the side of the vessel shall intercept the view of the coin, which would only become visible by removing it to A. Retaining the eye in this position, let the vessel be filled with water; the object c will then be visible, and will be seen as if it were at A; a fact which proves that the ray C B proceeding from the coin is bent into the direction B E, at the point where it emerges from the surface of the water, and the eye accordingly sees the coin in the direction of the line E B A.

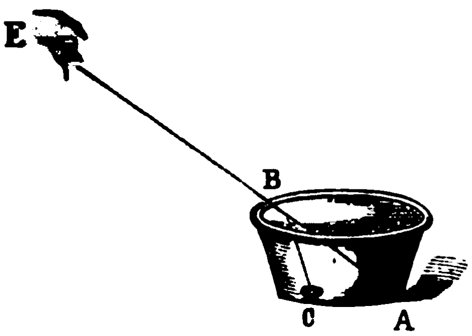

Fig. 57.

Let A B (*fig.* 58.) be the surface which separates the two media. Let I be the point at which a ray E I is incident, and let I R be the course which this ray takes after entering the second medium. Let N I N′ be a perpendicular to the surface A B, drawn through the point of incidence I. A B is called the *refracting surface*, E I N is called the *angle of incidence*, and R I N′, is called the *angle of refraction.*

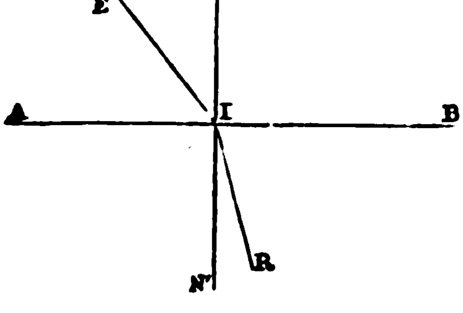

Fig. 58.

93. The following law of refraction has been established by experiment: —

I. The angles of refraction and incidence are in the same plane perpendicular to the refracting surface.

II. The sine of the angle of incidence has to the sine of the angle of refraction always the same ratio for the same medium.

It will appear hereafter that, under certain circumstances, a single ray of light entering a refracting medium will be divided into several, which follow different directions; but for the present we shall limit our observations to such light only as, after refraction, follows a single direction. To such light the above law is strictly applicable.

To explain the preceding law more fully, and to indicate the manner of verifying it by experiment, let A M B be a piece of glass, having the form of a semi-cylinder, as represented in *fig.* 59.

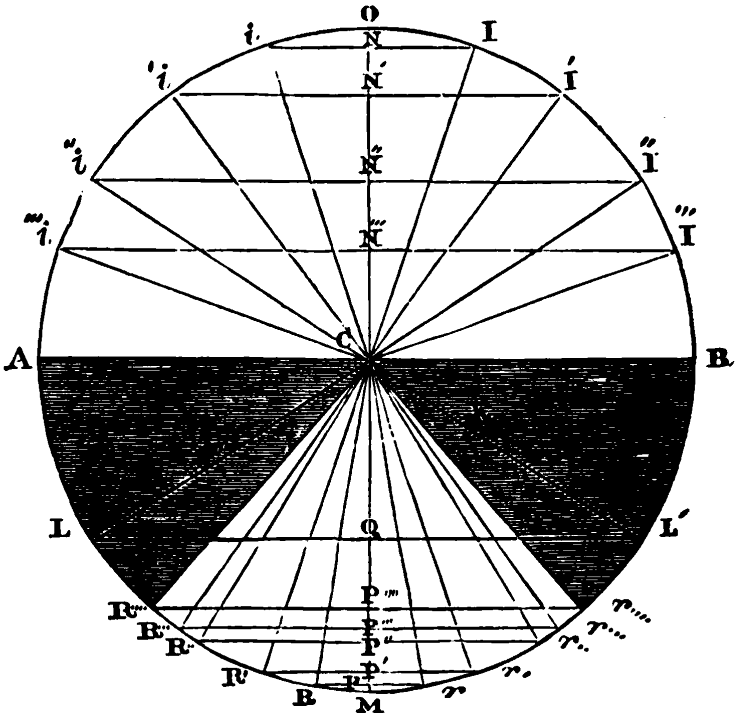

Fig. 59.

Let C be the centre, and A B the diameter of the semi-cylinder. Let the semicircle A O B be imagined to be drawn on a vertical card, so as to complete the circle. Let O C M be the diameter perpendicular to A B, and let the surface A B be covered with an opaque card, with a small hole to admit light at C.

If the flame of a candle, or any other bright object, be held at O, it will be visible to an eye placed at M. It follows, therefore, that a ray of light striking the refracting surface in a direction perpendicular to it, such as O C, will suffer no change of direction after it enters it, but will proceed in the same straight line C M as it would have done if it had passed through no refracting medium. Let the luminous point be now transferred to I, and let the line I N be drawn perpendicular to C O. This line I N is the sine of the angle of incidence I C N. Let the eye be now moved along the arc M A from M towards A, until it see the luminous point I.

Let R be the place at which the luminous point thus becomes visible, C R will then be the direction of the refracted ray. Draw R P perpendicular to C M. This line R P will be the sine of the angle of refraction R C M.

Now if I N and R P be respectively measured, it will be found that R P is exactly two thirds of I N. Therefore, in this case, the sine of the angle of incidence will be to the sine of the angle ot refraction as 3 to 2; that is to say, we shall have $\frac{\mathrm{IN}}{\mathrm{RP}}=\frac{3}{2}$.

Let the luminous point be now moved to I′, and let the eye be moved towards A until it see it. Let R′ be the point at which it becomes visible; C R′ will then be the refracted ray, I′ C being the incident ray.

Draw I′ N′ perpendicular to C O, and R′ P′ perpendicular to C M; I′ N′ will then be the sine of the angle of incidence, and R′ P′ will be the sine of the angle of refraction. If these two lines be respectively measured, it will be found that R′ P′ will be two thirds of I′ N′, so that we shall have, as before, $\frac{\mathrm{I'N'}}{\mathrm{R'P'}}=\frac{3}{2}$.

In the same manner, if the luminous point be moved to any other point, such as I″, and the eye be moved towards A until it see it, the lines I″ C and C R″ will be the incident and refracted rays, I″ N″ and R″ P″ will be sines of the angles of incidence and refraction respectively; and we shall find, as before, by measurement, that $\frac{\mathrm{I''N''}}{\mathrm{R''P''}}=\frac{3}{2}$.

Thus, in general, in whatever manner the position of the luminous point may be viewed, it will always be found that the sine of the angle of incidence will be to the sine of the angle of refraction as 3 to 2; that is to say, in one constant ratio.

In this case, the incident ray is supposed to pass through air, and the refracted ray through glass. If the semi-cylinder A M B, instead of glass, be water, then the ratio of the sine of the angle of incidence to the sine of the angle of refraction will be 4 to 3; so that we shall have

$$\frac{\mathrm{IN}}{\mathrm{RP}}=\frac{4}{3},\quad \frac{\mathrm{I'N'}}{\mathrm{R'P'}}=\frac{4}{3},\quad \frac{\mathrm{I''N''}}{\mathrm{R''P''}}=\frac{4}{3};$$

and so on.

Thus each transparent medium has its own particular refracting power, but for the same transparent medium the ratio of the sines of the angles of incidence and refraction is always the same.

94. **Index of refraction.** — The number which thus expresses the ratio of the sine of the angle of incidence to the sine of the angle of refraction, and which in the case of air and glass is $\frac{3}{2}$ or 1·5, and in the case of air and water is $\frac{4}{3}$ or 1·333, is called the *index of refraction.* From what has been stated, it is evident that each transparent medium will have its own index of refraction, which constitutes one of its most important physical properties.

95. **Case of light passing from denser into rarer medium.** — If the luminous point, instead of being moved along the arc O B, be moved along the arc M A, and the eye be transferred to the arc O B, then the incident ray will pass through the denser medium, and the refracted ray through the rarer medium. In this case it will be found that the direction of the incident and refracted rays, described in the former case, will be interchanged. Thus, if the luminous point be applied at M, it will be visible at O, showing that a ray of light incident perpendicularly on the surface of a rarer medium will suffer no change in its direction. If the luminous point be placed at R, it will be visible at I, showing that if R C be the incident ray, C I will be the refracted ray; and in the same manner, if the luminous point be placed at R′ and R″, it will be visible at I′ and I″.

96. **Directions of incident and refracted rays interchangeable.** — Hence it follows, that if a ray of light passing from one transparent medium into another transparent medium be refracted in a particular direction, a ray of light passing from the latter into the former in the direction in which it was refracted, will, after entering the former, follow the direction in which the former ray was incident; or, in general, it may be stated that the direction of the incident and refracted rays passing between the media are interchangeable.

97. It follows from this that the indices of refraction between the media are reciprocals; that is to say, if the index of refraction from air into glass be $\frac{3}{2}$, the index of refraction from glass into air will be $\frac{2}{3}$; the latter number being what is called in arithmetic the reciprocal of the former. In the same manner, the index of refraction from air into water being $\frac{4}{3}$, the index of refraction from water into air will be $\frac{3}{4}$.

It appears, in the two cases which have been stated of water and glass, that when a ray passes from air into either of these media it will be bent *towards* the perpendicular; and that, on the other hand, when it passes out of either of these media into air, it will be bent *from* the perpendicular. This will be evident by reference to *fig.* 59. The rays I C, I′ C, I″ C, entering water or glass, are bent in the directions C R, C R′, C R″ *towards* the perpendicular C M; and, on the other hand, the rays R C, R′ C, R″ C, passing from glass or water into air, are bent in the directions C I, C I′, C I″ *from* the perpendicular C O.

98. This result, being too hastily generalised, is sometimes announced as follows:—When a ray of light passes from a rarer into a denser medium, it is bent towards the perpendicular, and from a denser into a rarer from the perpendicular, which is by no means generally true. Such a proposition is based upon the supposition that the reflecting power always increases with the density, whereas numerous instances will be produced in which media of greater density will have a less refracting power.

99. **Index of refraction increases with the refracting power.**—The refracting power is estimated by the index of refraction, one medium being said to have a greater or less refracting power, according as its index of refraction is greater or less than that of the other. Thus, glass is said to have a greater refracting power than water, because its index of refraction being 1·50, is greater than the index of refraction of water, which is 1·33.

The propriety of this test of the refracting power will be easily understood. If the index of refraction of one medium be greater than that of another, the angle of refraction which corresponds to a given angle of incidence will be less in the former than in the latter; and, consequently, the same incident ray would be bent more out of its course in the one case than in the other; that is to say, it would be more refracted.

100. Although, however, the refracting power of a transparent medium increases with every increase of its index of refraction, this power does not increase in proportion to such index, but in proportion to a number found by subtracting 1 from the square of the index. Thus, in the case of glass, where the index of refraction is $\frac{3}{2}$, its square is $\frac{9}{4}$, from which 1 being subtracted leaves $\frac{5}{4}$, which represents the refracting power. In the same manner, the index of water being $\frac{4}{3}$, its square is $\frac{16}{9}$, from which 1 being subtracted leaves $\frac{7}{9}$, which represents the refracting power of water; or, in general, if n be the index, n^2-1 will represent the refracting power.

The principle upon which this number n^2-1 is shown to be proportional to the refracting power, does not admit of an explanation sufficiently elementary for this work. We must, therefore, adopt it as a datum without demonstration.

In the following table are given the indices of refraction of those transparent substances which are of most usual occurrence:—

101. *Table of the Indices of Refraction for Light passing from a Vacuum into various Media.*

SOLIDS AND LIQUIDS.

Chromate of lead (maximum)	2·974
,, (minimum)	2·500
Sulphur, native	2·115
Carbonate of lead (maximum)	2·084
,, (minimum)	1·813
Felspar (Spinelli)	1·764
Chryso beril	1·760
Nitrate of lead	1·758
Carbonate of strontian (maximum)	1·700
,, (minimum)	1·543
Boracite	1·701
Aragonite (ordinary* refraction)	1·693
,, (extraordinary* refraction)	1·535
Calcareous spar (ordinary refraction)	1·654
,, (extraordinary refraction)	1·483
Sulphate of barytes	1·647
,, (ordinary refraction)	1·620
,, (extraordinary refraction)	1·635
Colourless topaz	1·610
Topaz of Brazil (extraordinary refraction)	1·640
,, (ordinary refraction)	1·633
Anhydrite (extraordinary refraction)	1·622
,, (ordinary refraction)	1·577
Euclase (extraordinary refraction)	1·663
,, (ordinary refraction)	1·643
Quartz (ordinary refraction)	1·548
,, (extraordinary refraction)	1·558
Sulphate of lime	1·525
Saltpetre (nitrate of potash) (maximum)	1·514
,, (minimum)	1·335
Sulphate of potash	1·509
,,	1·495
Sulphate of ammonia Sulphate of magnesia	1·483
Carbonate of potash	1·482
Spermaceti, melted	1·446
Albumen	1·360
Ether	1·358
Aqueous humour of eye	1·337
Vitreous do.	1·339
External coating of the crystalline	1·377
Middle coating do.	1·379
Central coating do.	1·399
Entire crystalline	1·384

GASES,

Atmospheric air	1·000294
Oxygen	1·000272
Hydrogen	1·000138
Azote	1·000300
Ammonia	1·000385
Carbonic acid	1·000449
Chlorine	1·000772
Hydrochloric acid	1·000449
Nitrous oxide	1·000503
Nitrous gas	1·000303
Carbonic oxide	1·000340
Cyanogen	1·000834
Olefiant gas	1·000678
Carburetted hydrogen	1·000443
Muriatic ether (vapour)	1·001095
Hydrocyanic acid	1·000451
Oxychloro-carbonic gas (Phosgen)	1·001159
Sulphurous acid	1·000665
Sulphuretted hydrogen	1·000644
Sulphuric ether (vapour)	1·001530
Vapour of sulphuret of carbon	1·001500
Protophosphuret of hydrogen	1·000789

102. The indices of refraction given in the preceding table relate to rays of light passing from a vacuum into the several media indicated. If it be required to find the index of refraction for a ray passing from one medium to another, it is only necessary to divide the index of the medium into which the ray is supposed to pass by the index of the medium from which it passes, and the quotient will be the required index. Thus, if it be desired to determine the index of refraction for a ray passing from atmospheric air into any medium indicated in the table, it will be only necessary to divide the index of the medium whose relative index is required by 1·000294, the index of refraction of atmospheric air.

103. It follows from this, that if a ray pass from any medium successively through several transparent media with parallel surfaces, its course in the last of the series will be the same as it would be if it had been incident directly on the surface of the last

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* Ordinary and extraordinary refraction will be explained hereafter.

without having passed through the preceding media. This is easily proved; for let I be the angle of incidence upon the surface of the first medium, and R the angle of refraction. This angle R will be the angle of incidence on the second medium, in which the angle of refraction is R′. This angle of refraction R′ will be the angle of incidence on the surface of the third medium, in which the angle of refraction is R″.

If n be the index of refraction of the original medium through which the ray passes, and n', n'', and n''' be the indices of refraction of the three successive media by which it is refracted, then the index of refraction from the first medium into the second will be $\frac{n'}{n}$, and consequently we shall have

$$\frac{\sin. \mathrm{I}}{\sin. \mathrm{R}}=\frac{n'}{n};$$

and in like manner we shall have

$$\frac{\sin. \mathrm{R}}{\sin. \mathrm{R}'}=\frac{n''}{n'}, \qquad \frac{\sin. \mathrm{R}'}{\sin. \mathrm{R}''}=\frac{n'''}{n''}.$$

By multiplying all these together, we shall have

$$\frac{\sin. \mathrm{I}}{\sin. \mathrm{R}''}=\frac{n'''}{n};$$

which is the index of refraction from the original medium through which the ray passed to the last medium by which it has been refracted. The angle of refraction, therefore, R″, in this latter medium, would be the same if the original ray had been directly incident upon it with the same angle of incidence.

104. It follows from this, that if a ray of light, after passing through several successive media separated by parallel surfaces, pass finally into the medium from which it was originally incident, it will issue in a direction parallel to the original ray. Thus, in the preceding example, if the original ray of light A B, after passing successively through the three media, issue again into the medium through which it originally passed, its direction C D will be parallel to its original direction A B; for, according to what has been already proved, its course, after passing through the three media and into the fourth, will be the same as if it passed directly from the first medium into the fourth; but in this case, the first medium being the same as the fourth, the ray would not be deflected from its course. It must, therefore, after passing through the parallel media, preserve its original direction.

105. It is for this reason that plates of glass with parallel surfaces, such as window glass, produce no distortion in the objects seen through them; the rays from such objects, after passing through the glass, preserve their original direction.

106. The law of refraction which has been just explained and illustrated is attended with some remarkable consequences in the transmission of light through media of different refracting powers.

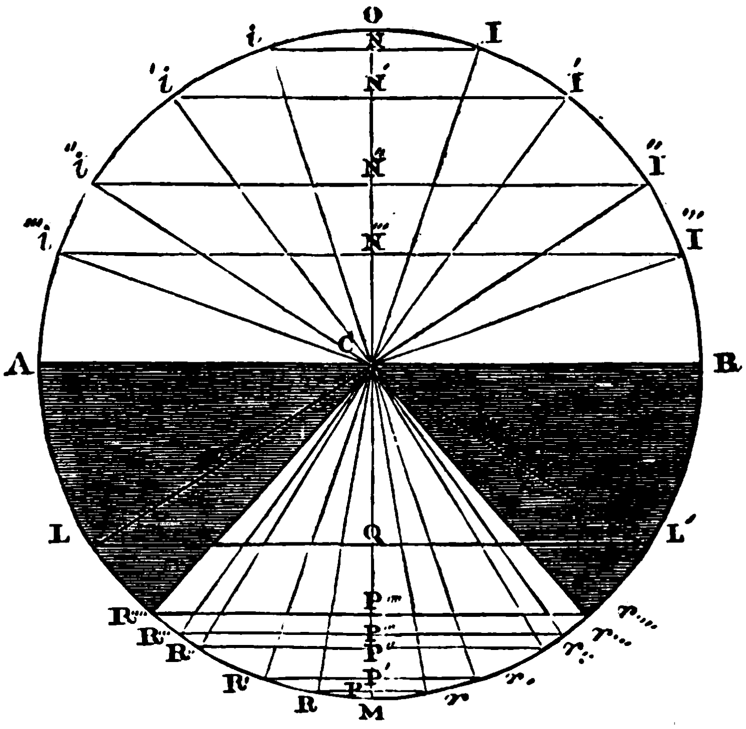

Fig. 60.

Let A B (*fig.* 60.) represent, as before, the surface which separates a medium of air A O B from a medium of glass A M B. According to what has been already explained, any incident ray, such as I C, will be deflected towards the perpendicular C M, so that its angle of refraction shall have a sine equal to two thirds of that of its angle of incidence. Now, let us suppose the angle of incidence gradually to increase, so as to approach to a right angle. It is evident that the sine of the angle of incidence I N will also gradually increase until it approach to equality with the radius C B. This will be evident on inspecting the diagram, in which I′ N′, I″ N″, I‴ N‴, &c., are the sines of the successive angles of incidence; and if we suppose the direction of the incident ray to approximate as closely as possible to that of the line B C, the sine of the angle of incidence will approach as close as possible to the magnitude of B C.

Now, let us consider what corresponding change the angles of refraction will suffer. Their sines will be respectively, in the case of glass here supposed, two thirds of the sine of the angle of incidence; thus the sine R P of the angle of refraction corresponding to I C will be two thirds of I N; the sine R′ P′ of the angle of refraction corresponding to I′ C will be two thirds of I′ N′; the sine R″ P″ of the angle of refraction corresponding to I″ C will be two thirds of I″ N″; and so on. When the incident ray approaches to coincidence with B C, the sine of the angle of incidence will approach to equality with B C, and consequently the sine of the angle of refraction will be equal to two thirds of B C. If, therefore, it were possible that a ray passing directly from B to C could enter the glass at C, such ray would have an angle of refraction whose sign would be two thirds of the radius B C. Now, if we draw C R'''' to such a point that the sine of the angle of refraction R'''' P'''' shall be two thirds of the radius B C, it is evident that all the incident rays whose directions lie between O C and B C will be refracted in directions lying between C R'''' and C M. In like manner it may be shown that all incident rays whose directions lie between O C and A C will be also included after refraction between the lines C M and C r'''', corresponding in position to C R''''. Thus it appears that rays of light converging from all directions to the point C, will be after refraction included within a cone whose angle is R'''' C r''''.

Hence follows the remarkable consequence, that light entering the glass at C, from whatever direction it may proceed, will be totally excluded from the space A C R'''' and B C r'''', all such light being included, as has been observed, within the cone whose angle is R'''' C r''''.

107. This may be verified experimentally in the following manner. Let an opaque covering be placed on the surface A B, a small circular aperture being left uncovered at C. Let a light be moved round the semicircle B O A. This light will enter the aperture C, and will successively illuminate the points of the arc R'''' M r''''. Commencing from B, it will produce an illuminated spot near R''''; as it is moved successively from B to O, it will illuminate the points successively from R'''' to M; and as it is moved successively from O to A, it will illuminate successively the points from M to r''''.

In the same manner it will be found that if the luminous point be placed at R'''', its light, after passing from the point C, will fall near B, taking the direction C B. If the light be moved successively over the parts of the arc R'''' M, it will successively illuminate the points of the arc from B to O; and being moved in like manner from M to r'''', it will successively illuminate the points of the arc from O to A.

108. Now a question arises as to what will happen if the light be placed between R'''' and A; for since, being at R'''', the sine of the angle of incidence R'''' P'''' is two thirds of C B, this sine will be more than two thirds of C B if the luminous point be placed between R'''' and A; and consequently it would follow, by the law of refraction, that the sine of the corresponding angle of refraction must be greater than the radius B C.

But since no angle can have a sine greater than the radius, it would follow that there can be no angle of refraction, and consequently that there can be no refraction, for a ray which shall make with the refracting surface at C a greater angle of incidence than R'''' C M. What then, it will be asked, becomes of such a ray as, for example, the ray L C, making an angle of incidence L C M, whose sine L Q is greater than two thirds of the radius C B?

109. The answer is, that such a ray being incapable of refraction at C will be reflected, and that such reflection will follow the common law of regular reflection, so that the ray L C will be reflected in the direction C L′, making the angle of reflection L′ C M equal to the angle of incidence L C M. Thus it follows that all rays which meet the point C, in any direction included between R'''' C and A C, will be reflected from C in corresponding directions between *r''''* C and B C, according to the common laws of reflection. This may be verified by observation; for if the flame of a candle be moved from R'''' to A, it will be seen in corresponding positions by an eye moved in the same way from *r''''* to B, and will be seen with a splendour of reflection far exceeding that produced by any artificially polished surface.

110. Hence it is that this species of reflection has been called *total reflection*. The angle R'''' C M, which limits the direction of the rays capable of being transmitted from C into the superior medium, and of being reflected, is called the *limit of possible transmission.* The rays C R'''' and C *r''''* separate the rays which are capable of refraction at C, from those which are reflected at C.

As in the case of glass, the limit of possible transmission is one whose sine is two thirds of the radius; so in the case of water, it would be three fourths of the radius, and, in general, it would be an angle whose sine is the reciprocal of the index of refraction.

It follows, therefore, that the limit of possible transmission diminishes as the refracting power of the medium increases.

Since the angle whose sine is ¾ is 48° 35′, and the angle whose sine is ⅔ is 41° 49′, it follows that these are the limits of possible transmission for water or glass into air.

111. *Table showing the Limits of possible Transmission, corresponding to the different Transparent Bodies expressed in the First Column.*

Names of Media.	Index of Refraction.	Limit of Transmission.		Names of Media.	Index of Refraction.	Limit of Transmission.	
		°	′			°	′
Chromate of lead	2·926	19	59	Ruby - - -	1·779	34	12
Diamond - -	2·470	23	53	Topaz - -	1·610	38	24
Sulphur - -	2·040	29	21	Flint glass - -	1·600	38	41
Zircon - -	2·015	29	45	Crown glass -	1·533	40	43
Garnet - -	1·815	33	26	Quartz - -	1·548	40	13
Felspar - -	1·812	33	30	Alum - - -	1·457	43	21
Sapphire - -	1·768	34	27	Water - -	1·336	48	28

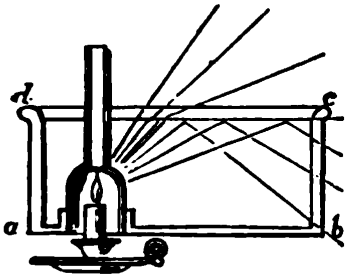

Fig. 61.

The properties here described may be illustrated experimentally by the apparatus represented in *fig.* 61.; let *a b c d* represent a glass vessel filled with water, or any other transparent liquid. In the bottom is inserted a glass receiver, open at the bottom, and having a tube such as a lamp-chimney carried upwards and continued above the surface of the liquid. If the flame of a lamp or candle be placed in this receiver, as represented in the figure, rays from it, penetrating the liquid, and proceeding towards the surface *d c*, will strike this surface with various obliquities. Rays which strike it under angles of incidence within the limits of transmission will issue into the air above the surface of the liquid, while those which strike it at greater angles of incidence will be reflected, and will penetrate the sides of the glass vessel *b c*.

An eye placed outside *b c* will see the candle reflected on that part of the surface *d c* upon which the rays fall at angles of incidence exceeding the limit of transmission; and an eye placed above the surface will see the flame, in the direction of the refracted rays, striking the surface with obliquities within the limit of transmission.

112. If a pencil of parallel rays be incident upon a plane surface S S′, *fig.* 62., which separates two refracting media M and M′, the rays of the pencil, provided they enter the medium M′ at all, will continue to be parallel.

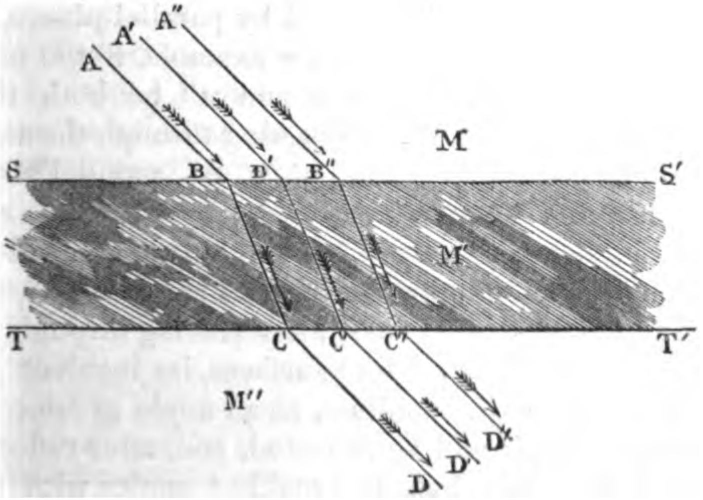

Fig. 62.

Whether the rays of the pencil enter the medium M′, will be determined by the relative refracting powers of the two media

M and M′, and the magnitude of the angle of incidence of the pencil upon the surface S S′.

If the medium M′ be more refracting than the medium M, then the pencil will enter the medium M′, whatever be the angle of incidence; but if the medium M′ be less refracting than the medium M, then the pencil will enter the medium M′ only when the angle of incidence is less than the limit of transmission. If it be greater than that limit, it will be reflected from the surface S S′, according to the common laws of reflection.

If a pencil of parallel rays be incident successively upon parallel plane surfaces separating different media, its rays will, if transmitted at all through them, preserve their parallelism; for, from what has been already proved, the pencil, if parallel in the medium M, will be parallel in the medium M′; and being parallel in the medium M′, it will for the same reason be parallel in the medium M″; and the same will be true for every successive medium through which the pencil passes, provided the surface separating the media be parallel.

But whether the pencil be transmitted at all through the successive media will depend, as before, upon the relative refracting powers of the media and the angles of incidence. If, for example, at any surface, such as T T′, the medium M″ have less refracting power than the medium M′, the pencil will only enter it provided the angle at which the rays strike the surface T T′ be less than the limit of transmission, otherwise the rays will be reflected.

If a refracting medium M′, bounded by parallel planes, have the same medium at each side of it, as, for example, if the medium M′ be a plate of glass, and the media M and M″ be both the atmosphere, the pencil of rays A B, after passing through the medium M′, will emerge in the direction C D, C′ D′, C″ D″, parallel to the original direction A B, A′ B′, A″ B″, &c.

This has been already proved for a single ray, and will therefore be equally true for any number of parallel rays.

113. If a pencil of parallel rays, after passing through a succession of media bounded by parallel surfaces, be incident upon the surface of a less refracting medium, at an angle greater than the limit of transmission, it will be reflected, and, after reflection, will return through the several media, making angles with the other surfaces equal to those which it made in passing through them, but on the other side of the perpendicular.

For example, let A B, *fig.* 63., be a ray of the incident pencil, and let it be successively refracted by the media M, M′, M″ in the directions B C, C D, and D E; and let it be supposed that, the medium M‴ having a less refracting power than the medium M″, the ray D E is incident upon its surface at an angle greater than the angle of transmission.

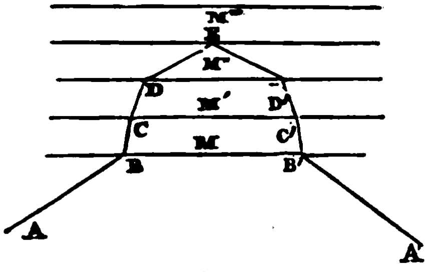

Fig. 63.

This ray will consequently be reflected in the direction E D′, making an angle with the surface at E equal to that which D E makes with it. The rays E D′ and E D, being equally inclined to the surface separating the media M″ and M′, the ray E D′ will be refracted by the medium M′ in the direction D′ C′, inclined at the same angle as D C to the surface D D′, but on the other side of the perpendicular; and in the same way, in passing through the medium M, it will take a direction C′ B′ inclined to C C′ at the same angle as the ray C B is inclined to it. In fine, it will issue from the medium M in the direction B′ A′, inclined to the surface B B′, at the same angle as the incident ray A B is inclined to such surface.

If an eye were placed, therefore, at A, it would see the object from which the ray A B proceeds in the direction A′ B′, the phenomenon being in all respects similar to that of common reflection.

114. **Mirage, Fata Morgana, &c. explained.** — These principles serve to explain several atmospheric phenomena, such as Mirage, the Fata Morgana, &c.

In climates subject to sudden and extreme vicissitudes of temperature, the strata of air are often affected in an irregular manner as to their density, and consequently as to their refracting power. If it happen that rays proceeding from a distant object directed upwards after passing through a denser be incident upon the surface of a rarer stratum of air, and that the angle of incidence in this case exceeds the limit of transmission, the ray will be reflected downwards; and if it be received by the eye of an observer, an inverted image of the object will be seen at an elevation much greater than that of the object itself.

To explain this, let S, *fig.* 64., be an object, which if viewed from E would be seen in the direction E S.

Let M and M′ be two atmospheric strata, of which M′ is much more rare than M, and let the ray S M be incident upon the surface separating these strata at an angle greater than the angle of transmission. Such ray will in this case be reflected in the direction M E, making with the surface an angle equal to that which S M makes with it. The eye, therefore, will see an image of S, exactly as it would if the surface separating M and M′ were a mirror, and consequently the image S′ of the object S will be inverted. If no opaque obstacle lie in the line E S, the object S and the inverted image will be seen at the same time; but if any object be interposed between the eye and s, such as a building, or elevated

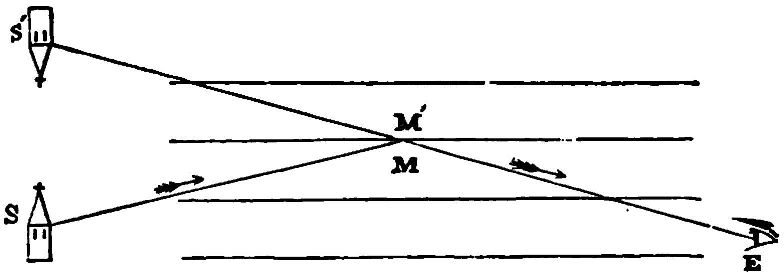

Fig. 64.

ground, or the curvature of the earth, then the object s will be invisible, while its inverted image s′ will be seen.

It sometimes happens that the reflection takes place from a lower stratum of air towards the eye in an upper stratum, and in such case the inverted image is seen below the object, as in *fig.* 65., which shows a frequent effect of mirage.

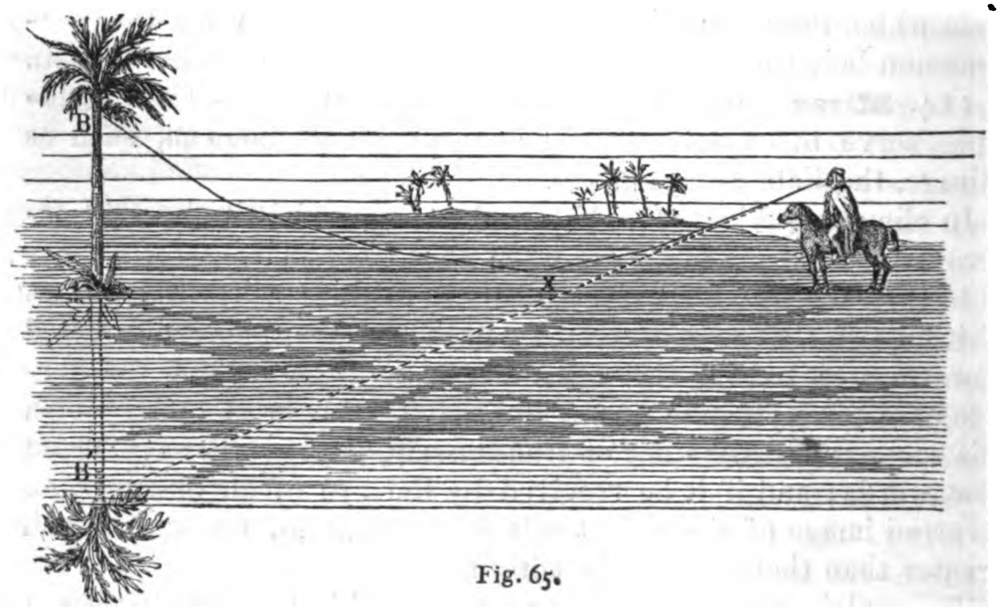

Fig. 65.

115. Various fantastic optical effects of this kind are recorded as having been observed during the campaign of the French army in Egypt. On this occasion, a corps of savans accompanied the army, in consequence of which, the particulars of the phenomena were accurately observed and explained.

When the surface of the sands was heated by the sun, the land seemed terminated at a certain point by a general inundation. Villages standing at elevated points seemed like islands in the middle of a lake, and under each village appeared an inverted image of it. As the spectator approached the boundary of the apparent inundation, the waters seemed to retire, and the same illusion appeared round the next village.

116. If a pencil of parallel rays be transmitted successively through several transparent media bounded by plane surfaces which are not parallel, its rays will preserve their parallelism throughout its entire course, whether they strike the successive surfaces at an angle within the limit of transmission or not.

If they strike them at angles within the limit of transmission, they will pass successively through the media, and the preservation of their mutual parallelism may be established by the same reasoning as was applied to parallel surfaces; for the angles of incidence of the parallel rays upon the surface of the first medium being equal, the angles of refraction will also be equal, and therefore the rays through the first medium will be parallel. They will therefore be incident at equal angles on the surfaces of the two media, and the angle of refraction through the strata within the limits of transmission will be also equal, and therefore the rays in passing through the second medium will be parallel; and the same will be true of every successive medium through which the rays would be transmitted. But if they strike upon the surface of any medium at an angle beyond the limit of transmission, they will be reflected, and being reflected at the same angle at which they are incident, the reflected rays must be parallel. In returning successively through the media they will be subject to the like observation, and will therefore preserve their parallelism whether they be refracted or reflected.

In these observations, it is assumed that all the rays composing the parallel pencil are equally refrangible by the same refracting medium, and to such only the above inferences are applicable. It will, however, appear hereafter that certain pencils may be composed of rays which are differently refrangible, a case not contemplated here.

117. The deflection of a pencil from its original course by its successive transmission through refracting surfaces which are not parallel, is attended with consequences of great importance in the theory of light, and it will therefore be necessary here to explain these effects with some detail.

If two plane surfaces be not parallel, they may be considered as forming two sides of a prism, which is a solid, having six sides, three of which are rectangular, and the two ends triangular. Such a solid is represented in *fig.* 66., A B C and A′ B′ C′ are the triangular ends, which are at right angles to the length of the prism, and therefore parallel to each other. The three rectangular sides are A B B′ A′, B C C′ B′, and A C C′ A′.

Such a prism is shown in perspective in *fig.* 67.

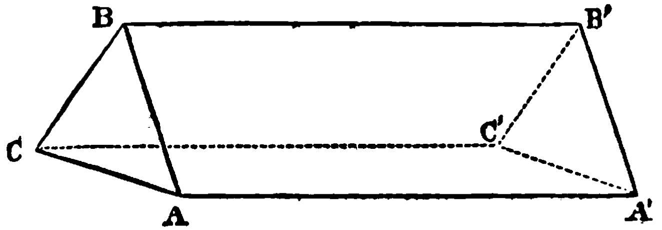

Fig. 66.

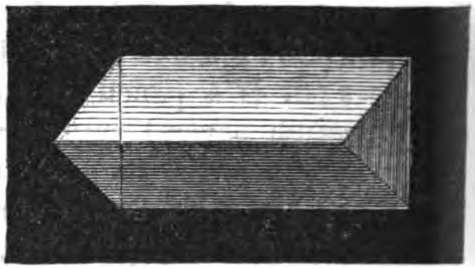

Fig. 67.

118. The refracting angle of the prism is that angle through the sides of which the refracted light passes. Thus, if the light enter at any point of the side A B B′ A′, and emerge from a point of the side B C C′ B′, then the angle of the prism whose edge is B B′ is called the *refracting angle*, and the opposite side A C C′ A′ is called the base of the prism.

Prisms are distinguished according to the properties of the triangles which form their base. Thus, if the triangle A B C be equilateral, the prism is said to be equilateral; if it be right-angled, the prism is said to be rectangular; if the sides A B and B C of the refracting angle be equal, the prism is said to be isosceles; and so forth.

119. It is usual to mount such prisms for optical purposes on a pillar, as represented in *fig.* 68., having a sliding tube *t* with a tightening screw, by which the elevation may be regulated at pleasure, and a knee-joint at *g*, by which any desired inclination may be given to the prism.

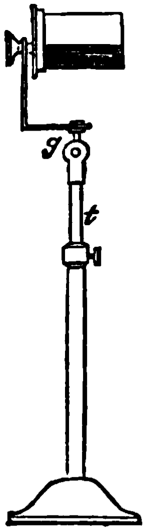

Fig. 68.

By the combination of these arrangements, the apparatus may always be adjusted, so that a pencil may be received in any desired direction with reference to its refracting angle.

If the transparent medium composing the prism be a solid, the prism may be formed by cutting and polishing the solid in the form required; if it be a liquid, the prism may be formed of glass plates hollow, so as to be filled by the liquid.

120. Let a pencil of parallel rays be supposed to be incident at O (*fig.* 69.), upon one side A B of the refracting angle A B C of a prism. Let it be required to determine under what conditions such a pencil entering the prism and traversing it will be transmitted through the other side B C.

We shall here assume that the refracting power of the prism is greater than that of the surrounding medium. This being the case, the pencil incident upon the surface A B will enter the prism, whatever be its angle of incidence. From O draw O M perpen-

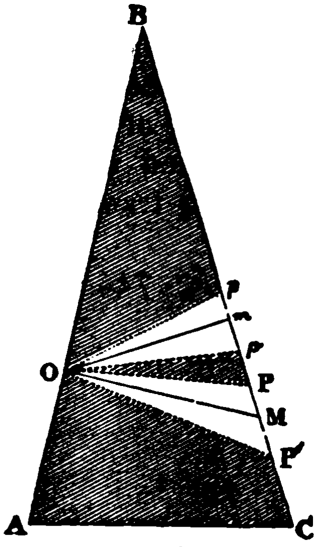

Fig. 69.

dicular to A B, and O *m* perpendicular to B C, draw O P and O P′, making with O M the angles P O M and P′ O M, each equal to the limit of transmission; and also draw the lines O *p* and O *p′*, making angles with O *m* also equal respectively to the limit of transmission. It is evident, from what has been already explained, that in whatever direction the incident ray would fall at O, it will, when refracted, fall within the angle P O P′. It follows also, from what has been explained, that no ray proceeding from O and incident upon the surface B C can be transmitted through it unless it fall between *p* and *p′*, that is, within the angle *p* O *p′*.

It is evident, then, that if these two angles P O P′ and *p* O *p′* lie altogether outside each other, as represented in *fig.* 69., no ray incident at O could pass through the surface B C; and that, consequently, every such ray must be reflected by such surface. In order that any of the rays transmitted through the prism, and therefore falling within the angle P O P′, should be transmitted, it would be necessary that the angle *p* O *p′*, or some part of it, should fall upon or within the angle P O P′.

To determine the conditions which would ensure such a result, we are to consider that the lines O M and O *m*, which are perpendicular respectively to the sides of the refracting angle, must form with each other the same angle, that is, the angle *m* O M must be equal to the refracting angle B.

This angle *m* O M is, as represented in *fig.* 69., equal to the sum of the three angles M O P, *m* O *p′*, and *p′* O P. Therefore, the angle *p′* O P will be equal to the angle *m* O M, diminished by twice the limit of transmission, because the two angles *m* O *p′* and M O P are respectively equal to the limit of transmission.

It follows, therefore, that the angle *p′* O P, which separates the rays transmitted through the prism from the direction of these rays which it would be possible to transmit through the surface B C, is equal to the difference between the refracting angle B, and twice the limit of transmission. If, therefore, the refracting angle of the prism be greater than twice the limit of transmission, the rays which enter the prism cannot be transmitted through the two surfaces of the refracting angle, but will be reflected by it. If the angle M O *m* be equal to twice the limit of transmission, then the commencement O P of the rays which pass through the prism will coincide with the co[illegible]nt O *p′* of those rays which it would be possible to transmit through the surface B C. This case is represented in *fig.* 70. In this case, none of the rays which pass through the prism can be transmitted through the surface B C, and the line O P is the limit which separates the two cones of rays, one consisting of the rays which traverse the prism, and the other including those directions which would render the transmission possible.

If, in fine, the angle *m* O M, as represented in *fig.* 71., be less than twice the limit of transmission, then a portion of the cone *p* O *p′* will lie within the cone P O P′, and all the refracted rays which are included between O P and O *p′* will fulfil the condition of transmission, and will consequently pass through the surface B C; but all the others which strike the surface B C between P′ and *p′*, and between P and *p*, will be reflected. The rays, therefore, incident at the point O, which are capable of being transmitted through the two surfaces B A and B C of the prism, will be those whose angles of refraction are greater than *p′* O M, and less than P O M.

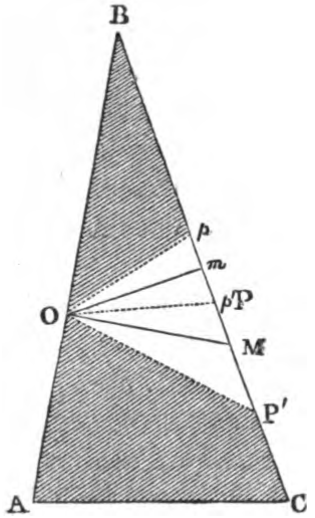

Fig. 70.

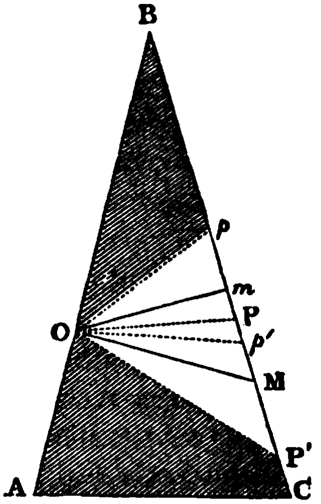

Fig. 71.

But if L express the limit of transmission, and B the refracting angle of the prism, we shall have

$$p'\,\mathrm{OM}=\mathrm{L}-p'\,\mathrm{OP}=\mathrm{B}-\mathrm{L}.$$

The condition, therefore, of transmission at the two surfaces is that the refracting angle of the prism shall be less than twice the limit of transmission, and the rays which in this case are capable of transmission are those whose angles of refraction at the first surface are greater than the difference between the refracting angle of the prism and the limit of transmission.

To explain the course of a ray which, passing through the prism,

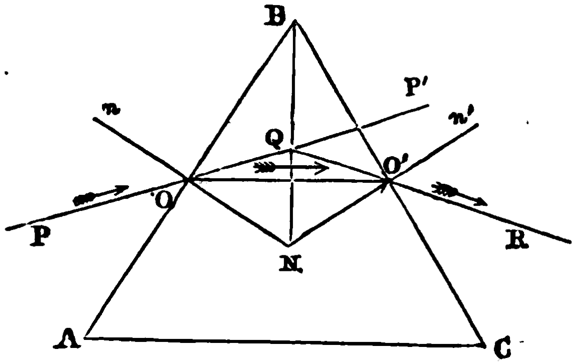

Fig. 72.

fulfils these conditions of transmission, let A B C (*fig.* 72.) be the refracting angle, and P O the incident ray.

The prism being supposed to be more dense than the surrounding medium, or to have a greater refracting power, the ray P O, in passing through it, will be bent towards the perpendicular O N, so that the angle of refraction O′ O N will be less than the angle of incidence at O. Thus the ray will be bent out of its course through the angle Q O O′, which is the first deviation of the ray from its original direction. The refracted ray O O′, being incident on the second surface at O′ at the angle O O′ N, will pass through this surface, and will emerge in the direction O′ R deflected from the perpendicular.

Since O Q is the direction of the original incident ray P O, and Q R the direction of the emergent ray O′ R, it follows that the total deflection of the ray from the original direction, produced by the two refractions, is the angle P′ Q R.

121. If the angle of incidence of the original ray P O be such that the refracted ray O O′ shall make equal angles with the sides of the prism, that is to say, so that the angles B O O′ and B O′ O shall be equal, then the deflection of the emergent ray O′ R from its original direction will be less than it would be for any other angle of incidence of the original ray P O.

In this case it is easy to see that the angles which the incident and emergent rays P O and O′ R make with the sides of the prism, and with the refracted ray O O′, are equal; for since the angles B O O′ and B O′ O are equal, the angles N O O′ and N O′ O are also equal.

But

$$\frac{\sin. \mathrm{PO}n}{\sin. \mathrm{NOO'}} = \text{index of refraction},$$

$$\frac{\sin. \mathrm{RO'}n'}{\sin. \mathrm{NO'O}} = \text{index of refraction}.$$

And since the angles N O O′ and N O′ O are equal, it follows that the angles P O *n* and R O′ *n*′ are also equal. Therefore the incident and emergent rays make equal angles with the perpendiculars to the two surfaces, and therefore with the two surfaces themselves.

It is easy to show experimentally that in this case the deflection of the direction of the emergent from that of the incident ray is a minimum, for the direction of these rays can be determined by observation, and the deviation directly measured. By turning the prism P (*fig.* 73.) on its axis, so as to vary the angle which the first surface makes with the incident ray by increasing or diminishing it, it will be found that the deflection of the direction of the emergent from that of the incident ray will be augmented in whatever way the prism may be turned from that position in which the incident and emergent rays are equally inclined to the sides of the prism.

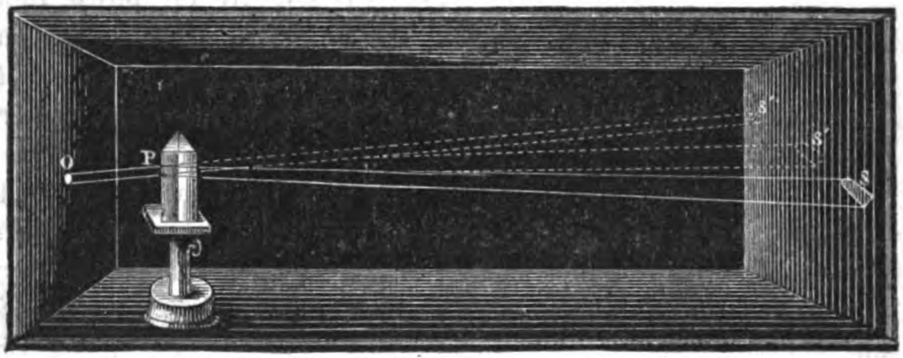

Fig. 73.

122. Means are thus obtained, by observing the minimum deviation produced upon a ray transmitted through a prism, of determining, by a simple observation, the index of refraction; for the angle of refraction N O O′, being equal to the angle N B O, is one half the refracting angle of the prism, and the angle of incidence P O *n* is equal to the angle of refraction N O O′, or one half the angle of the prism, together with the angle O′ O Q, or one half the deflection O′ Q P′. Thus, if I be the angle of incidence, and R the angle of refraction at the first surface O, and if B be the refracting angle of the prism, and D the deflection, we shall have

$$\text{I}=\tfrac{1}{2}\,\text{D}+\tfrac{1}{2}\,\text{B},\quad \text{R}=\tfrac{1}{2}\,\text{B}.$$

Therefore we shall have

$$\frac{\sin.\ \frac{1}{2}\,(\text{D}+\text{B})}{\sin.\ \frac{1}{2}\,\text{B}}=\text{index of refraction.}$$

By knowing, therefore, the angle of the prism, and by measuring the angle of minimum deflection, the index of refraction of the material composing the prism can be found.

If the ray transmitted through the prism do not fulfil the conditions of transmission at the second surface, it will be reflected, and will therefore return to the first surface, and pass through it into the medium from which it came, or will return to the base, and be transmitted through it, or reflected by it, according as the angle at which it strikes it is within the limit of transmission or not.

In the case represented in *fig.* 74., the incident ray P O, striking upon the surface B C at O′, is reflected from it and passes to the base at O″, through which it is transmitted.

123. A rectangular isosceles prism of glass is often used for an oblique reflector. Such a prism is represented in *fig.* 75. The sides A B and A C being equal, the angles A B C and A C B must be each 45°. If a parallel pencil of rays, of which P O is one, be incident upon B A perpendicularly, it will enter the medium of the prism without refraction, and will proceed to the surface B C, on which it will be incident at O′ at an angle of 45°. Now, the limit of transmission of glass being but 40°, such a ray must suffer total reflection, and will accordingly be reflected from B C at an angle of 45°, that is, in the direction of O′ R, at right angles to the original direction P O′.

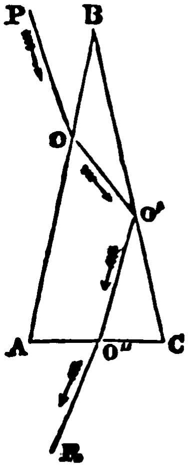

Fig. 74.

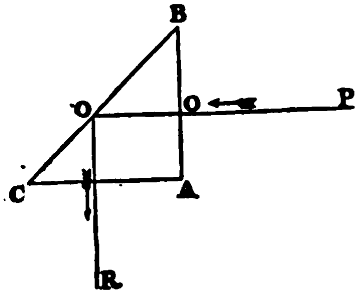

Fig. 75.

An object, therefore, placed at R would be seen by an eye placed at P in the direction P O′, and an object placed at P would be seen by an eye placed at R in the direction R O′.

An object seen by reflection with such a prism would, as will appear hereafter, be reversed from its natural position. This circumstance is obviated by using such a four-sided prism as is shown in *fig.* 76. The ray A B proceeding from the object,

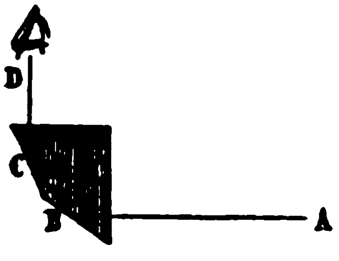

Fig. 76.

enters the prism perpendicularly, and after being reflected twice successively at B and C, emerges perpendicularly in the direction C D.

124. Let I, *figs.* 77, 78., be the focus from which a pencil of diverging rays proceeds, and is incident upon the refracting surface A B C, separating the media M and M.

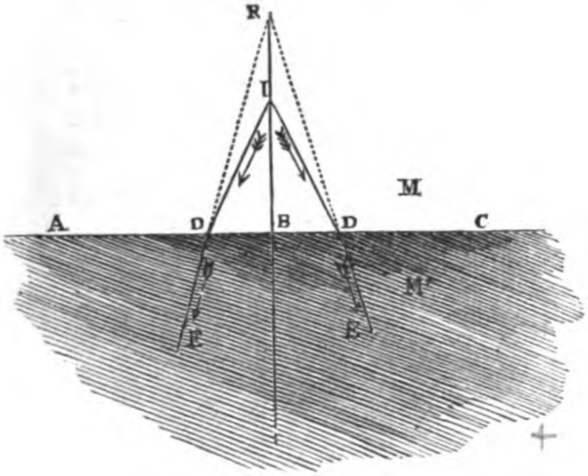

Fig. 77.

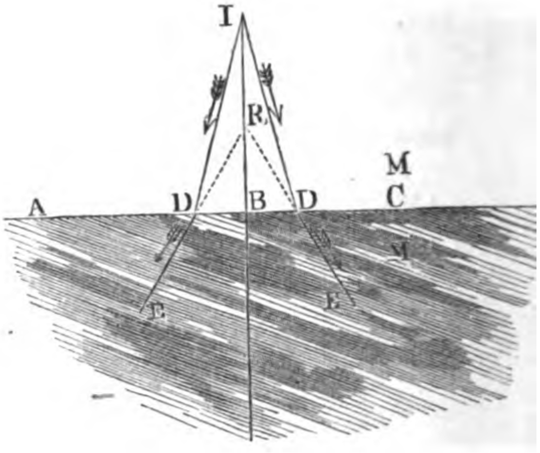

Fig. 78.

Let I B be that ray of the pencil which, being perpendicular to the surface, is its axis, and will therefore pass into the medium M' without having its direction changed. Let I D be two other rays equidistant from B, falling obliquely on the surface so near the point B as to bring them within the scope of the principle explained in (71.). Let D E be the directions of the refracted rays which being continued backwards meet the line B I at R. *Fig.* 77. represents the case in which the medium M' is more dense than M, and in which, therefore, the refracted rays are deflected towards the perpendicular. *Fig.* 78. represents the case in which the medium M' is less dense than M, and where, therefore, the refracted rays are deflected from the perpendicular. In the former case, the point R falls above I, in the latter below it. The point R will then be the focus at which the rays I B and D E, or their continuations, meet.

This will therefore be the focus of the refracted rays. The angle D I B which the incident ray makes with the perpendicular I B, is equal to the angle of incidence; and the angle D R B, which the direction of the refracted ray makes with the perpendicular, is the angle of refraction.

Let the distance I B of the focus of incident rays from the surface be expressed by f and R B, that of the focus of refracted rays from the surface by f'. Since the angles which R D and I D make with I B are so small as to come within the scope of the principle expressed in (71.), we shall have

$$\mathrm{I} = \frac{\mathrm{D\,B}}{f}, \qquad \mathrm{R} = \frac{\mathrm{D\,B}}{f'};$$

and consequently,

$$\frac{\text{I}}{\text{R}}=\frac{f'}{f}.$$

But since the angles I and R are small, their sines, by the principle explained in (71.), may be taken to be equal to the angles themselves; and, consequently, we shall have, by the common law of refraction, $\frac{\text{I}}{\text{R}}$ equal to the index of refraction n. Thus we shall have

$$\frac{f'}{f}=n,\quad f'=n\times f\ .\ .\ .\ \text{(c)}.$$

In this case, n is the index of refraction of the rays proceeding from the medium M to the medium M′, and is consequently greater than 1 when M′ is more dense than M, and less than 1 when M′ is less dense than M.

The formula (c) is equivalent to a statement that the distances of the foci of refraction and incidence from the refracting surface are in the proportion of the index of refraction to 1; that is to say,

$$f' : f :: n : 1.$$

125. The cases represented in *figs.* 77. and 78. are those of diverging rays. Let us now consider the case of converging rays. Let the rays E D be incident upon the surface A B C, *figs.* 79, 80., converging to the point I.

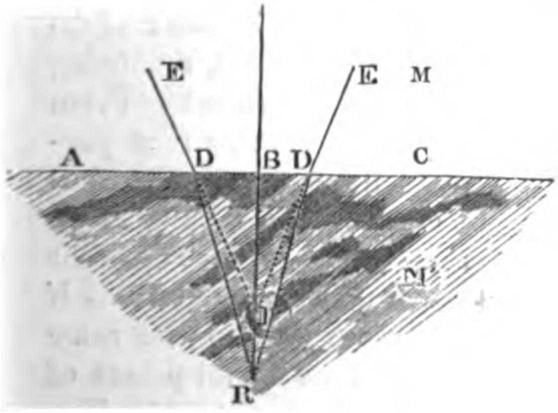

Fig. 79.

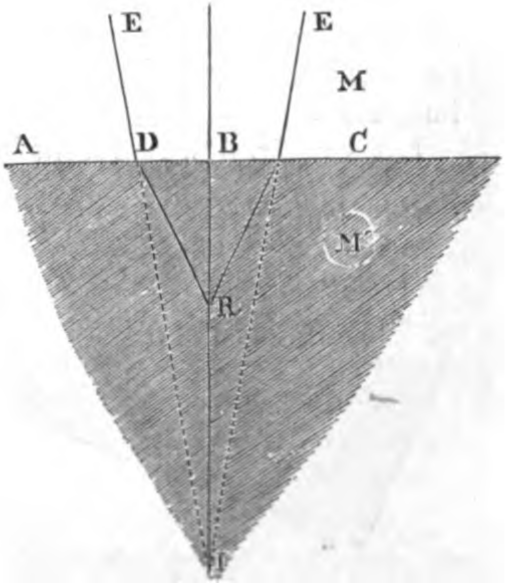

Fig. 80.

If the medium M′ be more dense than M, the rays, being deflected towards the perpendicular, would meet the axis B I at the point R, more distant than I from B; and if M′ be less dense than M, being deflected from the perpendicular they will meet the axis at the point R, less distant from the surface than I. In this case, the same reasoning will be applicable as in the former, and the same formula (c) for the determination of the relative distances of I and R from B will result.

If I, *figs.* 77, 78., be any point in an object seen by an eye placed within the medium M′, the point I will appear at R, because the rays D E proceeding from it enter the eye as if they came from R. The point will therefore seem to be more distant from the surface A C than it really is in the case represented in *fig.* 77., and less distant in that represented in *fig.* 78.

126. This explains a familiar effect, that when objects sunk in water are viewed by an eye placed above the surface, they appear to be less deep than they are, in the proportion of 3 to 4, this being the index of refraction for water. If thick plates of glass with parallel surfaces be placed in contact with any visible object, as a letter written on white paper, such object will appear, when seen through the glass, to be at a depth below the surface only of two thirds the thickness of the glass, the index of refraction for glass being $\frac{3}{2}$.

If a straight wand be immersed in water in a direction perpendicular to the surface, the immersed part will appear to be only three fourths of its real length, for every point of it will appear to be nearer to the surface than it really is, in the proportion of 3 to 4. If the wand be immersed in a direction oblique to the surface, it will appear to be broken at the point where it meets the surface, the part immersed forming an angle with the part not immersed.

Let A C, *fig.* 81., represent in this case the surface of the water, and let L B L′ be the real direction of the rod, B L′ being the part immersed. From any point P, draw P M perpendicular to the surface A C, and let M p be equal to three fourths M P. The point P will therefore appear as if it were at p; and the same will be true for all points of the rod from B to L′. The rod, therefore, which really passes from B to L′, will appear as if it passed from B to l', this line B l' being apparently at a distance from the surface of three fourths the distance B L′.

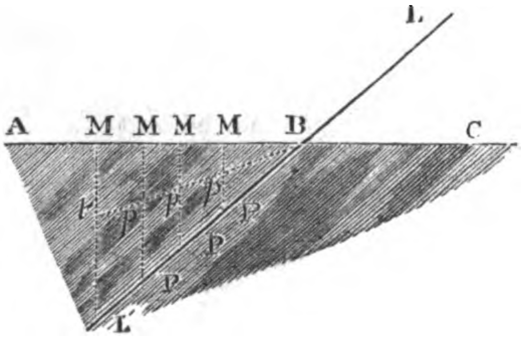

Fig. 81.

127. Much confusion and consequent obscurity prevails in the works of writers on optics of all countries, arising from the uncertain and varying use of the terms refracting or refractive power, as applied to the effect of transparent media upon light transmitted through them.

It is evident that if rays of light incident at the same angle on the surfaces of two media be more deflected from their original course in passing through one than in passing through the other, the *refracting power* of the former is properly said to be greater than the *refracting power* of the latter. But it is not enough for the purposes of science merely to determine the *inequality* of refracting power. It is necessary to assign numerically the amount or degree of such inequality, or, in other words, to assign the *numerical ratio* of the refracting powers of the two media.

In some works the *index of refraction* is adopted as the expression of the *refracting power*. Thus the first table in the Appendix to Sir David Brewster's Optics is entitled "Table of *Refracting Powers* of Bodies;" the table being, in fact, a table of the *indices of refraction*. The correct measure of the refracting power of a medium is, however, not the index of refraction itself, but the number which is found by subtracting 1 from the square of that index. Thus, if n express the index of refraction, n^2-1 would express the refracting power.

This measure is based upon a principle of physics not easily rendered intelligible without more mathematical knowledge than is expected from readers of a volume so elementary as the present. In the corpuscular theory of light, the number n^2-1 expresses the increment of the square of the velocity of light in passing from the one medium to the other; and in the undulatory theory it depends on the relative degrees of density of the luminous ether in the two media. In each case there are mathematical reasons for assuming it as the measure of the refractive power.

Taking the *refractive power* in this sense, it may be expressed for any medium, either on the supposition that light passes from a vacuum into such medium, or that it passes from one transparent medium to another. If the refractive powers of two media be given, on the supposition that light passes from a vacuum into each of them, the refractive power, where light passes from one medium to the other, can be found by dividing their refractive powers from a vacuum one by the other. Thus the refractive power of glass from vacuum being 1·326, and that of water 0·785, the refractive power of glass, in reference to water, will be

$$\frac{1{\cdot}326}{0{\cdot}785}=1{\cdot}69.$$

128. The term "absolute refracting power" has been adopted to express the ratio of the refracting power of a body to its density. Thus, if D express the density of a medium, and A express its absolute refracting power, we shall have

$$A=\frac{n^2-1}{D}.$$

When an elastic fluid or gaseous substance suffers a change of density, its refracting power undergoes a corresponding change, increasing with the density; but in this case the "absolute refracting power" remains sensibly constant, the index of refraction varying in such a manner that n^2-1 increases or diminishes in the same ratio as the density.

CHAP. V.

REFRACTION AT SPHERICAL SURFACES.

129. It has been already explained that a ray of light incident upon a curved surface suffers the same effect, whether by refraction or reflection, as it would suffer if it were incident upon a plane surface touching the curved surface at the point of incidence; and consequently the perpendicular to which such ray before or after refraction must be referred, will be the normal to the curved surface at the point of incidence. But as the curved surfaces which are chiefly considered in optical researches are spherical, this normal is always the line drawn through the centre of the sphere of which such curved surface forms a part. When a ray of light, therefore, is incident upon any spherical surface separating two media having different refracting powers, its angles of incidence and refraction are those which the incident and refracted rays respectively make with the radius of the surface which passes through the point of incidence.

Thus if A B C, *fig.* 82., be such a surface, of which O is the centre, a ray of light Y P, being incident upon it at P, and refracted in the direction P F, the angle of incidence will be the angle which Y P makes with the continuation of O P, and the angle of refraction will be O P F. The sine of the angle of incidence will be, according to the common law of refraction, equal to the sine of the angle of refraction multiplied by the index of refraction.

We shall first consider the case of pencils of parallel rays incident on spherical surfaces; and, secondly, that of divergent or convergent rays.

It may be here premised, once for all, that in what follows such pencils of rays only will be considered as have angles of incidence or refraction so small as to come within the scope of the principle explained in (71.), so that in these cases the angles of incidence and refraction themselves may be substituted for their sines, and *vice versâ;* and the arcs which subtend these angles, and the perpendiculars drawn from the extremity of either of their sides to the other, may indifferently be taken for each other. The retention of this in the memory of the reader will save the necessity of frequent repetition and recurrence to the same principle.

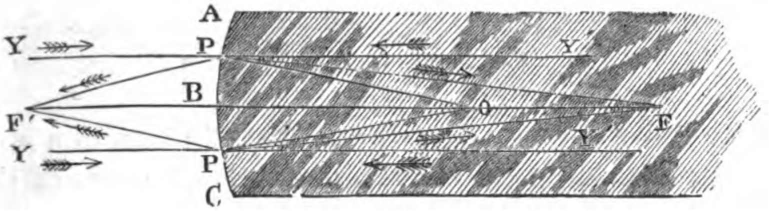

Fig. 82.

130. **Parallel rays.**—Let Y P, *fig.* 82., be two rays of a parallel pencil whose axis is F O B, and which is incident at P upon a spherical surface A B C, whose centre is O.

There are two cases presenting different conditions:

I. When the denser medium is on the concave, and the rarer on the convex side of the refracting surface.

II. When the denser medium is on the convex, and the rarer medium on the concave side of the refracting surface.

131. **First case.—Convex surface of denser medium.**—The rays Y P (*fig.* 82.), incident at P, entering a denser medium, will be deflected towards the perpendicular O P, and will consequently meet at a point F beyond O. The angle P O B is equal to the angle of incidence. Let this be called I. The angle O P F is the angle of refraction, which we shall call R.

By the common principles of geometry (Euclid, Book 1. Prop. 32.), we have

$$R = I - B F P.$$

If the distance B F, of the focus F, from the vertex B be expressed by F, and the radius B O by r, we shall have

$$I = \frac{BP}{r}, \quad R = \frac{BP}{r} - \frac{BP}{F}.$$

But since I is equal to $n \times R$, we shall have

$$\frac{BP}{r} = n \times \frac{BP}{r} - n \times \frac{BP}{F}.$$

Omitting the common numerator B P, we shall have

$$\frac{1}{r}=\frac{n}{r}-\frac{n}{F};$$

and consequently

$$F=\frac{n\,r}{n-1}\quad . \; . \; (A).$$

132. By this formula, when the index of refraction n, and the radius r of the surface A B C, are known, the distance of the point F from B can always be computed, as it is only necessary to multiply the radius by the index of refraction, and to divide the product by the same index diminished by 1.

To find the distance of the focus F from the centre O, it is only necessary to subtract from the formula expressing its distance from B, the radius r. Thus we have

$$F\,O=\frac{n\,r}{n-1}-r=\frac{r}{n-1}\quad . \; . \; (B).$$

133. In the case contemplated above, the rays Y P pass from the rarer to the denser medium. If they pass from the contrary direction, that is to say, in the direction Y′ P, then the index n from the denser to the rarer medium will be less than 1, and the expression for F, formula (A), will be negative, showing that in this case the focus lies to the left of the vertex B at F′. The same formula, however, expresses its distance from B, only that the index of refraction n is in this case the reciprocal of the index for the rays passing in the contrary direction. If, then, we express by n' the index of refraction from the denser to the rarer medium, the distance of F′ from B will be expressed by

$$F'=\frac{n'\,r}{n'-1}.$$

It is easy to show that the distance F′ B of the focus of the rays Y′ P from the vertex B is equal to the distance F O of the focus F of the rays Y′ P from the centre. To show this, it is only necessary to substitute $\frac{1}{n}$ for n', which is its equivalent, and we find

$$F'=\frac{r}{1-n},$$

which is the same as the expression already found for the distance of F from O, but having a different sign, inasmuch as it lies at a different side of the vertex B.

134. The two foci F and F′ of parallel rays incident upon the

refracting surface A B C in opposite directions, are called the *principal foci*, one F of the convex surface, and the other F′ of the concave surface. It follows from what has been just proved that the distance of each of these foci from the vertex B is equal to the distance of the other from the centre O, and that parallel rays, whether incident upon the convex surface of a denser, or the concave surface of a rarer, medium, will be refracted, converging to a point upon the axis in the other medium, determined by the formulæ above obtained.

135. **Second case.—Concave surface of a denser medium.** — The formulæ (A) and (B) are equally applicable to the case in which the denser medium is on the convex side of the surface A B C. It is only necessary, in this case, to consider that the value of n, for the rays Y P, is less than 1. This condition shows that the value of F, given by the formula (A), was negative, and consequently that the focus would lie to the left of the vertex B, as at F′. Now, since the rays Y P, after passing the surface A B C, have their focus at F′ they must be divergent, and the focus F′ would be imaginary.

In like manner, if the rays pass from the rarer to the denser medium, in the direction Y′ P, the value of F will be positive, because in this case n will be greater than 1, and consequently the focus will lie to the right of the vertex B, as at F, the rays diverging from it being those which, by refraction, pass into the medium to the left of the surface A B C. The focus F, therefore, in this case, is also imaginary.

The same *fig.* 82., therefore, will represent the circumstances attending the case in which the denser medium is at the convex side of the surface, the only difference being that in this latter case F is the focus of the rays Y′ P, and F′ the focus of the rays Y P. The distances of F and F′ from B and O respectively will be the same as in the former case.

136. To illustrate the application of the preceding formulæ, let us suppose, for example, that the denser medium is glass, and the rarer air, and that consequently the value of n, for rays passing from the rarer to the denser, is $\frac{3}{2}$, and its value for rays passing from the denser to the rarer is $\frac{2}{3}$.

We have, consequently, in the case represented in *fig.* 82.,

$$\text{F B} = \frac{n r}{n-1} = 3 r;$$

that is to say, the distance of the principal focus of the parallel rays Y P from B is three times the radius O B, and consequently its distance F O from O is twice its radius.

In like manner, to find the distance F′ B, we have

$$n' = \frac{2}{3},$$

and consequently

$$F' = -2r;$$

that is to say, the distance F′ B is equal to twice the radius, and is negative, since it lies to the left of B.

In like manner, it will follow that when the surface of the denser medium is concave, B F′ and F O are each equal to twice the radius O B.

137. Since the directions of the incident and refracted rays are in all cases reciprocal and interchangeable, it follows that if, when the denser medium is on the concave side of the surface, rays are supposed to diverge from either of the foci F or F′ (*fig.* 82.), they will be refracted parallel to the axis F B in the other medium; and in the second case, if rays be incident upon the refracting surface in directions converging to F or F′, they will be refracted parallel to the axis in the other medium.

It may be asked what utility there can be in considering the case of incident rays converging, inasmuch as rays which proceed from all objects, whether shining by their own light, or rendered visible by light received from a luminary, must be divergent, each point of such objects being a radiant point, which is the focus of a pencil of rays radiating or diverging from it in all directions.

It is true that the rays which proceed immediately from any objects are divergent, and therefore, in the first instance, all pencils of rays which are incident upon reflecting or refracting surfaces are necessarily divergent pencils; but in optical researches and experiments, pencils of rays frequently pass successively from one reflecting or refracting surface to another, and in these cases pencils which were originally divergent often are rendered convergent, and in this form become pencils incident upon other reflecting or refracting surfaces. In such cases the pencils have imaginary foci behind the surface upon which they are incident, such foci being the points to which they would actually converge if their direction were not changed by the reflecting or refracting surfaces which intercept them.

138. It appears from the preceding investigation that a spherical refracting surface, having a denser medium on its concave side, always renders parallel rays convergent, in whatever direction they are incident upon it; and that, on the contrary, a spherical surface, having a denser medium at its convex side, always renders parallel rays divergent in whatever direction they are incident upon it. As these two surfaces possess these distinguishing optical

properties, it will be convenient to express the former as a convergent refracting surface, and the latter as a divergent refracting surface.

139. Having explained the conditions which determine the position of the foci of parallel rays incident on spherical reflecting surfaces, we shall now proceed to investigate those by which the focus to which diverging or converging pencils of incident rays are refracted is determined.

Let ABC, (*figs.* 83, 84.), be a spherical refracting surface, of which the centre is O, and the vertex B. Let I be the focus of the pencil of incident rays, whether diverging or converging; and let R be the conjugate focus of refracted rays, so that the incident pencil may after refraction be converted into another pencil, diverging from or converging to the point R. The angle OPI will be the angle of incidence, and the angle OPR the angle of refraction.

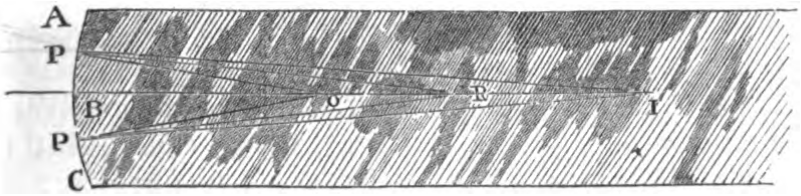

Fig. 83.

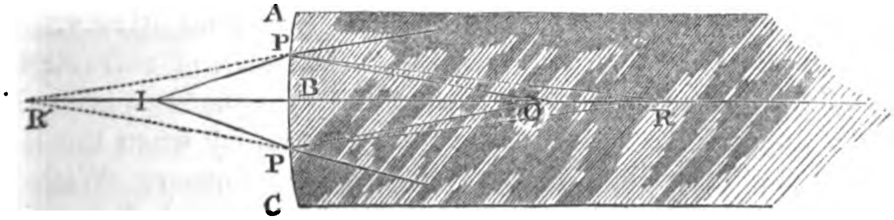

Fig. 84.

Let the radius BO be expressed as before by r, and let IB and RB be expressed respectively by f and f'.

We shall have, by the principles of geometry* (*fig.* 83.),

$$\text{OPI}=\text{BOP}-\text{BIP}=\frac{\text{BP}}{r}-\frac{\text{BP}}{f},$$

$$\text{OPR}=\text{BOP}-\text{BRP}=\frac{\text{BP}}{r}-\frac{\text{BP}}{f'}.$$

But since the angle of incidence, being small, is equal to the angle of refraction multiplied by the index of refraction, we shall have

$$\frac{\text{BP}}{r}-\frac{\text{BP}}{f}=n\times\left(\frac{\text{BP}}{r}-\frac{\text{BP}}{f'}\right).$$

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* Euclid. Book 1. Prop. 32.

Omitting the common numerator B P, we shall have

$$\frac{1}{r}-\frac{1}{f}=n\times\left(\frac{1}{r}-\frac{1}{f'}\right).$$

From this we infer,

$$\frac{1}{f}-\frac{n}{f'}=\frac{1-n}{r}\quad . \ . \ (\text{c}).$$

140. By this formula, when the distance of the focus of incident rays from the vertex, the radius of the surface, and the index of refraction, that is f, n, and r, are known, the position of the focus of refracted rays, that is, its distance f' from the vertex, can always be determined. It is only necessary to observe, that when the value of f' obtained from the formula (c) is positive, it is to be measured to the right of the vertex B, and consequently lies on the concave side of the surface; and that when negative it should be measured to the left of B, and consequently lies on the convex side of the surface.

When the focus of incident rays I lies to the right of B, and therefore on the concave side of the surface, the distance f is positive; but if I lie to the left of B, or on the convex side of the surface, then f in the formula (c) must be taken negatively. The index n is understood in all cases to be the index of refraction of the medium from which the ray proceeds to the medium into which it passes; and is, consequently, greater than unity when the latter is denser, and less when it is rarer than the former. With this qualification, the formula (c) will determine the relative position of conjugate foci in every possible case, whether of convergent or divergent rays, and at whichever side of the surface the denser medium may lie.

As an example of the application of this formula, let us take the most common case of a pencil of rays passing from air into glass.

If the pencil be divergent and the refracting surface be convex, (as represented in *fig.* 84.), the distance of I B, the focus of incident rays from the vertex, will be negative, and the value of n will be $\frac{3}{2}$. Hence the formula (c) will become

$$-\frac{1}{f}-\frac{3}{2f'}=\frac{-1}{2r}.$$

From whence we infer,

$$f'=\frac{3fr}{f-2r}\quad . \ . \ (\text{D}).$$

If I B, or f, therefore, be greater than twice the radius, f' will be positive, and will therefore lie within the surface A B C at a distance from B determined by the formula (D). In this case the rays diverging from I, *fig.* 84., will be made to converge after refraction to R. But if the distance IB or f be less than twice the radius, then the preceding value of f' will be negative, and must, consequently, be taken to the left of B, as at R′ (*fig.* 84.). Consequently, in this case, rays after refraction will diverge, as if they had proceeded from R′.

In fine, if IB be equal to $2r$, then the value of f will be infinite, which indicates that in such case the refracted rays are parallel, their points of intersection being at an infinite distance.

By like reasoning, the position of the focus of refracted rays which corresponds to every other variety of position of the focus of incident rays may be determined.

141. In the preceding observations, the focus of incident rays is supposed to be placed upon the axis of the spherical surface. Such pencil is, as in the case of reflectors, called the *principal pencil*, and the axis the *principal axis*.

When the focus of a pencil of rays is not on the axis of the refracting surface, or if it be a parallel pencil when its rays are not parallel to such axis, it is called a *secondary pencil;* and its axis, which is the ray passing through the centre of the refracting surface, is called a *secondary axis*. The focus of refracted rays of a secondary pencil lies upon its axis, and is determined in the same manner as in the case of a principal pencil. The rays, however, from such a pencil will only be refracted to the same point, provided the distance of its extreme rays from the axis, measured on the spherical surface, does not exceed a few degrees. If the rays be refracted beyond this limit, they will not be collected into a single point, but will, as in the case of reflectors, be dispersed over a certain space, and produce an aberration of sphericity.

PROPERTIES OF LENSES.

142. When a transparent medium is included between two curved surfaces, or a curved surface and a plane surface, it is called a *lens*.

Lenses are of various species, according to the characters of the curved surfaces which bound them; but those which are almost exclusively used in optical instruments and in optical experiments, are bounded by spherical surfaces, and to these, therefore, we shall here limit our observations.

Spherical surfaces, combined with each other and with plane surfaces, produce the following six species of lens, which are denominated converging and diverging lenses, because, as will be explained hereafter, the first class render a pencil of parallel rays incident upon them convergent, and the second class render such a pencil divergent.

143. Converging lenses are of the three following species: —

I. **The meniscus.** — The form of this lens may be conceived to be produced as follows: —

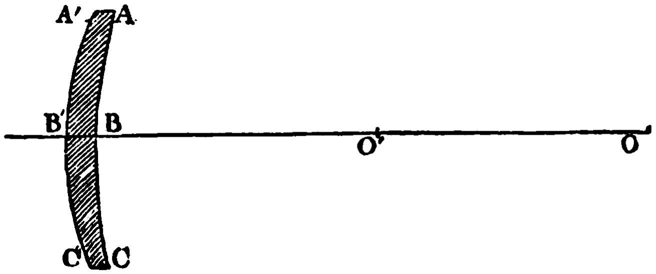

Fig. 85.

Let ABC and A'B'C', *fig.* 85., be two circular arcs, whose middle points are B and B', and whose centres are O and O', the radius OB being greater than the radius O'B'. Let the two arcs be supposed to revolve round a line OO'BB' as an axis, and they will in their revolution produce a solid of the form of the meniscus lens.

It is evident from this that the convexity A'B'C' of such a lens is greater than its concavity ABC, the radius O'B' of the convexity being less than the radius OB of the concavity.

II. **Double convex lens.** — The form of this lens may in like manner be conceived to be produced as follows: —

Two circular arcs, ABC and A'B'C' (*fig.* 86.), whose middle

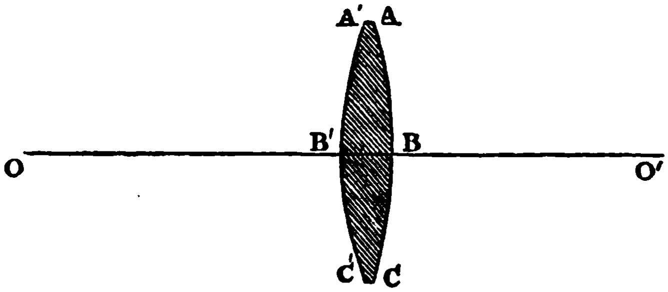

Fig. 86.

points are B and B', and whose centres are O and O', being conceived to revolve round a line OB' BO' as an axis, will, by their revolution, produce the form of this lens. The convexities of the sides will be equal or unequal, according as the radii OB and O'B' are equal or unequal.

III. **Plano-convex lens.** — The form of this lens may be conceived to be produced as follows: —

Let A'B'C' (*fig.* 87.) be a circular arc, whose middle point is B', and whose centre is O'; and let A B C be a straight line at right angles to B' O', whose middle point is B. If a figure thus formed revolve round the line O' B B' as an axis, it will produce the form of

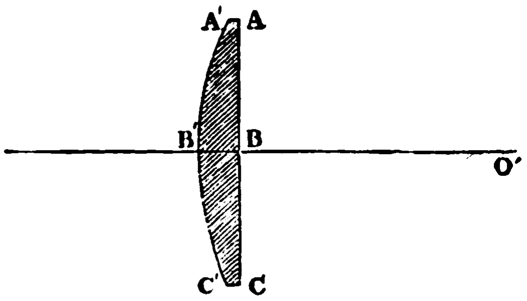

Fig. 87.

a plano-convex lens, the side A B C being plane, and the side A′ B′ C′ being convex.

144. Diverging lenses are of the three following species: —

I. **Concavo-convex lens.** — To form this lens, as before, proceed as follows: —

Let A B C and A′ B′ C′ (*fig.* 88.) be two circular arcs, whose middle points are B and B′, whose centres are O and O′, and whose radii are O B and O′ B′; the latter being greater than the former. If this be supposed to revolve round the line O′ O B B′ as an axis, it will produce the form of a concavo-convex lens. Since the radius of the concave side A B C is less than the radius of the concave side A′ B′ C′, the concavity will be greater than the convexity.

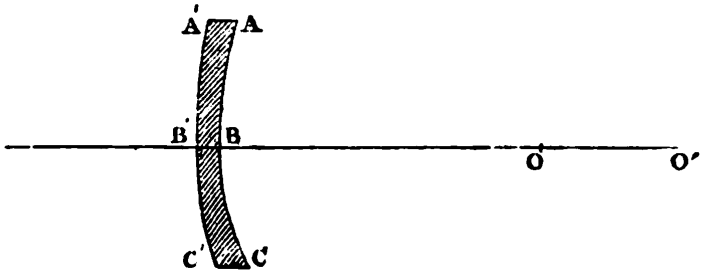

Fig. 88.

II. **Double concave lens.**—The form of this lens may be supposed to be produced as follows: —

Let A B C and A′ B′ C′ (*fig.* 89.) be two circular arcs, whose middle points are B and B′, and whose centres are O and O′. Let this figure be supposed to revolve round the line O O′ as an axis, and it will produce the form of a double concave lens. The convexities will be equal or unequal, according as the radii O B and O′ B′ are equal or unequal.

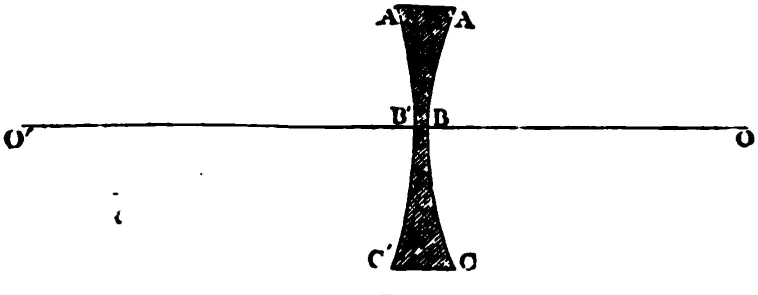

Fig. 89.

III. **Plano-concave lens.**—This lens may be conceived to be produced as follows:—

Let A B C, *fig.* 90., be a circular arc, whose middle point is B, and whose centre is O. Now let A′ B′ C′ be a straight line perpendicular to O B, whose middle point is B′. Let this figure be supposed to revolve round O B B′ as an axis, and it will produce the form of a plano-concave lens.

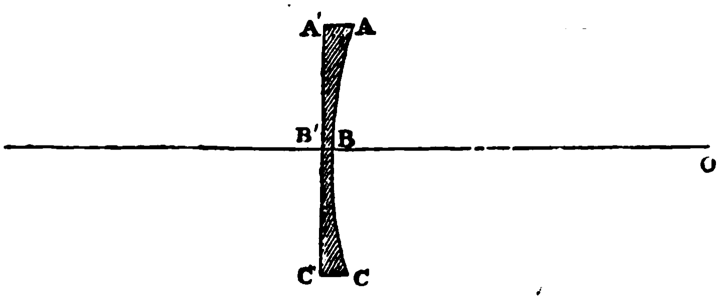

Fig. 90.

Examples of double convex lenses are presented by spectacle glasses, which are adapted to weak sight, and of double concave lenses by those which are adapted to short sight.

Meniscus lenses are sometimes used for weak sight, and concavo-convex for short sight; the concave side being always turned towards the eye. These are called by opticians periscopic glasses, from the circumstance of objects being seen when viewed obliquely through them with more distinctness.

145. In all these forms of lens the line O B B′ is called the *axis of the lens.*

146. To determine the effect produced on a pencil of rays by a lens, we shall first take the case of the meniscus.

Let O, *fig.* 91., be the centre, and O B the radius of the concave surface A B C. Let O′ be the centre, and O′ B′ be the radius of the convex surface A′ B′ C′. Let I be the focus of a pencil of rays incident upon the surface A B C. Let R′ be the focus to which the rays of this pencil would be refracted by the surface A B C, independently of the surface A′ B′ C′.

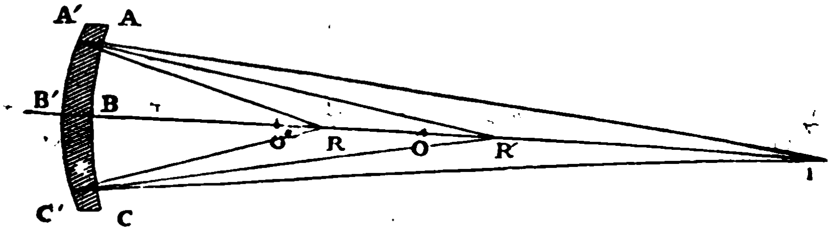

Fig. 91.

The pencil whose focus is this point R′ will then be incident upon the second surface A′ B′ C′ of the lens, and the rays from this pencil being again refracted by the second surface will have another focus R, which will be the definitive focus of the rays after refraction by both surfaces of the lens. In this, and in all other cases of lens, it will be necessary that the thickness B B′ of the lens may be disregarded, being inconsiderable compared with the other magnitudes which enter into computation.

Now let the distances of the foci I, R′, and R from the middle point B or B′ of the lens be expressed respectively by f, f', and f''; and let the radii O B and O′ B′ be expressed by r and r'; we shall then have, by what has been already explained respecting refracting surfaces, the following conditions: —

$$\frac{1}{f}-\frac{n}{f'}=\frac{1-n}{r}.$$

$$\frac{1}{f'}-\frac{n'}{f''}=\frac{1-n'}{r'}.$$

In this case n is the index of refraction from air into the medium of the lens, and n' is the index of refraction from the medium of the lens into air. By what has been already explained, these two indices are reciprocals, and consequently their product is equal to unity, so that we shall have $n\,n'=1$.

Now if we multiply the latter equation by n, we shall have

$$\frac{n}{f'}-\frac{n\,n'}{f''}=\frac{n-n\,n'}{r'};$$

but since $n\,n'=1$, this will become

$$\frac{n}{f'}-\frac{1}{f''}=\frac{n-1}{r'};$$

by combining this with the first equation we shall have

$$\frac{1}{f}-\frac{1}{f''}=\frac{n-1}{r'}-\frac{n-1}{r} \quad . \; . \; (\text{E}).$$

By these conditions the distance f'' can always be determined when f, r r', and n are known; that is to say, the position of the focus of refracted rays can always be determined when the position of the focus of incident rays, the radii of the lens, and the index of refraction are known.

This formula (E), by a due attention to the signs of the quantities which compose it, may be applied to lenses of every species. If the focus of incident rays lie to the right of the lens, as in *fig.* 91., f must be taken to be positive; if to the left of the lens, f must be taken negatively. If the centre of either surface lie to the right of the lens, the radius will be taken positively; and if to the left of the lens, it will be taken negatively. If one of the surfaces of the lens be a plane surface, it may be considered as having an infinite radius; and accordingly, the term of the equation (E) in the denominator of which such radius enters will become equal to o, and will therefore disappear from the equation.

When the value of f', which determines the distance of the focus of refracted rays from B, shall have been found by the equation (E), it must be taken to the right of the point B if it be positive, and to the left if it be negative.

147. If the incident rays whose focus is I be refracted parallel, then the distance f' of the focus of refraction from B will be infinite, and consequently, we shall have $\frac{1}{f'}=0$. Now, in this case, I will be the *principal focus* of the surface A B C. Let this be expressed by F, and we shall have by the equation (E)

$$\frac{1}{F}=\frac{n-1}{r'}-\frac{n-1}{r},$$

from which we infer,

$$F=\frac{r\,r'}{(n-1)\,(r-r')}\quad . \; . \; (F).$$

a formula by which the distance of the focus of parallel rays incident upon A B C can always be calculated.

If the incident rays be parallel, their focus I will be at an infinite distance, and we shall have $\frac{1}{f}=0$. In this case, the focus R will be the principal focus of the parallel rays, incident upon the surface A′ B′ C′.

Let the distance of this focus from B be expressed by F′, and we shall find as before from equation (E),

$$F'=\frac{-r\,r'}{(n-1)\,(r-r')}\quad . \; . \; (G).$$

Thus it appears that F and F′ differ in nothing save in their sign, the one being positive, and the other negative; the inference from which is, that parallel rays, whether incident on the one or the other surface of a lens, will be refracted to points equally distant from the lens, but on opposite sides of it.

148. The common distance of these principal foci from the lens is called the *focal distance* or *focal length of the lens.*

149. If the lens be a meniscus, and composed of a refracting substance more dense than air, it will render a parallel pencil incident upon either of its surfaces convergent, and its principal foci will consequently be real. This follows as a consequence from the formulæ (F) and (G); for in the case of a meniscus, r' is less

than r, and, therefore, the value of F given by the formula (F) is positive, and the value of F′ given by the formula (G) is negative; consequently, the focus of parallel rays incident upon A B C lies to the right of the lens, and the focus of parallel rays incident on A′ B′ C′ lies to the left of it. Parallel rays are therefore rendered convergent after refraction, and the foci are real in whichever direction they may pass through such a lens.

It is easy to show that the same will be true for double convex and plano-convex lenses. In the case of double convex lenses, the radius r is negative and r' positive; the consequence of which is, that the value of F is positive, and F′ negative. In the case of plano-convex lenses, the radius r is infinite, and the formulæ (F) and (G) become

$$F=\frac{r'}{n-1}, \quad F'=\frac{-r'}{n-1}.$$

Thus it appears, that in all the three forms of convergent lens parallel rays, whether incident on the one surface or on the other are refracted, converging to a focus on the other side of the lens, and the foci in all such cases are consequently real.

150. It is easy to show, by the same formula, that parallel rays incident on every species of divergent lens will be refracted diverging from a point on the same side of the lens as that at which they are incident.

In the case of the concavo-convex lens, the radius r is less than the radius r'; and since n is greater than 1, the value of F (given in the formula F) will be negative, and the value of F′ (given in the formula G) positive. Thus it appears that the principal focus of parallel rays incident on the surface A B C, *fig.* 88., will be to the right of B, and the principal focus of the rays incident on the surface A′ B′ C′ to the left of B, the foci in each case being at the same side of the lens with the incident rays; and, consequently, being in such case imaginary.

In the case of the double concave lens, the radius r' is negative; and since n is greater than 1, the value of F will be negative, and that of F′ positive.

In the case of the plano-concave lens, the value of r' is infinite; and since n is greater than 1, F will be negative, and F′ positive.

Thus it appears that in all the forms of divergent lenses, parallel rays incident upon their surfaces are refracted, diverging from a focus on the same side of the lens as that at which they are incident.

It is from this property that the two classes of convergent and divergent lenses have received their denomination; and it is evident, therefore, that the meniscus and plano-convex lens are optically equivalent to a double convex lens, and that the concavo-convex and plano-concave lens are optically equivalent to a double concave lens.

151. Among the varieties presented by the preceding formulæ, there is an exceptional case which requires notice. If the radii of the two surfaces of a lens be equal, and their centres be both at the same side of the lens, the lens will hold an intermediate place between a meniscus and a concavo-convex. In the former the radius of the convex surface is less than that of the concave surface; and in the latter, the radius of the concave surface is less than that of the convex surface. These radii might, however, be in each case as nearly equal as possible, the lenses actually retaining their specific characters. Each species, therefore, would approach indefinitely to an intermediate lens whose surfaces would have equal radii.

It is evident that the condition which would render equal the radii r and r', and give them the same sign, would render both the focal distances F and F′ infinite, their denominators being nothing.

To comprehend this it is only necessary to consider that in the case of the meniscus and the concavo-convex lens, the more nearly equal the radii r and r' are, the less will be the denominators of the values of F and F′; and, consequently, the greater will be these values themselves, and if we suppose the difference between the radii to be infinitely diminished, the values of F and F′ will be infinitely increased. These conditions lead to the inference that if the radii of the two surfaces be equal, the focus of parallel rays incident upon these two surfaces will be infinitely distant from the lens; that is to say, parallel rays will be refracted parallel.

Thus it appears that a lens formed by spherical surfaces, whose radii are equal, and whose centres lie at the same side of the lens, will have no effect on the direction of rays proceeding through it, and that such lens will be equivalent to transparent plates with parallel surfaces.

An example of such a lens as this is presented in the usual form of a watch glass.

152. Lenses may be composed of any transparent substance, whether solid or liquid.

If they be composed of a solid, such as glass, rock crystal, or diamond, they must be ground to the required form, and have their surfaces polished; if they be composed of liquid, they must then be included between two lenses such as have been just described, having themselves no refracting power, and having the form required to be given to the liquid lens. Thus, two watch glasses, placed with their concavities towards each other, and so inclosed at the sides as to be capable of holding a liquid, would form a double convex liquid lens. If their convexities were presented towards each other, they would form a double concave liquid lens.

153. The material almost invariably used for the formation of lenses in optical instruments being glass, it will be useful here to give the principal formulæ, showing the position of the focus in lenses of this material.

In the case of glass, the index of refraction, the incident rays being supposed to pass from air into that medium, is $\frac{3}{2}$: the formulæ (E) and (F) therefore, in this case, become

$$\frac{1}{f}-\frac{1}{f'}=\frac{1}{2r'}-\frac{1}{2r} \quad . \; . \; (\text{E}'). \qquad \text{F}=\frac{2rr'}{r-r'} \quad . \; . \; (\text{F}').$$

By the latter formula, the focal length of a glass lens can always be found.

In its application, however, it is necessary to observe that when the convexities of the surface of the lens are turned in opposite directions, as in the cases of double convex and double concave lenses, the denominator will be the *sum* of the radii; and if they are turned in the same direction, as in the case of the meniscus, and the concavo-convex lens, it will be the *difference* of the radii. The following general rule will always serve for the determination of the focus when both surfaces of the lens are spherical: —

RULE.

Divide twice the product of the radii by their difference for the meniscus and concavo-convex lenses, and by their sum for the double convex and double concave lenses. The quotient will in each case be the focal length sought.

To find the focus of a plano-convex or a plano-concave lens, we are to consider that it has been already proved that the focal length is given by the formula

$$\text{F}=\frac{r}{n-1};$$

and since n is $\frac{3}{2}$ we shall have $\text{F}=2r$;

that is to say, the focal length of a plano-convex or plano-concave lens is double the radius of the convexity or concavity.

If a double convex or double concave lens have equal radii, then the formula (F′) becomes $\text{F}=r$.

The focal length, therefore, of such a lens is equal to the radius of either surface.

For the same class of lens the formula (E′) becomes

$$\frac{1}{f}-\frac{1}{f'}=\frac{1}{r};$$

where r expresses the common magnitude of the radii of the two surfaces. From this we infer,

$$f'=\frac{rf}{r-f};$$

which supplies the following rule for finding the focus of refracted rays, when the focus of incident rays is given:—

Rule.

Multiply the common radius of the two surfaces by the distance of the focus of incident rays from the lens, and divide the product by the difference between the radius and the distance of the focus of incident rays from the lens.

If the distance of the focus of incident rays from the lens in this case be less than the radius, the value of f' will be positive, and the focus of refracted rays will lie at the same side of the lens with the focus of incident rays; but if the value of f be greater than r, then the value of f' will be negative, and the focus of refracted rays will lie at the other side of the lens.

154. **Case of secondary pencils.**—We have here considered those cases only in which the focus of the incident pencil is placed upon the axis of the lens, or of pencils whose rays are parallel to that axis. The focus of the refracted rays may, however, be determined by the same formula for secondary pencils whose axes, passing through the centre of the lens B, are inclined to its axis, provided only the inclination be not so great as to produce such spherical aberration as may prevent the rays from having an exact, or nearly exact, focus.

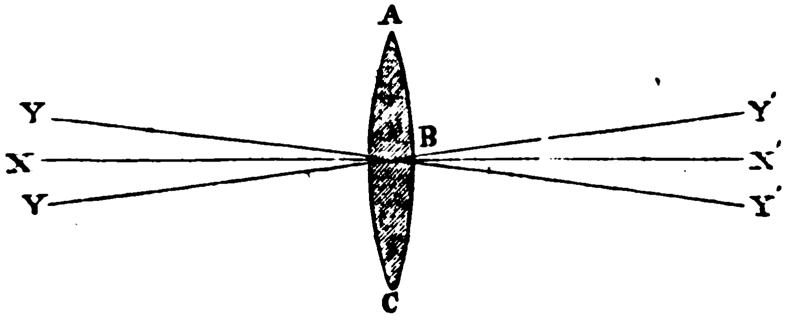

Fig. 92

155. If X X′, *fig.* 92., be the axis of the lens, and Y B X be the greatest angle at which the axis of the secondary pencil can be

inclined to X X′, so that the rays may have a nearly exact focus, the angle included between the two secondary pencils Y Y′ is called the *field of the lens.*

The angle formed by lines drawn from the edge of the lens to its principal focus is called the *aperture of the lens;* and this cannot in general exceed 10° or 12° without producing an aberration of sphericity, which would prevent the rays of the pencil incident upon it from having an exact focus.

156. **Images formed by lenses.**—The images of objects formed by lenses are explained upon the same principles as have already been applied to the case of spherical surfaces. If an object, whether it be self-luminous like the sun, or receive light from a luminary like the moon, be placed before a lens, each point upon its surface may be considered as a point from which light radiates in all directions. Such a point will be then the focus of a diverging pencil incident upon the lens, the bases of the pencil being the surface of the lens.

If the pencils which thus diverge from all points of the object be rendered, after refraction by the lens, convergent, they will have real foci on the other side of the lens, and the assemblage of such foci will form an *image* of the object. But if these pencils, after passing through the lens, be divergent, their foci will be imaginary, and will be placed at the same side of the lens with the object. These pencils would in such case be received by an eye on the other side of the lens as if they had originally proceeded from these points, which are the foci of the refracted pencils. The assemblage of these points would thus form an *imaginary image.*

All these circumstances are analogous to those which have been already explained in the case of reflectors. They will, however, be rendered still more intelligible by explaining their application to glass lenses.

157. Since all converging lenses, having equal focal lengths, are optically equivalent, a double convex lens with equal radii can always be assigned, which is the optical equivalent of any proposed converging lens, whether it be meniscus, double convex with unequal radii, or plano-convex.

Since, in like manner, all diverging lenses having equal focal lengths are optically equivalent, a double concave lens with equal radii may always be assigned, which is the optical equivalent of any proposed diverging lens, whether it be concavo-convex, double concave with unequal radii, or plano-concave.

158. **Image formed by double convex lens.**—It will therefore be sufficient to investigate the effects of double convex and double concave lenses with equal radii.

Let A B C, *fig.* 93., therefore, be a double convex lens, with equal radii; and let L M be an object, the centre of which is upon the axis of the lens, and placed beyond the principal focus F. Let the

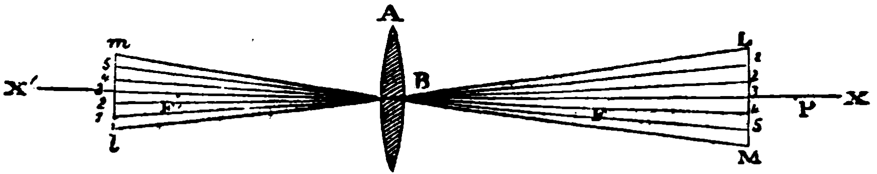

Fig. 93.

distance of this object from B be expressed by f; let the distance of its image be f', and the focal length of the lens, or its radius, be r. By what has been already explained, we shall have

$$\frac{1}{f'}=\frac{1}{f}-\frac{1}{r};$$

and, therefore,

$$f'=\frac{rf}{r-f}.$$

Since the distance of the object from the lens is supposed to be greater than B F, we shall have f greater than r; and consequently f' will be negative, which indicates that the image of L M will lie on the other side of the lens. It appears, also, by the preceding formula, that the distance f' of the image from the lens will be greater than r, and the image *l m* will therefore lie beyond the point F′.

If we draw L B *l*, this line will be the secondary axis of the pencil whose focus is at L, and consequently the focus of refracted rays will be at *l*; so that an image of the point L will be formed at *l*. In like manner it may be shown that an image of the point M will be formed at *m*; and in like manner the images of all the points of the object, such as 1, 2, 3, 4, 5, between L and M, will be formed at corresponding points 1, 2, 3, 4, 5, between *l* and *m*. It is evident, therefore, that in this case the image will be inverted.

159. Since the axes of the extreme secondary pencils L *l* and M *m* intersect at the centre of the lens, we shall have the following proportion:—

$$\text{LM} : lm :: \text{LB} : l\text{B};$$

or, which is the same,

$$\text{LM} : lm :: f : f';$$

that is to say, the magnitudes of the object and its image are as their distances respectively from the lens. The image, therefore, will be greater, equal to, or less than the object, according as *f′* is greater, equal to, or less than *f*.

To determine the manner in which the magnitude of the image varies with the distance of the object from the lens, it is only necessary to consider how the value of *f′* varies with respect to that of *f* as determined by the formula established above. Let L L, *fig.* 94., be a double convex lens with equal radii, and let these radii be expressed by *r*; it appears, from what has been stated, that if the object O be placed at a distance from L L greater than twice *r*, its image I will be nearer to the lens than the object, and less than the object in exactly the same proportion. If the object O be supposed gradually to approach the lens, its image I will gradually recede from it, and will be enlarged as it recedes, as shown in the figure, where O′ O″ O‴ are successive positions of the object, and I′ I″ I‴ the corresponding positions and magnitudes of the image.

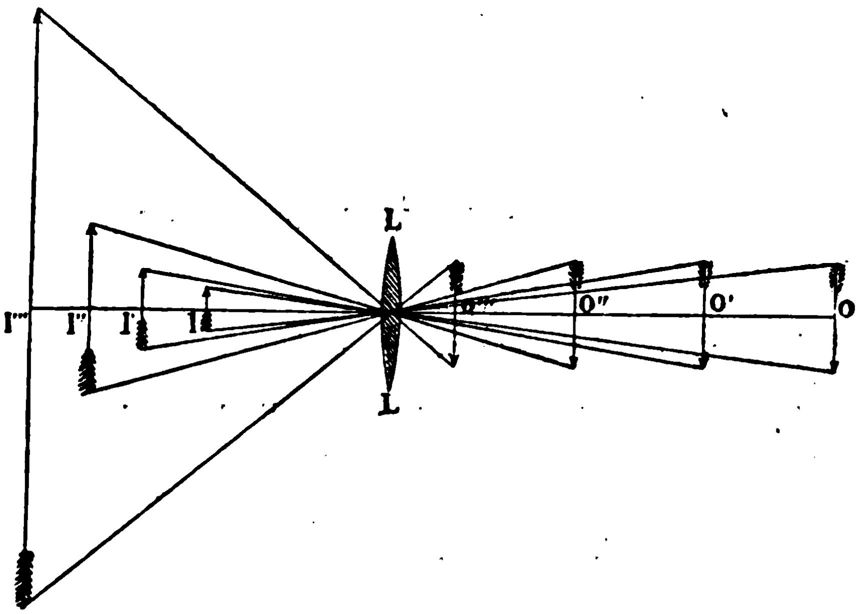

Fig. 94.

When O′, approaching the lens, arrives at a distance from it equal to 2 *r*, the object and image will be equal, the latter being also at the distance 2 *r* from the lens.

When the object approaches still closer to the lens, its distance being less than 2 *r*, but greater than *r*, the distance of the image from the lens will be greater than 2 *r*, and the image will be much greater than the object. As the object approaches the lens, the image recedes from it, and becomes rapidly larger; and this increase of the image, both in distance and magnitude, is enormously augmented as the object approaches the distance *r*; and when it actually arrives at that distance, the image altogether disappears, having receded to an infinite distance, and increased to an infinite magnitude.

If, on the other hand, the object O, instead of approaching the lens, be supposed to recede from it, its image I will continually approach the lens, and will continually decrease in magnitude. It might, therefore, be imagined that this decrease in its distance from the lens and in its magnitude would go on indefinitely; but such is not the case, for, as the object recedes from the lens, the image continually approaches the distance *r*, but never comes within that distance; and, in fact, if the distance of the object from the lens be considerable, the distance of the image from the lens will not sensibly differ from *r*.

It must not be forgotten that in the case of a double convex lens with equal radii, the points upon the axis of the lens at the distance *r* are its principal foci, and if the lens have unequal radii, its principal foci, determined by the formulæ, have similar properties.

160. **Experimental illustrations.** — All these circumstances admit of easy experimental verification. Let P (*fig.* 93.) be a point on the axis at a distance from B equal to 2 B F, so that P F shall be equal to B F. Let the flame of a candle be held at L M between F and P, the lens A C being inserted in an aperture formed in a screen so as to exclude the light of the candle from the space to the left of the lens. If a white screen be held at right angles to the axis and behind the lens, and be moved to and fro, until a distinct inverted image of the candle shall be seen upon it, its distance from the lens when this takes place will be found to be greater than twice the focal length, and to correspond exactly with that which would be computed by the formula. If the candle be moved towards P, the image will become indistinct upon the screen, but will recover its distinctness by moving the screen towards F′; and if the candle be placed at P, the screen being placed at a distance from B equal to twice B F′, a distinct image will be formed on the screen equal in magnitude to the object. If the candle be moved from P towards X, the screen must be moved towards F′ to preserve the image distinct; and if the candle be gradually moved in the direction P X, the screen must be continually moved towards F′. If the candle be moved to so great a distance from the lens that the diameter of the lens shall have an insignificant proportion to its distance, then a distinct image will be formed on the screen placed at the principal focus F′. If the candle be placed at the principal focus F, then the screen will show no image of it in whatever position it may be placed behind the lens, but will exhibit merely an illuminated disc formed by parallel

rays composing the refracted pencils into which the pencils proceeding from such point of the candle are converted by the lens.

The arrangement for performing these experiments is shown in *fig.* 95., where A is the candle, B the lens, and C the screen. Let us now suppose such object placed at L M (*fig.* 96.), before the principal focus F in the lens. In this case, f being less than r, the value of f' obtained by the preceding formula will be positive,

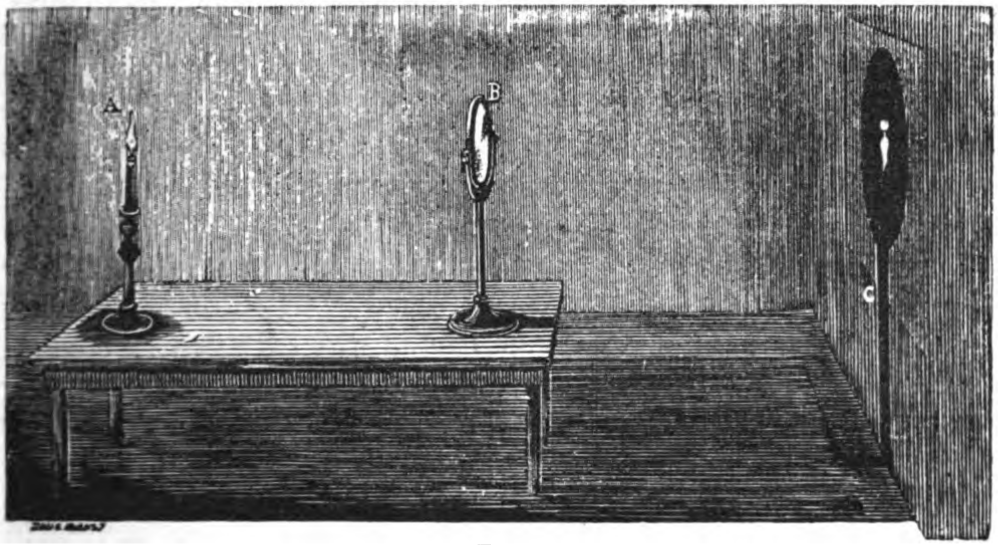

Fig. 95.

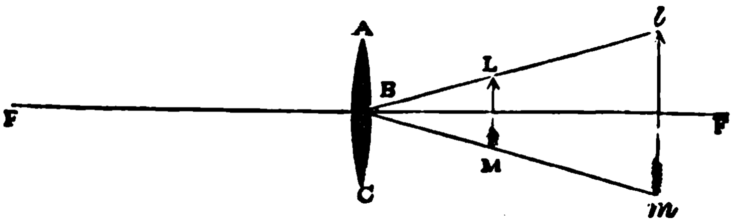

Fig. 96.

and, consequently, the focus of refracted rays will lie at the same side of the lens with the focus of incident rays. If, then, the pencil of rays diverging from L pass through the lens, it will, after refraction, diverge from the point *l*, more distant from the lens than L. In like manner, the pencil diverging from M will, after passing through the lens, diverge from *m*; and the same will be true of all the intermediate points of the object, so that the various pencils which diverge from different points of the object and pass through the lens will, after refraction, diverge from the corresponding points of *l m*. The image, therefore, in this case will be imaginary, and an eye placed to the left of the lens A B C would receive the rays of the various pencils as if they diverged, not from a point of the object L M, but from points of the imaginary image *l m*.

The magnitude of the image in this case will be greater than the object in the same proportion as *l* B is greater than L B.

As the object L M is moved towards F, its distance f from the lens will approach to equality with r, and the denominator of f' in the preceding formula diminishes, and consequently the distance of its image from the lens will be proportionately increased; therefore, as the object L M is moved towards F, its image *l m* will recede indefinitely from the lens, and would become infinite in distance and magnitude when the object arrives at F, which is consistent with what has been already explained of the principal focus.

It appears, therefore, that whenever the object is between the principal focus and the lens, its image will be at a greater distance from the lens on the same side of it, and will be erect, imaginary, and greater than the object.

161. If an object, L M (*fig.* 97.), be placed before a double concave lens A B C, the focus corresponding to the several points of the object will lie between the object and the lens at distances determined by the formula

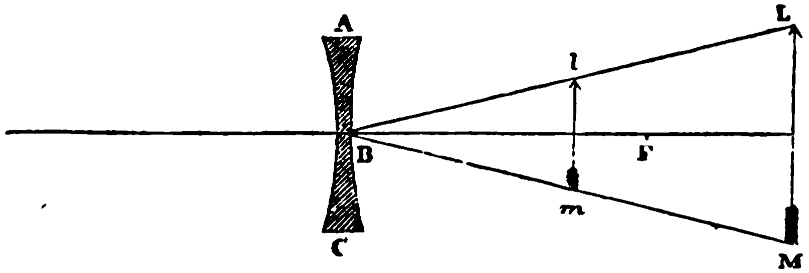

Fig. 97.

$$f' = \frac{r\,f}{r+f}.$$

It is evident from this formula that f' is less than f, and that consequently the distance of the image *l m* from the lens is less than the distance of the object from it. It appears also that the distances f and f' increase and diminish together, so that when the distance of an object from the lens L M is augmented, the distance of its image *l m* will also be augmented. But the distance of the image from the lens can never be greater than the focal length of the lens, because, as the distance of the object is indefinitely increased, the value of f' obtained from the formula approaches indefinitely to equality with r, though it can only become equal to it when the distance of the object becomes infinite.

162. If a radiant point be placed in the principal focus of a lens, the rays which diverge from it, after passing through the lens,

will be rendered parallel. This is a necessary consequence of the fact that the principal focus is the point to which parallel rays would be made to converge. It may be established experimentally by placing a candle or lamp with a very small flame in the principal focus of a lens, as shown in *fig.* 98. The pencil of rays, diverging from the flame after passing through the lens, and being projected upon a screen placed at right angles to them, will produce upon the screen an illuminated circle equal in magnitude to the lens.

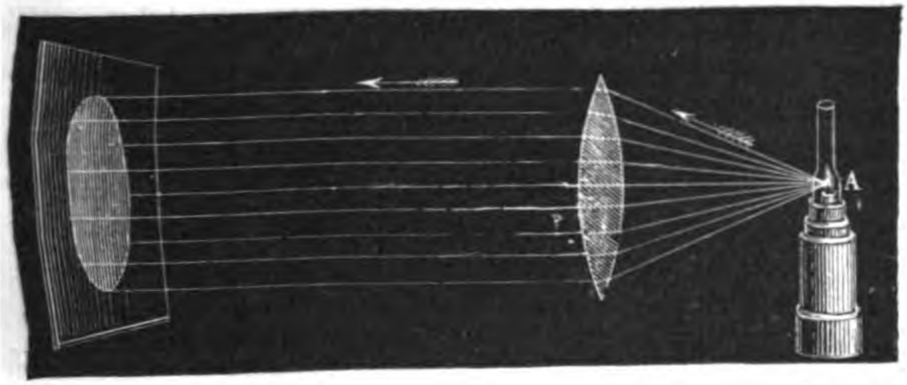

Fig. 98.

163. **Distortion of images.**—In the preceding paragraphs it has been assumed that the form of the image is in all respects similar to that of the object; and when the image is very small compared with the object, which is always the case when the distance of the object from the lens is considerable, this may be considered to be practically true. But otherwise it is easy to show that the picture of the object produced by a lens is always more or less distorted.

If, for example, the object be a flat surface placed at right angles to the axis of the lens, that point of it which is in the axis will be nearer to the centre of the lens than any other point of it; and all other points of the surface of the object will be so much the more distant from the centre of the lens as they are more distant from the point at which the axis meets the surface.

Now, it has been already shown that the more distant an object is from the lens the nearer to the lens will be its image; it follows, therefore, that in the case here supposed the images of those points of the object which are more remote from the axis of the lens will be nearer to the lens than are the images of those points of the object which are nearer to the axis of the lens.

It will be evident from this, that when the object is flat its image must necessarily be curved, having its concavity towards the lens. Thus, if o′ o′ (*fig.* 99.) be a straight or flat object, placed at a greater distance from the lens L L than its principal focus, its image I′ I′ will be curved as shown in the figure, the concavity of the curve being presented to the lens; for, according to what has been explained, the extreme points O′ O′, being more

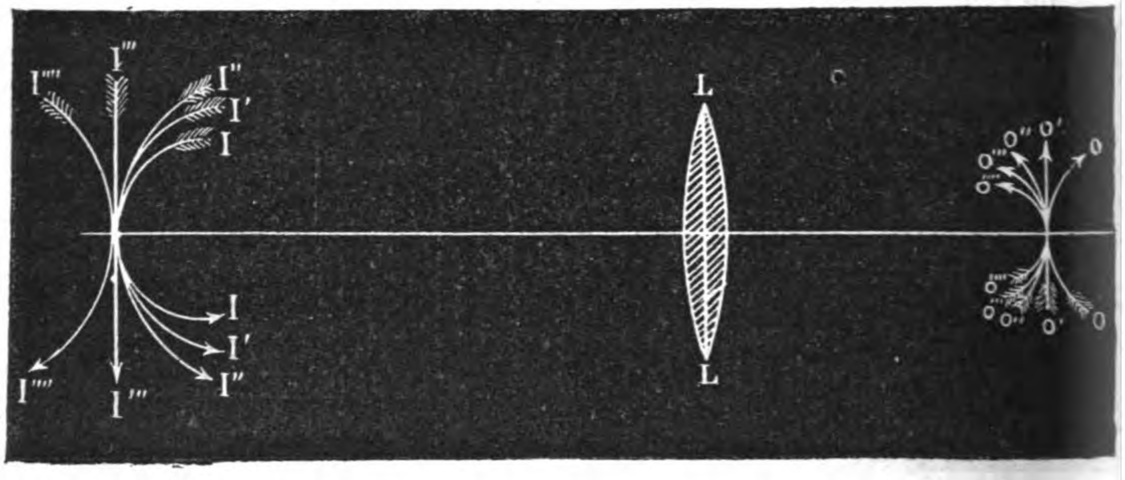

Fig. 99.

distant from the centre of L L than the central points, their images I′ I′ will be nearer to the centre of the lens than are the images of the central points.

Thus it appears that if the object O′ O′ be a straight line, its image will be a curved line; and if the object be a flat surface, its image will be a curved surface, the concavity in both cases being presented to the lens.

But if the object, instead of being straight, be curved, having its convexity towards the lens, as shown at O O, then its image I I will be still more curved, since the extreme points will be relatively brought closer to the lens, so that the concavity of the one presented to the lens will be greater than the convexity of the other.

If, on the other hand, the object be curved with its concavity towards the lens, as at O″ O″, the extreme points of its image being relatively more removed from the lens, by reason of the greater proximity of the extreme points of the object, the image I″ I″ will be less curved or less concave than the object.

Now, it is easy to imagine that since, by increasing the concavity of the object, the concavity of the image will be gradually diminished, the object may assume such a degree of concavity, O‴ O‴ for example, that its image I‴ I‴ shall be straight or flat; and that if the object be still more concave, as at O⁗ O⁗, the image I⁗ I⁗ will be convex towards the lens.

It will be understood, therefore, from what has been here explained, that when a real image of an object is formed by a convex lens, or by any equivalent converging lens, such image differs from the object, inasmuch as if the object be straight or flat, or if it be convex, the image will be concave towards the lens; and if the object be concave towards the lens, its image will be less concave, straight or convex, according to the degree of curvature of the object.

In this case we have supposed the object to be placed outside the principal focus of the lens; and, therefore, the image to be real. Similar conclusions may be deduced if the object be supposed, as in *fig.* 100., to be within the principal focus, and the image therefore to be imaginary.

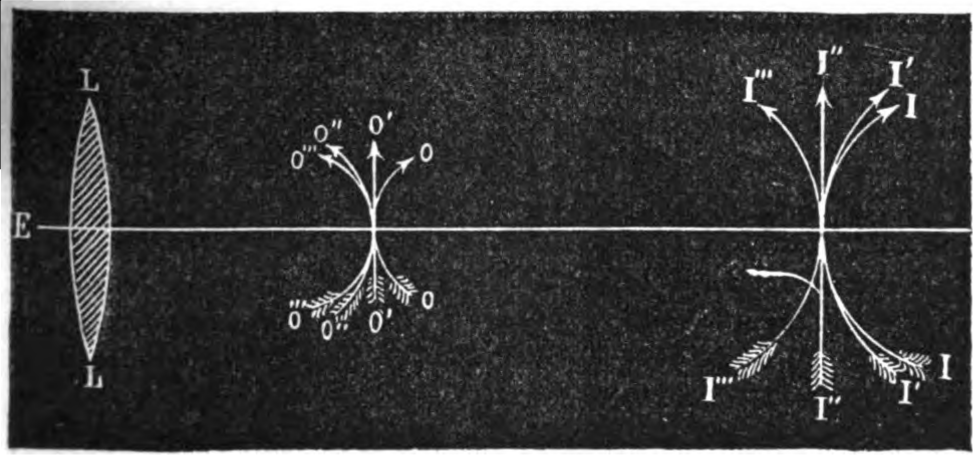

Fig. 100.

Let us suppose, first, the object $O\,O$ to be curved, with its convexity towards the lens; the rays, after passing through the lens, would be received by an eye at E, as if they had diverged from the points of an image $I\,I$ much more distant from the lens than those of the object. But as the extreme points $O\,O$ of the object are more distant from the centre of the lens than its central points, their images will be relatively still more distant than those of the central points, and the image $I\,I$ will consequently be convex towards the lens, and still more so than the object.

If the object be straight or flat, as at $O'\,O'$, the same reasoning will show that its image $I'\,I'$ will be convex towards the lens.

And in the same manner it may be inferred that if the object be concave towards the lens, as at $O''\,O''$ and $O'''\,O'''$, its image will either be straight, as at $I''\,I''$, or concave, but less so towards the lens, as at $I'''\,I'''$.

Similar conclusions may be inferred with like modifications respecting the imaginary images formed by concave lenses, or their optical equivalents.

164. **Spherical aberration.**—We have hitherto considered that the pencils of rays proceeding from the lens were brought to

an exact focus, and this will be practically the case if the angles of incidence of the extreme rays of the pencils do not exceed a certain limit; but if, from the magnitude of the lens, or the proximity of the object, this be not the case, effects will be produced which have been called *spherical aberration*, which it will be necessary here more clearly to explain.

Let A B C, *fig*. 101., be a plano-convex lens, having its plane side presented to the incident rays. Let a circular disc of card or sufficiently thick paper be formed, a little less in diameter than the lens, and let it be attached concentrically with the lens upon the plane side, so as to leave a narrow ring of the glass uncovered round the edge of the lens, as represented in *fig*. 102.

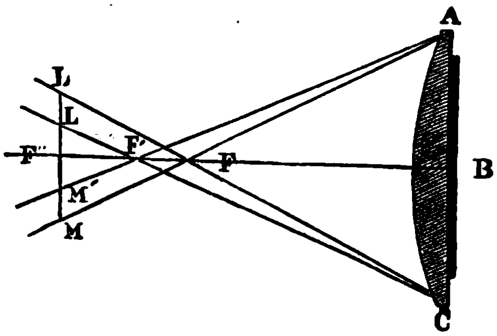

Fig. 101.

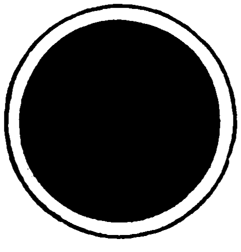

Fig. 102.

If this lens be now presented to a distant object, such as the sun, none but the extreme rays of each pencil will pass through it, and an image will be formed of the sun by these extreme rays at F, which will therefore be the principal focus of an annulus of parallel rays passing through the edge of the lens. Now let another circular piece of paper or card be cut so as to cover an annular surface surrounding the edge of the lens, and another to cover the central portion of it, so as to leave a ring of the surface uncovered at some distance within the edge, as represented in *fig*. 103. The lens being again presented to the sun, it will be found that an image will be formed at F′, *fig*. 101., somewhat more distant from the lens than F.

If, in fine, a disc of card be cut, equal in magnitude with the lens, having a small circular aperture at its centre, as represented in *fig*. 104., and be in the same manner attached to the lens, so as to allow only the central rays of each pencil to pass, an image of the sun will be formed at F″, *fig*. 101., still further from the lens.

It appears, therefore, that those rays of the pencil which are nearest the centre will have a focus further from the lens than those which are more distant from it, and the more distant the rays of each pencil are from the axis of the lens, the nearer their focus will be to the lens.

In fine, by continuing this process, it will be found that if the lens be resolved into a series of annular surfaces, concentric with each other and with the lens, a series of images will be produced

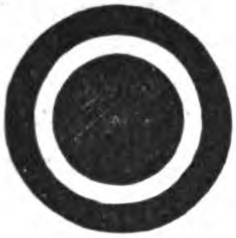

Fig. 103.

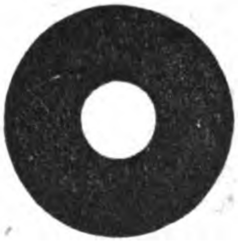

Fig. 104.

at distances *d*′, *d*″, *d*‴, *d*⁗, &c., gradually increasing, that produced by the external annulus being at the least distance, and that produced by the spot surrounding the centre at the greatest distance.

On comparing the series of distances *d*′, *d*″, *d*‴, *d*⁗ . . . . at which these images are placed, a very important circumstance will be observed in their distribution. It will be found that while those produced by the central annuli are crowded very closely together, those produced by the annuli near the edge of the lens are separated one from another by much more sensible spaces.

When the entire surface of the lens is uncovered and exposed at once to the object, it is evident that this series of images will be produced simultaneously. Some idea of their distribution along the axis of the lens may be formed by referring to *fig.* 105.

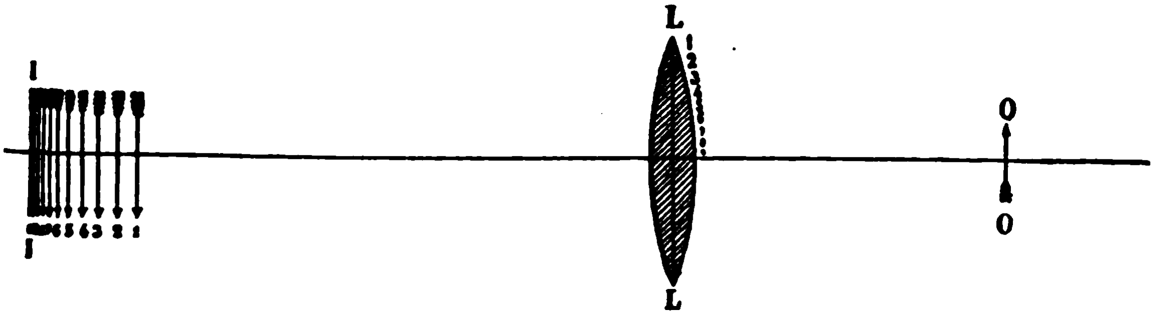

Fig. 105.

The object being o o, and the image produced by the small central spot of lenticular surface being at I I, the images formed by the rings of surface immediately contiguous to this spot will be crowded together so closely in front of a screen held at I I, that they will all be formed upon the screen with very little less distinctness than the image formed by the central spot itself, so that by their superposition upon the screen, all will contribute to augment the brightness of the image formed upon it, without producing injurious confusion or indistinctness. But not so with the much more distant and more widely separated images 1, 2, 3, 4, &c., produced by the exterior rings of the lenticular surface. These, being at very sensible distances from the screen held at the place of the central image, would produce a confused, cloudy, and indistinct picture on the screen, which, falling upon the more distinct picture produced by the central part, would give the whole a nebulous and misty appearance, when the object is a circular disc.

165. **Experimental illustration.**—These effects may be rendered apparent by holding a white screen at F″, *fig.* 101., at right angles to the axis of the lens. An image of the sun will be formed round F″, and beyond the edge of this image will be formed a ring or halo of light, growing fainter from the central image outwards, as represented in *fig.* 106.

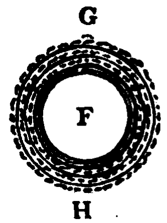

Fig. 106.

166. **Magnitude of spherical aberration in different forms of lenses.**—The distance F F″, *fig.* 101., measured on the axis between the focus of the extreme rays which pass through the edge of the lens, and the focus of the central rays along which the foci of all the intermediate rays are placed, is called the *longitudinal aberration:* the point F, which is the focus of the central rays, is called the *principal focus* of the lens; and the circle whose diameter is L M, over which the rays are spread, is called the *lateral aberration.*

Different lenses composed of the same material and having the same focal length will have different quantities of spherical aberration, according to the different curvatures given to their surfaces; thus the aberration of a double convex lens with equal convexities, will be different from that of a lens with equal focal length having unequal convexities, or of the plano-convex or meniscus, and the same observation will of course be applicable to divergent lenses, or those which are optically equivalent to a double concave lens. The following rules have been established to determine the relative amount of aberration produced by converging lenses of different forms: —

I. In a plano-convex lens, with its plane side turned to parallel rays, that is, turned to distant objects if it is to form an image behind it, or turned to the eye if it is to be used in magnifying a near object, the spherical aberration will be 4½ times the thickness.

II. In a plano-convex lens, with its convex side turned towards parallel rays, the aberration is only $1\frac{17}{100}$ of its thickness. In using a plano-convex lens, therefore, it should always be so placed that parallel rays either enter the convex surface or emerge from it.

III. In a double convex lens with equal convexities, the aberration is $1\frac{67}{100}$ of its thickness.

IV. In a double convex lens, having its radii as 2 to 5, the aberration will be the same as in a plano-convex lens in Rule I., if the side whose radius is 5 is turned towards parallel rays; and the same as the plano-convex lens in Rule II., if the side whose radius is 2 is turned to parallel rays.

V. The lens which has the least spherical aberration is a double convex one, whose radii are as 1 to 6. When the face whose radius is 1 is turned towards parallel rays, the aberration is only $1\frac{7}{100}$ of its thickness.

These results are equally true of plano-concave and double concave lenses.

If we suppose the lens of least spherical aberration to have its aberration equal to 1, the aberration of the other lenses will be as follows:—

Best form, as in Rule V. - - - - - -	1·000
Double convex or concave, with equal curvatures - -	1·561
Plano-convex or concave in best position, as in Rule II. -	1·093
Plano-convex or concave in worst position, as in Rule I. -	4·206

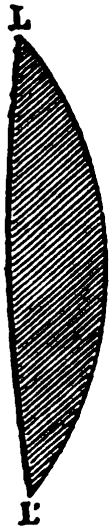

Fig. 107.

167. A lens of the form of least aberration, as explained in the fifth of the above rules, is shown in section in its proper proportions in *fig.* 107., and it is evident upon inspection that it differs little from a plano-convex lens.

168. If the object to which a converging lens is presented is very distant from it, and consequently the image proportionately close to it, as is the case, for example, with the object glasses of telescopes and opera-glasses, the more convex side of the lens must be turned to the object. But if, on the contrary, the object be very close to the lens, and consequently its image comparatively distant from it, as is the case in the compound microscope, the flatter side of the lens must be turned to the object.

The close approximation which the form of lens represented in *fig.* 107. has to a plano-convex lens, must render it evident that the aberration of the one cannot differ much from that of the other; and it appears in fact by the numbers given in the second and fifth of the above rules, that when two such lenses have equal thicknesses, the proportion of their aberrations will be that of 107 to 117, or, what is the same, 100 to 109; so that the aberration of the plano-convex exceeds that of the lens of least aberration by no more than an eleventh of its whole amount. The consequence of this has been that, in practice, especially in the case of the object-glasses of microscopes, the plano-convex lens has been used on account of the much greater facility of working it. The plane side of such a lens should be turned towards the object when near, and the convex side, when distant.

169. **Aberration diminished by compound lenses, proposed by Sir John Herschel.**—Although no expedient has been discovered by which the spherical aberration of single lenses can be rendered less than 1·07 of their thickness, yet, by combining different lenses in such a manner as to give their curvature contrary spherical influences, so far as relates to aberration, a great decrease, and even the total removal, of their imperfections may be accomplished.

Sir John Herschel has shown that if, instead of a double convex lens, two plano-convex lenses be used, so placed that their convexities shall be turned towards each other, the plane side of one being turned towards the object, and that of the other towards the eye, their combined aberration will be only 0·248, or a fourth of their thickness, provided that the focal length of one be 2·3 times that of the other. When this combination is used for the object-glass of a telescope, the lens of less curvature must be presented to the object, and when used as a simple microscope it must be turned towards the eye. It appears, therefore, that this combination reduces the spherical aberration to one fourth of its amount in a single lens of the best form.

If the two plano-convex lenses in this case have the same curvature, the spherical aberration will be 0·603 of the thickness, being a little more than half that of a single lens in its best form.

Sir John Herschel has also shown that the spherical aberration may be wholly effaced by the combination of a double convex lens C with a meniscus M, having suitable curvatures. In this case the convex side of M must be turned towards C, and when the lens is used as an object-glass, C must be turned towards the object; but if the combination be used as a simple microscope, C must be turned towards the eye.

170. **Table of their curvatures.**—The following are the radii and focal lengths of two combinations of these lenses, as computed by Sir John Herschel:—

	1st Combination.	2nd Combination.
C.		
Focal length - - - -	10·000	10·000
Radius of outer surface - - -	5·833	5·833
Radius of inner surface - - -	35·000	35·000
M.		
Focal length - - - -	17·829	5·497
Radius of outer surface - - -	3·688	2·054
Radius of inner surface - - -	6·291	8·128
Focal length of combinations -	6·407	3·474

171. From all that has been here explained it appears that the spherical aberration is augmented with the curvature of the lens and the shortness of its focal length. It follows, therefore, that any expedient by which a lens of a given focal length can be obtained with a less curvature will supply a means of diminishing the spherical aberration without diminishing the power of the lens. But since the focal length of a lens is diminished as the index of refraction of the substance of which it consists is increased, it follows that if two lenses of the same focal length be constructed of different materials, that of which the material has the greater refracting power will have less convexity, and, consequently, less spherical aberration.

172. **Gem lenses.** — One of the most obvious expedients, therefore, to diminish the effects of spherical aberration is to find transparent media suitable for lenses, whose refracting power is greater than that of glass. Several transparent substances having this important property are found among the precious stones. The diamond, more particularly, has a greater refracting power than any known transparent body. This advantage, and some other optical properties, induced Sir David Brewster and some other scientific men to cause lenses to be made of diamond, sapphire, ruby, and other precious stones; and sanguine hopes were entertained that vast improvements in microscopes would result from their substitution for glass lenses.

173. These hopes have, nevertheless, proved delusive; for, notwithstanding all that enterprise, skill, and perseverance could accomplish, as well on the part of scientific men, such as Sir David Brewster, and practical opticians, such as Pritchard and Charles Chevalier, the attempt has been abandoned. Independently of the cost of the material, difficulties almost insuperable arose from the heterogeneous nature of the gems, their double refraction, and the imperfect transparency and colour of some of them. The improvement of simple microscopes composed of glass lenses by the invention of doublets, and by the proper combination and adaptation of their curvatures, was also such as to render their performance little, if at all, inferior even to the gem lenses, while their cost is not much more than a twentieth of that of the latter.

Although it is not possible to efface altogether the effects of spherical aberration, yet they have been so considerably diminished by the adaptation of the curvatures of the lenticular surfaces, that in well-constructed optical instruments they may be regarded as entirely removed for all practical purposes. This is accomplished by giving to the two sides of the lens different curvatures, so adapted that the aberration produced by one shall be more or less counteracted by the aberration produced by the other.

174. **Aplanatic lenses.**—Lenses, or combinations of lenses, which thus practically efface the effects of spherical aberration are said to be APLANATIC, from two Greek words α (a) and πλάνη (plánē), which signify *no straying*.

175. **Illumination of image.**—For optical purposes it is not enough that the image of an object produced by a lens shall be distinct in its lineaments, which it will be in proportion as the spherical aberration is effaced; it must also be sufficiently illuminated to affect the eye in a sensible manner. Now the intensity of the illumination of such an image will, *cæteris-paribus*, be proportional to the number of rays proceeding from each point of the object, which are collected upon the corresponding point of the image, and it is easy to show that this will depend upon the angle formed by lines drawn from any point of the object to the extreme edges of the lens.

Thus, for example, let L L and L′ L′, (*fig.* 108.), be two lenses of

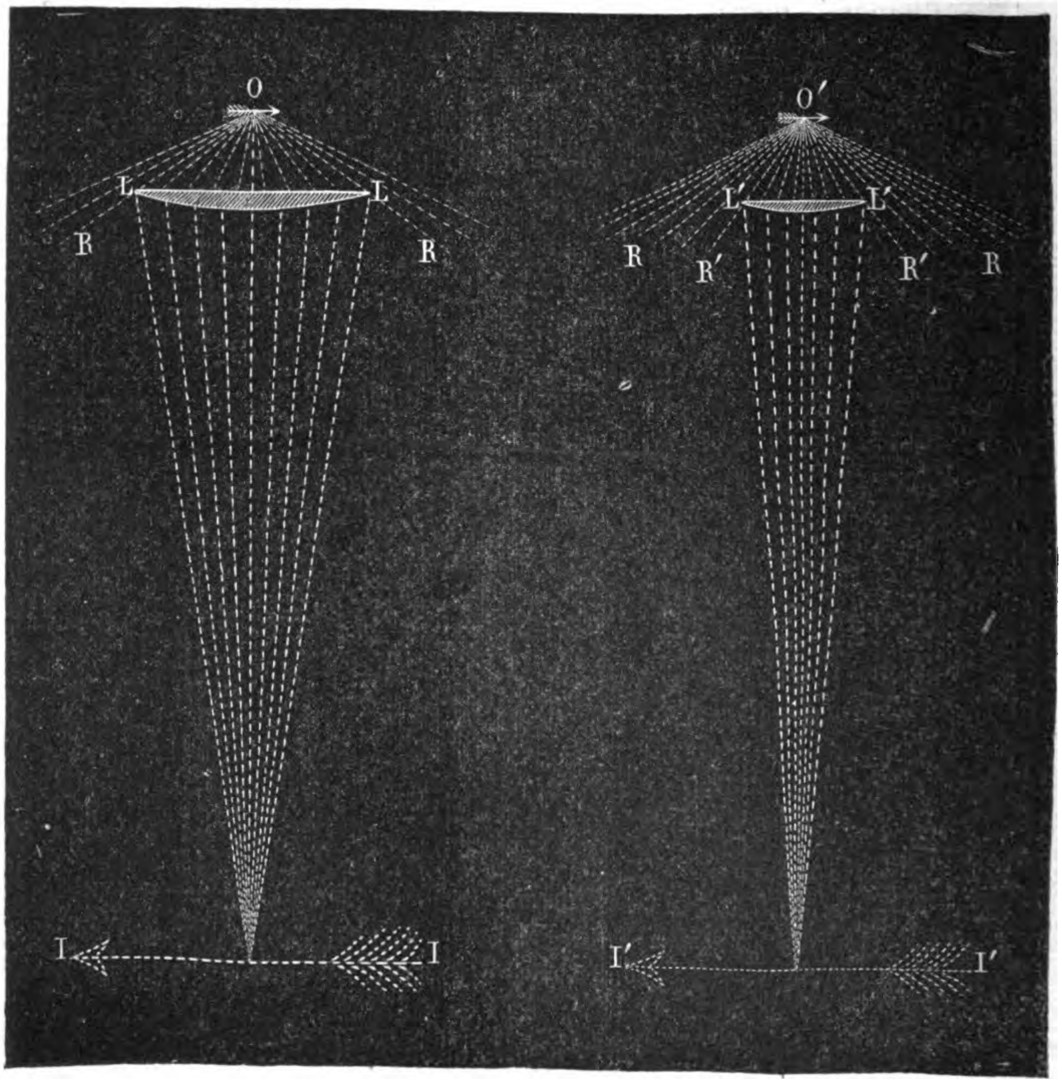

Fig. 108.

equal power which will produce images of the same object, O or O′, placed at the same distance from them, such images having equal magnitudes.

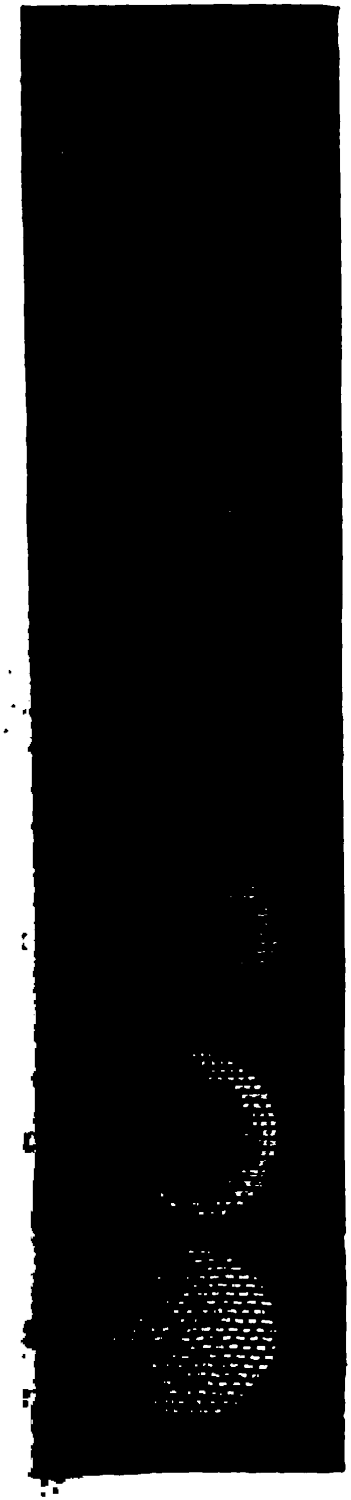

Fig 109.

Now it is evident, by mere inspection of the figure, that the number of rays which converge upon a point of the image I I will be those included within a cone whose vertex is the point upon the object and whose base is the lens L L. In the same manner, the number of rays which converge upon the corresponding point of the image I′ I′ will be those included within a cone whose vertex is a point upon the object and whose base is the lens L′ L′.

But the number of these rays will obviously be proportional to the square of the angle L O L or L′ O′ L′ formed by lines drawn from a point of the object to the extremities of the lens. This angle is the *angular aperture* of the lens.

In the case presented in the figure, therefore, the illumination of the image I I will be greater than that of I′ I′ in the same proportion as the square of the angle L O L is greater than the square of the angle L′ O′ L′.

We have here supposed the object to be very near the lens, as it always is in microscopes. If it be very distant from the lens, as it is in telescopes, the rays which proceed from any point of it, and which are received upon the lens, are parallel; and in that case the number of rays collected on each point of the image will be in the exact proportion of the area of the lens, or, what is the same, of the square of its diameter. Thus, for example, if images of the sun be produced by two lenses having equal focal lengths, the diameter of one being twice that of the other, the images will be equal in magnitude, but the illumination of one of them will be four times more intense than that of the other.

176. **Effects of increased aperture.** — How much the distinctness with which the image of an object is rendered perceptible is increased by augmenting the angular aperture of the lens by which the image is produced, will be understood by reference to *fig.* 109., in which seven drawings are given made from the image of the same object produced by the same lens, to which different angular apertures were successively given by covering more or less of its edges. These drawings were made by the late Dr. Goring to illustrate the advantage of large angular apertures in the case of the object-glasses of microscopes. With the smallest aperture the image appeared as shown at A, and as the aperture was gradually increased, it assumed the successive appearances shown at B, C, D, E, F, G.

177. **Objects invisible to the naked eye rendered visible.** — Independently of the effect they produce by magnifying the images of distant objects, lenses and reflectors are capable of rendering distant objects visible which would be invisible to the naked eye, by increasing the quantity of light proceeding from them which enters the eye. The light which produces vision, as will be more fully explained hereafter, enters the eye through a circular aperture called the pupil, which is the black circular spot surrounded by a coloured ring appearing in the centre of the front of the eye. It is clear that when the eye receives the rays diverging from a distant object, as shown in *fig.* 110., the number of rays which enter the pupil will be those included within a cone whose summit is the luminous point and whose base is the pupil.

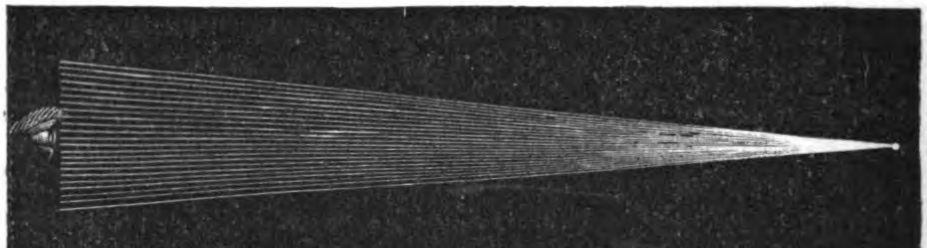

Fig. 110.

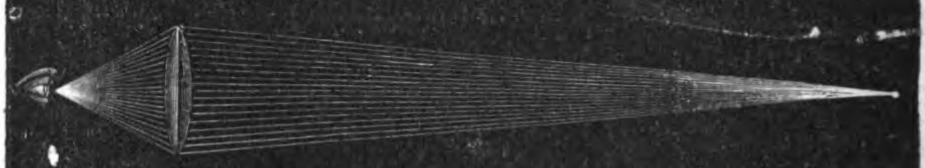

Fig. 111

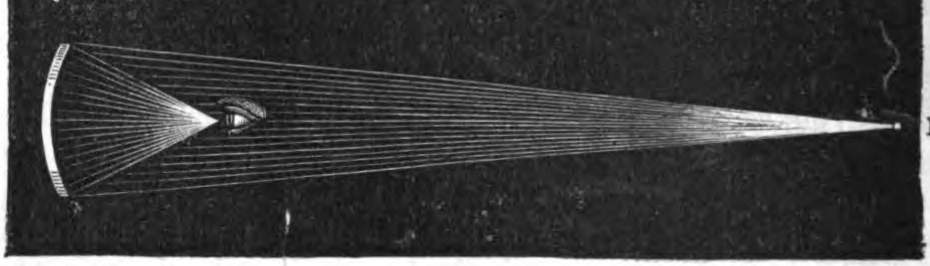

Fig. 112.

None of the rays which fall outside that cone can enter the eye or contribute in any way to produce vision. But if a convex lens, as in *fig.* 111., be interposed, so large as to receive all the rays of the cone shown in *fig.* 110., and if this lens be capable of converging these rays to a focus at a short distance beyond it, the eye placed at or very near the focus will receive all the rays into the pupil. Putting aside, therefore, all consideration of the magnifying power of the lens, it will obviously have the effect of augmenting the quantity of light received by the eye from each point of the object in the proportion of the superficial magnitude of the lens to that of the pupil; or, what is the same, in the proportion of the square of the diameter of the lens to the square of the diameter of the pupil.

Since the diverging rays may be equally rendered convergent by a concave reflector, the latter may be used to produce the same effect, as shown in *fig.* 112.

CHAP. VI.

ANALYSIS OF LIGHT. — CHROMATIC ABERRATION.

178. **Solar light compound.** — In the preceding chapters light has been regarded, in relation to transparent media, as a simple and uncompounded principle, each ray composing a pencil being subject to the same effects. That all light is not thus subject to uniform effects, is rendered manifest by the following experiment of Newton: —

Let a pencil of parallel rays of solar light be admitted through a circular opening P (*fig.* 113.), about half an inch in diameter, made in a screen or partition S T, all other light being excluded from the space into which the pencil enters. If a white screen X Z be placed parallel to S T, and at a distance from it of about 12 feet, a circular spot of light nearly equal in diameter to the hole will appear upon it at P′, the point where the direction of the pencil meets the screen. Now let a glass prism be placed at A B C, with the edge of its refracting angle B in a horizontal direction, and presented downwards so as to receive the pencil upon its side A B at Q. According to what has been already explained, the pencil would be refracted, in passing through the surface A B, in the direction Q L towards the perpendicular; and it would be again refracted in emerging from the surface C B from the perpendicular in the direction L K. It might therefore be expected that the effect of the prism would be merely to move the spot of light from

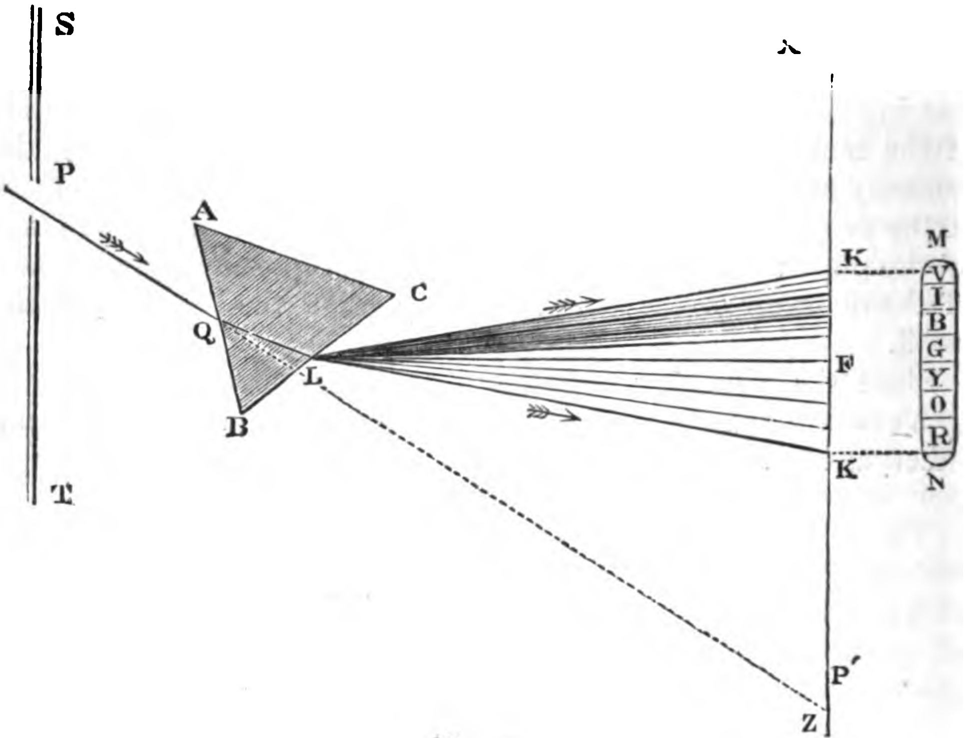

Fig. 113.

P′ to some point, such as K, more elevated upon the screen. The phenomenon, however, will be very different. Instead of a spot of light, the screen will present an oblong coloured space, the outline of which is represented at M N as it would appear when viewed in front of the screen.

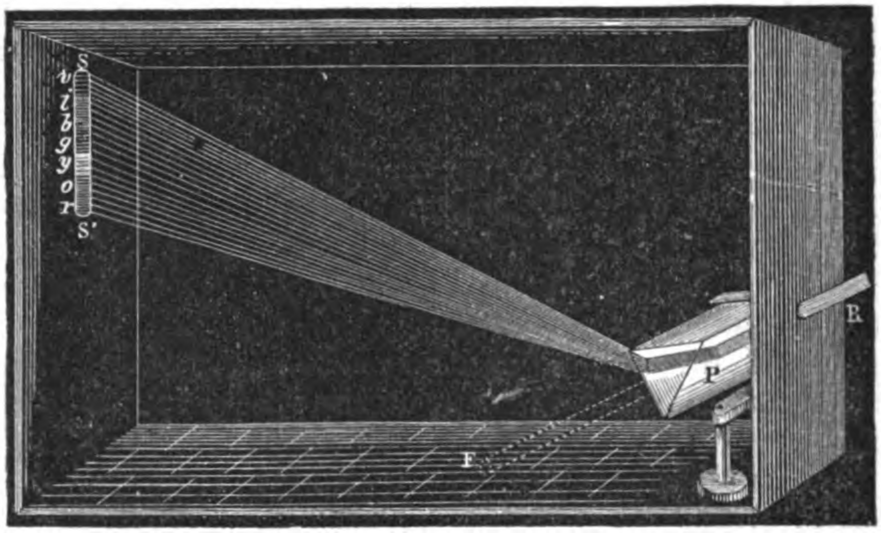

Fig. 114

The arrangement for making this celebrated experiment is shown in perspective in *fig.* 114.

179. **The prismatic spectrum.** — The sides of this oblong figure are parallel, straight, and vertical; its ends are semicircular, and its length consists of a series of seven spaces, vividly coloured, the lowest space being red, R; the next in ascending orange, O; and the succeeding spaces yellow, Y; green, G; light blue, B; dark blue or indigo, I; and, in fine, violet, V.

These several coloured spaces are neither equal in magnitude nor uniform in colour. The red space R, commencing at the lowest point with a faint red, increases in brilliancy and intensity upwards. The red, losing its intensity, gradually melts into the orange, so that there is no definite line indicating where the red ends and the orange begins. In the same manner, the orange, attaining its greatest intensity near the middle of the space, gradually melts into the yellow; and in the same manner, each of the succeeding colours, having their greatest intensities near the middle of the spaces, melts towards its extremities into the adjacent colours.

The proportion of the whole length occupied by each space will depend upon the sort of glass of which the prism is composed. If it be flint glass, and the entire length M N be supposed to consist of 360 equal parts, the following will be the length of each succeeding colour, commencing from the violet downwards: —

Violet	109
Indigo	47
Blue	48
Green	46
Yellow	27
Orange	27
Red	56
	360

It appears, therefore, that the ray of light P Q, after passing through the prism, is not only deflected from its original course P Q P′, but it is resolved into an infinite number of separate rays of light which diverge in a fanlike form, the extreme rays being L K and L K′, the former being directed to the lowest point of the coloured space upon the screen, and the latter to the highest point. The coloured space thus formed upon the screen is called the *prismatic spectrum.*

180. **Composition of solar light.** — From this experiment the following consequences are inferred: —

I. Solar light is a compound principle, composed of several parts differing from each other in their properties.

II. The several parts composing solar light differ from each other in refrangibility, those rays which are directed to the lowest part of the spectrum being the least refrangible, and those directed to the highest part being the most refrangible; the rays directed to the intermediate parts having intermediate degrees of refrangibility.

III. Rays which are differently refrangible are also differently coloured.

IV. The least refrangible rays composing solar light are the red rays, which compose the lowest division R of the spectrum. But these red rays are not all equally refrangible, nor are they precisely of the same colour. The most refrangible red rays are those which are deflected to the lowest point of the red space R, and the least refrangible are those which are directed to the point where the red melts into the orange. Between these there are an infinite number of red rays having intermediate degrees of refrangibility. The colour of the red rays varies with their refrangibility, the most intense red being that of rays whose refrangibility is intermediate between those of the extreme rays of the red space.

The same observations will be applicable to rays of all the other colours.

V. Each of these components of solar light having a different refrangibility will have for each transparent substance a different index of refraction. Thus the index of refraction of the red rays will be less than the index of refraction of the orange rays, and these latter will be less than the index of refraction of the yellow rays, and so on; the index of refraction of violet rays being greater than for any other colour.

But the rays of each colour being themselves differently refrangible, according as they fall on different parts of the coloured space, they will, strictly speaking, have different indices of refraction. The index of refraction, therefore, of any particular colour must be understood as expressing the index of refraction of the middle or mean ray of that particular colour. Thus, the index of refraction of the red rays will be the index of refraction of the middle ray of the red space; the index of refraction of the orange rays will be the index of refraction of the middle ray of the orange space; and so on.

It must not, however, be supposed that a pencil of solar light consists of separate and distinct rays of the different colours which form the spectrum, so that it might be possible by any mechanical division of such a pencil to resolve it into such rays. Each individual ray of such a pencil is composed of all the rays of the spectrum, just as the gases oxygen and hydrogen, which are the chemical constituents of water, enter into the composition of each particle of that liquid, no matter how minute it be.

181. **Experiments which confirm the preceding analysis of light.** — The validity of the preceding analysis of light is confirmed by the following observations and experiments: —

If the spectrum produced by the decomposition of the ray R, (*fig.* 115.), by the prism P, be thrown upon a screen S, the spectrum may be made to ascend by turning the prism P upon its axis, so

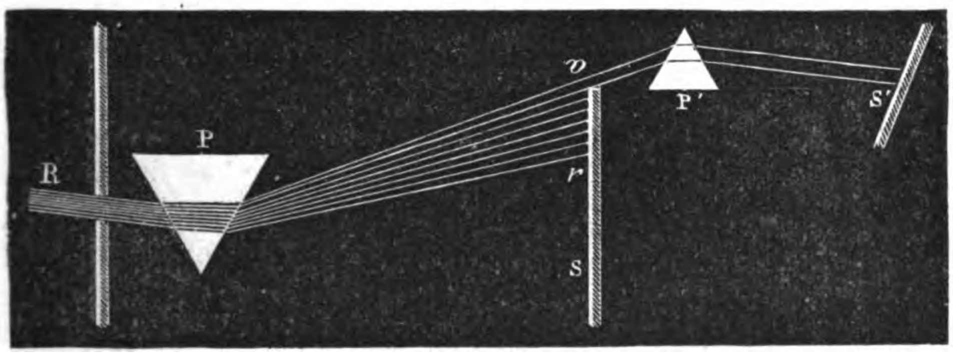

Fig. 115.

that the several component colours shall pass successively over the edge of the board, or the experiment may be modified by making a hole in the board through which the colours shall be successively transmitted. A prism P′, similar to P, is placed behind the board, upon which the rays thus transmitted are successively received; the two prisms are so placed that the ray shall fall on P′ at the same angle of incidence as that with which it fell upon P. In this case it will be found that the deflection of each ray by the prism P′ will be exactly equal to that produced by the prism P. The rays which thus successively fall upon the prism P′, will not be dilated by the second prism as the original compound ray was by the first, and no second spectrum will be formed.

Let a band of white paper be divided into seven equal spaces, and let those spaces be coloured red, orange, yellow, green, light blue, indigo, and violet severally, each colour being of uniform tint, and resembling as closely as possible the seven colours of the spectrum. Let this band be placed vertically upon a black ground, as shown in *fig.* 116., and let it be viewed through a prism having the edge of its refracting angle vertical. The images of the several coloured spaces, seen through a prism, will then be in the positions *r o y g b i* and *v*, shown in the figure, each successive colour being more and more

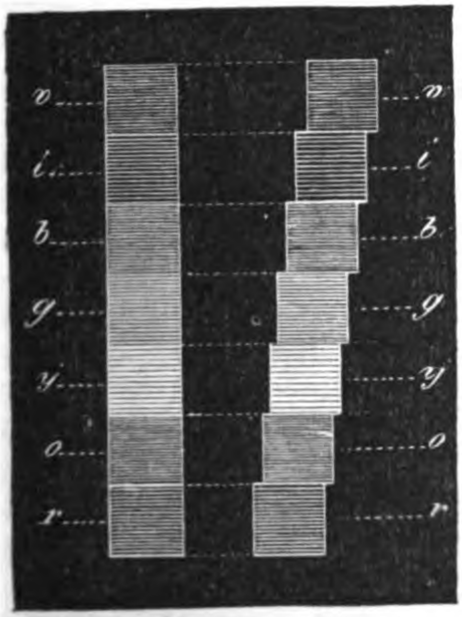

Fig. 116.

removed from its true position in ascending from the red to the violet. This phenomenon is obviously the result of the relative refrangibilities of the different colours.

Instead of artificial colours, let the spectrum itself be thrown upon a screen, as shown in *fig.* 117., so that its position will be that indicated by the dotted lines.

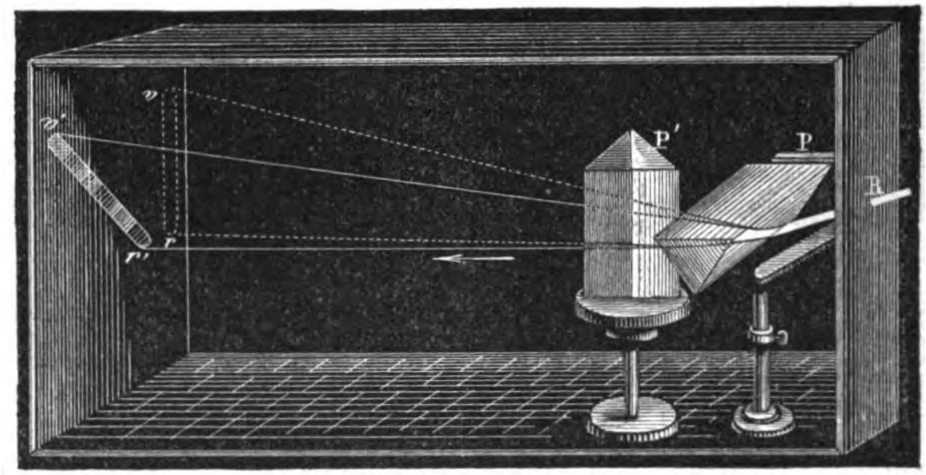

Fig. 117.

Let a second prism, (*fig.* 117.), having its refracting angle vertical, be now interposed, and the spectrum will be thrown into the oblique position *r' v'*, shown in the figure. The coloured space occupied by the spectrum in this case will not form, as in *fig.* 116., a series of ascending steps, but will be bounded by uniform and parallel lines, which is explained by the fact already stated, that the light composing each of the coloured spaces R, O, Y, &c. of the spectrum is not uniformly refrangible.

The rays which illuminate the red space R increase gradually in refrangibility from the extremity A to the boundary of the orange space; and in like manner, the rays which illuminate the orange space O increase gradually in refrangibility to the boundary of the yellow space; and so on.

Hence it is that the boundary of the image of the spectrum is a line uniformly inclined to A L. The divisions of the coloured spaces in the image correspond, however, with those of the spectrum, each colour in the image being vertically above the corresponding colour in the spectrum.

182. As the solar light is resolved by the prism into the various coloured lights exhibited in the spectrum, it might be expected that, these coloured lights being mixed together in the proportion in which they are found in the spectrum, white light would be reproduced. This is accordingly found to be the case. If the spectrum formed by the prism A B C, *fig.* 118., instead of being thrown

upon a screen, be received upon a concave reflector MN, the rays which diverged from the prism and form the spectrum will be reflected converging to the focus F; and after intersecting each other at that point, they will again diverge, the ray RF passing in the direction FR′, and VF in the direction FV′.

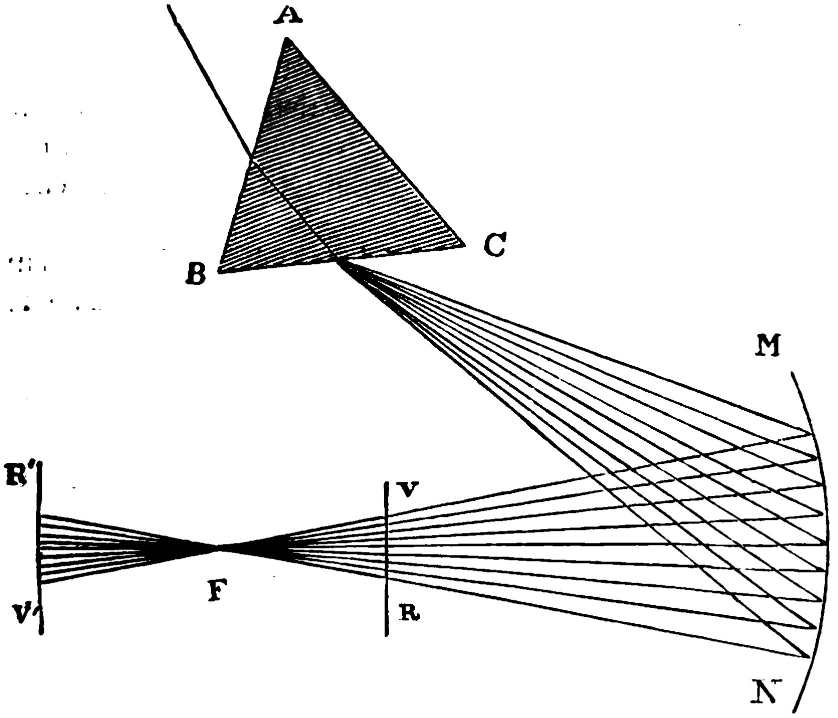

Fig. 118.

Now, if a screen be held between F and the reflector, the spectrum will be seen upon the screen. If the screen be then moved from the reflector towards the focus F, the spectrum upon the screen will gradually diminish in length, the extreme colours R and V approaching each other. When it comes so near to F that the extreme limits of the red and violet touch each other, the central point of the spectrum will become white; and when the screen arrives at the point F, the coloured rays being all mingled together, the spectrum will be reduced to a white colourless spot.

Just before the screen arrives at F, it will present the appearance of a white spot, fringed at the top with the colours forming the upper end of the spectrum, — violet, blue, and green; and at the bottom with those forming the lower end of the spectrum, — red, orange, and yellow. This effect is explained by the fact that until the screen is brought to the focus F, the extreme rays of the other end of the spectrum are not combined with the other colours.

If the screen be removed beyond F, the same succession of appearances will be produced upon it as were exhibited in its approach to F, but the colours will be shown in a reversed position.

As the screen leaves F, the white spot upon it is fringed as before, but the upper fringe is composed of red, orange, and yellow, while the lower is composed of violet, blue, and green; and when the screen is removed so far from the focus F as to prevent the superposition of the colours, the spectrum will be produced upon it, with the red at the top, and the violet at the bottom, the position being inverted with respect to that which the screen exhibited at the other side of the focus. These circumstances are all explained by the fact that the rays converging to F intersect each other there.

Similar effects may be produced by receiving the spectrum upon a double convex lens, as represented in *fig.* 119. The rays are made as before to converge to a focus F, where a white spot would be produced upon the screen. Before the screen arrives at F, and after it passes it, the same effects will be produced as with the concave reflector.

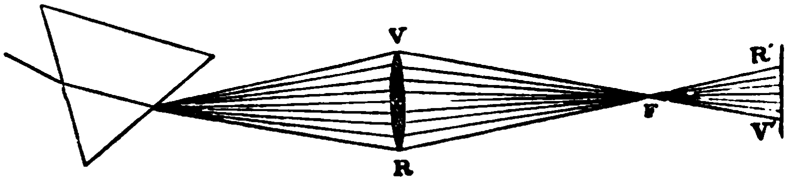

Fig. 119.

The proposition, that the combination of colours exhibited in the prismatic spectrum produces whiteness, may be further verified by the following experiment: —

Let a circular card be formed with a blackened circle, and its centre surrounded by a white circular band, and a black external border, as represented in *fig.* 120.

Let the circular disc be divided into seven spaces proportional in magnitude to the spaces occupied by the seven colours in the prismatic spectrum, those spaces being R, O, Y, G, B, I, and V. Let these spaces be respectively coloured with artificial colours resembling as near as practicable in their tints the colours of the spectrum. If the centre of this card be placed upon a spindle, and a very rapid motion of rotation be imparted to it, the ring on which the seven colours are painted will present the appearance of a greyish white. In this

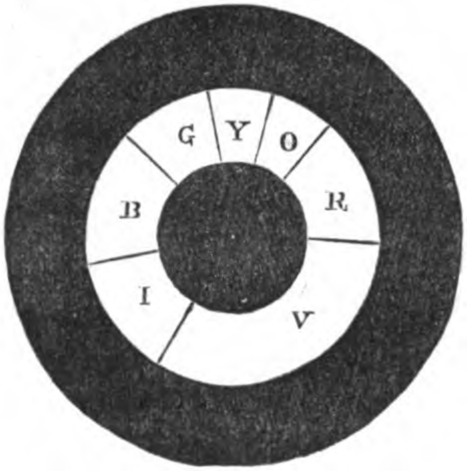

Fig. 120.

case, if all the colours except one were covered with black, the revolving card would present the appearance of a continuous ring of that colour; and, consequently, if all the coloured spaces be uncovered, seven continuous rings of the several colours would be produced; but these rings being superposed and mingled together will produce the same effect on the sight as if all the seven colours were mixed together in the proportion which they occupy on the card. If the colours were as intense and as pure as they are in the spectrum, the revolving card would exhibit a perfectly white ring; but as the colours of natural bodies are never perfectly pure, the colour produced in this case is greyish.

This experiment may be further varied by having uncovered any two, three, or more combinations of the colours depicted on the card. In such case the rotation of the card produces the appearance of a ring of that colour which would result from the mixture of the colours left uncovered: thus, if the red and yellow spaces remain uncovered, the card will produce the appearance of an orange ring; if the yellow and blue remain uncovered, it will produce the appearance of a green ring; and so on.

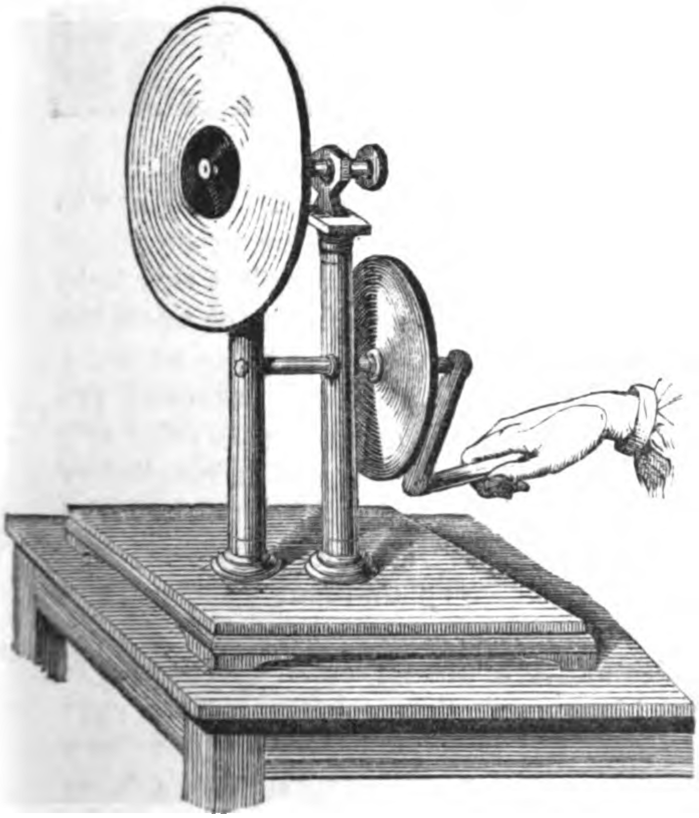

Fig. 121.

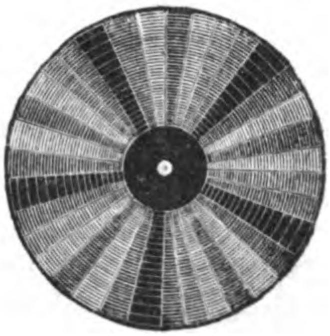

Fig. 122.

The usual apparatus for performing this experiment is shown in *fig.* 121., the coloured disc, before it is put in revolution, being represented in *fig.* 122. The disc is here supposed to be coloured in sectors diverging around a black central spot.

The following is a pretty experiment illustrating the recomposition of light, suggested by Newton: —

The spectrum is received upon seven plane reflectors, as shown in *fig.* 123., which are suspended in such a manner as to be capable of shifting the direction of their planes at pleasure. They are so adjusted as to receive the light proceeding from the prism which corresponds to the seven different colours, and to reflect this light to the same point upon a screen conveniently placed, or upon the ceiling of the room, the spot of light thus produced being white.

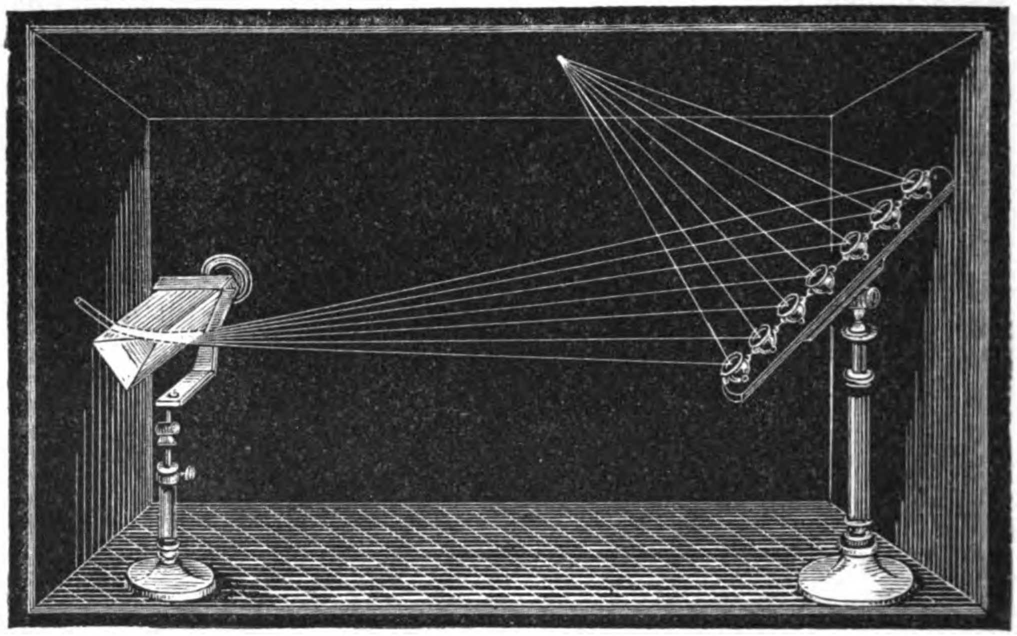

Fig. 123.

183. **Lights of the same colour may have different refrangibilities.** — Although the phenomena attending the prismatic spectrum prove that rays of light which differ in refrangibility also differ in colour, the converse of this proposition must not be inferred; for it is easy to show that two lights which are of precisely the same colour may suffer very different effects when transmitted through a prism.

Let us suppose two holes made in the screen in the middle of the space occupied by the blue and yellow colours, so that rays of these colours may be transmitted through the holes. Let these rays be received upon a double convex lens, and brought to a focus upon a sheet of white paper, so as to illuminate the spot G′ (*fig.* 124.). The colour that it produces then will be a green. Let another spectrum be now thrown by a prism upon the screen, and let a hole be made in the screen at that part of the green space where the tint is precisely similar to the colour produced at G′ on the

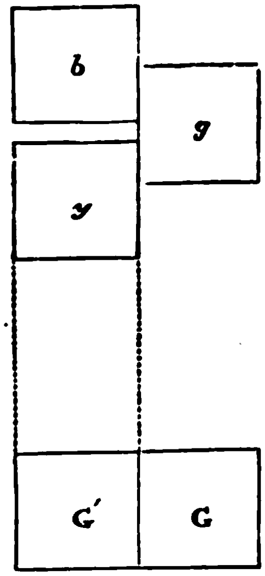

Fig. 124

white paper, and let the light which passes through this hole fall upon the spot G beside G′.

The spaces G and G′ will then be illuminated by lights of precisely the same colour; but it will be easy to show that these lights are not similarly refrangible. Let them be viewed through a prism having its refracting angle presented upwards. The image of the illuminated space G will be seen in a more elevated position at *g*; but two images will be produced of the space G′, one yellow and the other blue, at *y* and *b*, the yellow image *y* being a little below *g*, and the blue image *b* a little above it. Thus it is evident that the green light on the space G′ is a compound of yellow and blue, and is separable into its constituents by refraction, while the similar green light on the space G is incapable of decomposition by refraction.

184. **Colours produced by combining different rays of the spectrum.**—An endless variety of tints may be produced by combining in various ways the colours composing the prismatic spectrum; indeed, there is no colour whatever which may not be produced by some combination of these tints. Thus, all the shades of red may be produced by combining some proportion of the yellow and orange with the prismatic red; all the shades of orange may be produced by combining more or less of the red and yellow with each other and with the orange; all the shades of yellow may be produced by varying the proportion of green, yellow, and orange; and so on.

185. **Complementary colours.**—If two tints T and T′ be produced, the former T by combining a certain number of the prismatic colours, and the latter T′ by combining the remainder together, these two tints T and T′ are called *complementary*, because each of these contains just those colours which the other wants to produce complete whiteness; and, consequently, if the two be mixed together, whiteness will be the result. Thus, a colour produced by the combination of the red, orange, yellow, and green of the spectrum in their just proportions, will be complementary to another colour produced by the blue, indigo, and violet in their just proportions, and these two colours, if mixed together, would produce whiteness.

186. **Colours of natural bodies.**—Almost all colours, natural or artificial, except those of the prismatic spectrum itself, are more or less compounded, and their compound character belongs

to them equally when they have tints identical with the coloured spaces of the spectrum. Thus, a natural object whose colour is indistinguishable from the yellow space of the spectrum, will be found, when subjected to the action of the prism, to refract light in which there is more or less of green or orange; and an object which appears blue will be found to have in its colour more or less of green or violet.

187. Instead of receiving the spectrum on a screen, it may be viewed directly by placing the eye behind the prism A B C (*fig.* 125.), at L, so as to receive the light as it emerges. This mode of observing the prismatic effects is in many cases more convenient than by means of the screen, colours being thus rendered observable which would be too feeble to be visible after reflection from the surface of the screen. It is necessary, however, to consider that in this manner of viewing the prismatic phenomena, the colours will be seen in an order the reverse of that which they would hold on the screen; for if the eye be placed at L, it will receive the violet ray which enters in the direction L V as if such ray had proceeded from V′, and it will receive the red ray which enters it in the direction L R as if it had proceeded from R′; the red will therefore appear at the top, and the violet at the bottom of the spectrum, when the refracting angle B of the prism is turned downwards.

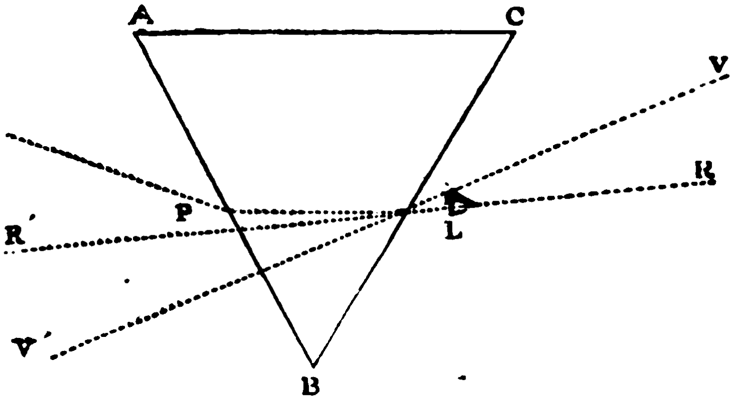

Fig. 125.

But if the refracting angle B be turned upwards, as represented in *fig.* 126., then the red will appear at the bottom, and the violet at the top of the spectrum, as will be perceived from the figure.

188. **Why objects seen through prisms are fringed with colours.** — In general, when objects are viewed through a prism they appear with their proper colours, except at their boundaries, where they are fringed with the prismatic tints in directions parallel to the edge of the reflecting angle of the prism.

Let A A M M (*fig.* 127.) be a small rectangular object seen upon a black ground, the sides A M being vertical, and A A and M M horizontal. Let us first suppose that this object has the colour of a pure homogeneous red. If this object be viewed through a

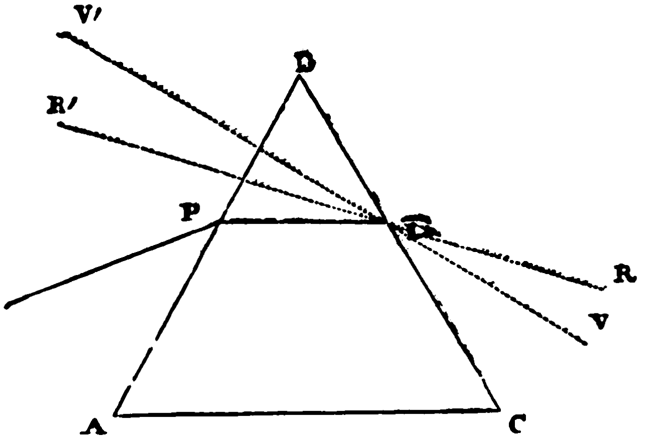

Fig. 126.

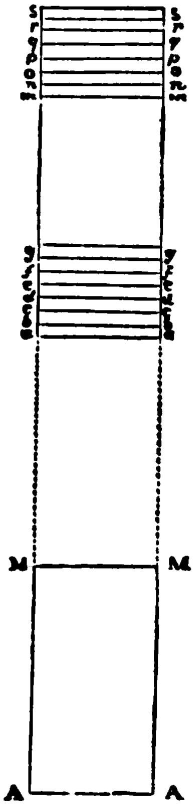

Fig. 127.

prism whose refracting angle is directed upwards with its edge horizontal, it will be seen in a more elevated position, such as *a a m m*, as already explained.

Let us next suppose that the object A A M M has the colour of a pure homogeneous orange. When viewed through the prism it will, as already explained, appear in a position *b b n n*, a little above *a a m m*.

If we next suppose the object A A M M to be coloured with homogeneous yellow, it will be raised by the prism to *c c o o*, a little above the orange image.

If it be next supposed to have the colour of a prismatic green, it will be seen at *d d p p*, a little above the yellow image; and if it be coloured light blue, its image will be seen at *e e q q*, above the green image; if it be dark blue or indigo, its image will be in the position *f f r r*; if it be violet, its image will be in the position *g g s s*.

Now, if we suppose the object A A M M to be white, that is to say, to have a colour which combines all the prismatic colours together, then all these several images will be seen at once through the prism in the respective positions already described. They will therefore be more or less superposed one upon the other, and the image will exhibit in its different parts those tints which correspond to the mixture of the colours thus superposed.

Hence it appears that the space between *a a* and *b b* from which all colour except the red is excluded, will appear red; in the space between *b b* and *c c*, in which the orange image is superposed upon the red image, a colour will be exhibited corresponding to the mixture of these two colours; in the space between *c c* and *d d*, the three images, red, orange, and yellow, are superposed, and a colour corresponding to the combination of these will be produced. In fine, the colours which are superposed between every successive division of the upper and lower edges of the combined images are as follows, where the prismatic colours are designated by the capital letters, and their mixture or superposition by the sign +:

Between *a a* and *b b*	R
„ *b b* „ *c c*	R + O
„ *c c* „ *d d*	R + O + Y
„ *d d* „ *e e*	R + O + Y + G
„ *e e* „ *f f*	R + O + Y + G + B
„ *f f* „ *g g*	R + O + Y + G + B + I
„ *g g* „ *m m*	R + O + Y + G + B + I + V = W.

Thus it appears that the space between *g g*, the bottom of the violet image, and the top *m m* of the red image is coloured with a white light, because in this space all the seven images are superposed.

In the space between *g g*, the bottom of the violet image, and *f f*, the bottom of the dark blue image, there is a space which is illuminated by all the prismatic colours except the violet, and this space consequently approaches so near a white as to be scarcely distinguishable from it. The space between *f f*, the bottom of the dark blue image, and *e e*, the bottom of the light blue image, is illuminated by all the colours except the dark blue and indigo, and it consequently has a yellowish tint. The succeeding divisions downwards towards *a a* become more and more red until they attain the pure prismatic red of the lowest division. The colours of the upper extremity of the image may in like manner be shown to be as follows: —

Between *s s* and *r r*	V
„ *r r* „ *q q*	V + I
„ *q q* „ *p p*	V + I + B
„ *p p* „ *o o*	V + I + B + G
„ *o o* „ *n n*	V + I + B + G + Y
„ *n n* „ *m m*	V + I + B + G + Y + O
„ *m m* „ *g g*	V + I + B + G + Y + O + R = W.

Thus it appears that the highest fringe at the upper edge is violet, that those which succeed it are formed by the mixture of violet and blue, to which green and yellow are successively added, until the colours become so completely combined that the fringe is scarcely distinguishable from a pure white. It is evident, therefore, that at the lower extremity the reds, and at the upper the blues, prevail.

If the object A A M M viewed through the prism be not white, then the preceding conclusions must be modified according to the analysis of its colour. Thus, if its colour be a green, it may be either a pure homogeneous green, or one formed by the combination of blue and yellow or other prismatic tints. In the former case the prism will exhibit the object without fringes, but in the latter it will be fringed according to the composition of its colour, determined by the same principles as those which have been applied to the object A A M M.

189. **Laws of refraction applied to compound solar light.** — The analysis of light, which has been here explained and illustrated, will enable us to generalise and extend the law of refraction explained in (93.).

Let AMB, *fig.* 128., be a transparent medium having a semi-cylindrical form, C being its centre. Let IC be a ray of solar light incident at C, the angle of incidence being ICO. This ray, on entering the transparent medium, will, according to what has been already explained, be resolved into an infinite number of other rays differently refracted, that which is least refracted being CR, and that which is most refracted being CV. The ray CR is red, and the ray CV is violet; the rays of intermediate colours and intermediate refrangibilities being included between them. The angle RCM is the angle of refraction of the extreme red ray corresponding to the angle of incidence ICO, and the angle VCM is the angle of refraction of the extreme violet ray corresponding to the same angle of incidence.

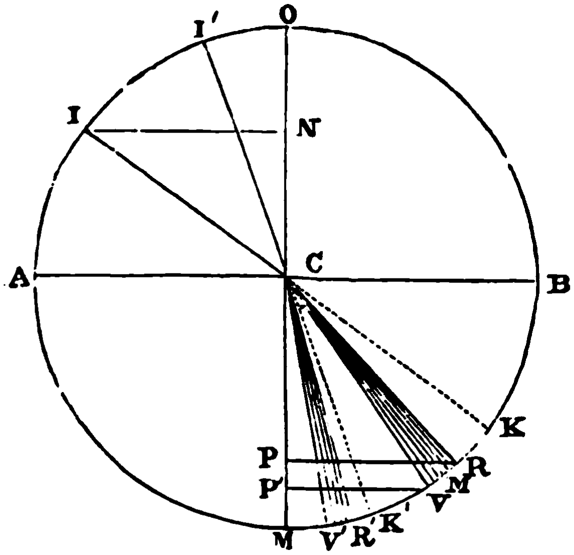

Fig. 128.

The index of refraction of the former will be found by dividing IN by RP; and the index of refraction of the latter will be found by dividing IN by VP'.

It is evident that the indices of refraction for the intermediate rays will be included between these two, being greater than the index of the extreme red, and less than the index of the extreme violet.

If the angle of incidence I C O be diminished, the angles of refraction R C P and V C P′ will be both diminished, since their sines will still bear the same ratio to the sine of the angle of incidence. Thus, if I′ C, *fig.* 128., be the incident ray, and I′ C O the angle of incidence, then C R′ will be the extreme red, and C V′ the extreme violet refracted rays, and the intermediate rays, into which the incident ray is resolved, will lie between these as before.

In this case, the angle R′ C V′ which measures the divergence of the extreme rays into which the incident ray is resolved, will be less than the angle R C V, which measures their divergence with the greater angle of incidence I C O. Thus it appears that the divergence of the decomposed rays is diminished as the angle of incidence is diminished, and increased as the angle of incidence is increased; but with the same angle of incidence this divergence is always the same in the same transparent medium.

The angle K C R, formed by the direction of any ray, such as C R, with the direction C K, which it would have followed had it not been refracted, is called the *refraction* of that ray.

Now it is necessary to distinguish carefully this term from the *angle of refraction* already defined.

Thus it appears that the refraction of the different rays into which the ray C I is resolved is different; that of the extreme red being K C R, and that of the extreme violet being K C V.

190. **Dispersion.** — The difference between the refraction of these extreme rays, or the angle of divergence R C V of the rays into which the original solar ray I C has been resolved by refraction, is called the *dispersion* produced upon the solar ray I C by the process of refraction. It follows from what has been just explained, that this dispersion in the same medium diminishes and increases as the angle of incidence, or the angle of refraction, or, in fine, as the refraction itself diminishes or increases.

191. **Mean refraction.** — But the term refraction, to have a definite meaning, in this case, must be applied to some one of the rays into which the solar ray is resolved, since each of these rays has a different refraction, varying from K C R to K C V. The middle ray, therefore, C M, of the rays diverging from C, is adopted for this purpose; and, accordingly, the ray C M is called the mean ray, and the angle K C M the mean refraction.

The refraction produced by any transparent medium upon a given ray at a given angle of incidence, is the measure of the refracting power of the medium on such ray; but as this refraction is always the difference between the angles of incidence and refraction, and as this difference may be taken to be proportional to the difference between their sines, we shall have the refractive power of the medium expressed thus:

$$\frac{\sin.\,\text{I}-\sin.\,\text{R}}{\sin.\,\text{R}}=n-1$$

where n expresses the index of refraction.

The measure, therefore, of the refracting powers of different media, is the number found by subtracting 1 from their index of refraction.

It follows from what has been explained, that in the same medium the dispersion increases and diminishes as the mean refraction increases or diminishes.

192. **Dispersive power.**—When different media are compared together, it is found that with the same mean refraction there will be different dispersions,—a fact which supplies a characteristic of different media, which has been called their *dispersive power;* one medium being said to have a greater or less dispersive power than another medium, according as the dispersion it produces with the same mean refraction is greater or less than that produced by the other medium.

The dispersion, therefore, produced by any medium being expressed by the difference of the indices of refraction n'' and n' of the extreme rays, and the refracting power being expressed by $n-1$, the absolute dispersive power is the quotient obtained by dividing the dispersion by the refracting power, and will be

$$\text{D}=\frac{n''-n'}{n-1}.$$

In the tables of refraction which have been given in page 68. the indices of refraction must be understood to refer to the mean ray of the spectrum, produced by the various media indicated in the tables.

To illustrate the application of this formula, let us take the case of crown glass and diamond. The index of refraction of the extreme and mean rays of crown glass are as follows:—

$$n''=1{\cdot}5466,\ n'=1{\cdot}5258,\ n=1{\cdot}5330;$$

consequently we shall have for crown glass,

$$\text{D}'=\frac{208}{5330}=0{\cdot}0390.$$

In like manner, the indices for diamond are

$$n''=2{\cdot}4670,\ n'=2{\cdot}4110,\ n=2{\cdot}4390;$$

therefore, we shall have

$$D' = \frac{56}{1439} = 0\cdot0389.$$

From whence it appears that although the refracting powers of the diamond and crown glass are as 3 to 1, their dispersive powers are the same.

This identity of their dispersive powers may be proved experimentally by taking two prisms, one of diamond and the other of crown glass, and producing with them two spectra in the manner represented in *fig.* 113., so that the mean ray L F of each shall be equally inclined to the direction P P′ of the incident ray. It will be found that the two spectra thus produced will have equal lengths, and consequently that the dispersions which correspond to equal refractions are equal.

Transparent media differ from each other, not only in the dispersive powers which they have on solar light, but also in the dispersive powers with which they act on the different elements which compose such light. Thus, for instance, it will happen that although two media, such as the diamond and crown glass, may have equal dispersive powers in relation to the compound light of day, they will have very different dispersive powers upon the several coloured lights of which such compound light is made up.

This may be rendered experimentally apparent by producing two spectra of equal lengths, with prisms of different materials.

If these two spectra be placed in juxta-position, so that their extremities shall coincide, although their coloured spaces will succeed each other invariably in the order already described, yet the boundaries which separate these coloured spaces will not coincide. The red in the one will be more or less extensive than in the other, and the same will be true of the other colours.

Let two spectra, A B and C D, *fig.* 129., be produced in this manner, equal in length, by two hollow prisms, one filled with the oil of cassia, and the other with sulphuric acid. In the spectrum A B, produced by the oil of cassia, the red, orange, and yellow spaces are less than in the spectrum C D, produced by the sulphuric acid, while in the latter the blue, indigo, and violet spaces are greater.

The middle ray *m n* in the spectrum A B passes through the blue space, while it passes through the green space in the spectrum C D. It appears, therefore, that in this case sulphuric acid has a greater

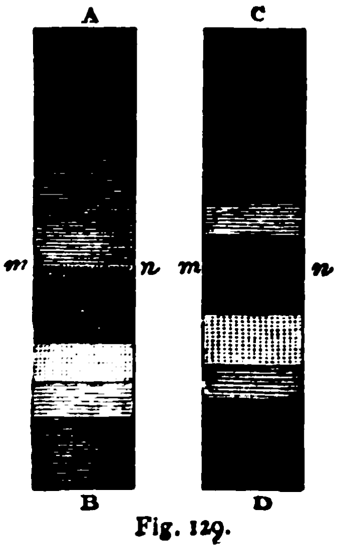

Fig. 129.

dispersive power upon the less refrangible rays, and a less dispersive power on the more refrangible rays, than the oil of cassia.

These effects are consequences of the fact, that although the indices of refraction of the extreme rays for any two media may be equal, the index of refraction of the intermediate rays may be unequal, and a difference of position of the corresponding colours in the spectrum will be the necessary consequence.

In the following table, the indices of refraction corresponding to the mean rays of the seven coloured spaces of the spectrum are given according to the experiments of Frauenhofer :—

193. *Table of the Indices of Refraction of the Mean Rays of each of the Prismatic Colours for certain Media.*

Refracting Substances.	n_1.	n_2.	n_3.	n_4.	n_5.	n_6.	n_7.
Flint glass, No. 13. -	1·627749	1·629681	1·635036	1·642024	1·648260	1·660285	1·671062
Crown glass, No. 9. -	1·525832	1·526849	1·529587	1·533005	1·536052	1·541657	1·546566
Water - - - -	1·330935	1·331712	1·333577	1·335851	1·337818	1·341293	1·344177
Water - - - -	1·330977	1·331709	1·333577	1·335849	1·337788	1·341261	1·344162
Solution of potash -	1·399629	1·400515	1·402805	1·405632	1·408082	1·412579	1·416368
Oil of turpentine - -	1·470496	1·471530	1·474434	1·478353	1·481736	1·488198	1·493874
Flint glass, No. 3. -	1·602042	1·603800	1·608494	1·614532	1·620042	1·630772	1·640373
Flint glass, No. 30. -	1·623570	1·625477	1·630585	1·637356	1·643466	1·655406	1·666072
Crown glass, No. 13. -	1·524312	1·525299	1·527982	1·531372	1·534337	1·539908	1·544684
Crown glass, letter M.	1·554774	1·555933	1·559075	1·563150	1·566741	1·573535	1·579470
Flint glass, No. 23. prism of 60° -	1·626596	1·628469	1·633667	1·640495	1·646756	1·658848	1·669686
Flint glass, No. 23. prism of 45° -	1·626564	1·628451	1·633666	1·640544	1·646780	1·658849	1·669680

194. The dispersion proper to each successive colour will be found by taking the difference of the two adjacent indices, and the total dispersion produced by each medium by taking the difference between the extreme indices. Thus the total dispersion produced by each medium given in the above table will be as follows: —

Flint glass, No. 13. - - -	0·043313	Flint glass, No. 3. - - -	0·038331
Crown glass, No. 9. - - -	0·020734	Flint glass, No. 30. - - -	0·042502
Water - - - - - -	0·013242	Crown glass, No. 13. - - -	0·020372
Water - - - - - -	0·013185	Crown glass, letter M. - -	0·024696
Solution of potash - - -	0·016739	Flint glass, No. 23., prism 60° -	0·043090
Turpentine - - - - -	0·023378	Flint glass, No. 23., prism 45° -	0·043116

195. In all that precedes, it has been assumed that the light composing each part of the prismatic spectrum is simple and homogeneous. This conclusion, deduced by Newton, and adopted generally by all physical investigators since his time, is based on the assumption, that light which, being refracted by transparent media, cannot be resolved into parts differently refrangible, is simple and homogeneous.

Sir David Brewster has, however, published the results of a series of observations, from which it would follow that a pencil of light which does not consist of parts differently refrangible, may, nevertheless, be resolved into parts which have different colours; in other words, that the light of certain parts of the spectrum, such, for example, as orange and green, although simple so far as respects refraction, is compound so far as respects colour. Thus, the orange light may be resolved into two lights equally refrangible, but different in colour, one being red and the other yellow; and the green light may in like manner be resolved into two equally refrangible, one being yellow and the other blue.

196. In a word, the observations and experiments of Sir David Brewster have led him to the conclusion that the prismatic spectrum consists in reality of three spectra of nearly equal length, each of uniform colour, superposed one upon another; and that the colours which the actual spectrum exhibits arise from the mixture of the uniform colours of these three spectra superposed. The colours of these three elementary spectra, according to Sir David Brewster, are red, yellow, and blue. He shows that by the combination of these three, not only all the colours exhibited in the prismatic spectrum may be reproduced, but that their combination also produces white light. He contends, therefore, that the white light of the sun consists not of seven, but of three constituent lights, — red, yellow, and blue.

This conclusion is established by showing that there is another method by which light may be resolved into its components, besides the method of refraction by prisms. In passing through certain coloured media, it is admitted that a portion of the light incident is intercepted at the surface upon which it is incident, and in its passage through the medium; a part only is transmitted.

Now, this property of colours is taken by Sir David Brewster as another method, independently of refraction, of decomposing colours. He assumes that such a medium resolves the light incident upon it into two parts: first, the part which it transmits; and, secondly, the part which it intercepts. He concludes that these two parts are complementary, that is to say, that each contains what the other wants to make up white solar light; or, more generally, that the incident light, whatever be its nature, must be assumed to be a compound, consisting of the light transmitted and the light intercepted.

This being assumed, let a coloured medium, such as a plate of blue glass, be held between the eye and the spectrum. Certain colours of the spectrum will be transmitted and others intercepted. If the colours of the spectrum be simple and homogeneous light, such as they are assumed to be in the Newtonian theory of the decomposition of light, then the consequence would be that the appearance of the spectrum seen through the coloured medium would consist of dark and coloured spots; those simple lights intercepted by the glass appearing dark, and those transmitted by the glass having their proper colour. But if each colour of the prism be, as is assumed in the chromatic theory, simple, then the plate of glass can make no change in its colour by transmission.

It must therefore be wholly transmitted, partly transmitted, or wholly intercepted. If it be wholly transmitted, no change will be made, therefore, in its colour or intensity; if it be partly transmitted, its colour will remain the same, but its intensity will be diminished; if it be wholly intercepted, the space it occupied on the spectrum will be black. But these are not the effects, as Sir David Brewster states, which are observed. He finds, on the other hand, that the coloured spaces on the spectrum are not merely diminished in intensity, but actually changed in colour. Now, if any space of the spectrum be changed in colour, it follows, from what has been stated, that the light transmitted must be a constituent of the colour of that space, to which the light intercepted being added, would reproduce the colour of the spectrum. By such an experiment as this, Sir David Brewster found that the parts of the spectrum occupied by the orange and green lights produced yellow, from which he inferred that the glass intercepted the red, which, combined with the yellow, produced orange; and the blue, which, combined with the yellow, produced green. But if the glass have the power of thus intercepting the red and blue light, it might be expected that the red and the blue spaces of the spectrum would appear dark. He accordingly found that the light of the middle of the red space was almost entirely absorbed, as was also a considerable part of the blue space.

From experiments like these, which he made in great number, and under various conditions, Sir David Brewster deduced the conclusion to which we have adverted above.

He inferred that at each point of the spectrum, red, yellow, and blue light are combined in various proportions, the colour of each part being determined by the proportional intensities of these three colours in the mixture. In the red space, the proportions of blue and yellow are exactly those necessary to produce white light, but the red is in excess; a portion of it combined with the blue and yellow produces a white light, which is reddened by the surplusage of red. In the same manner, in the yellow space the proportion of blue and red is that which is proper to white light, but there is a greater than the just proportion of yellow. A part of this, combining with the blue and red, produces white light, which is rendered yellow by the surplus. In the same manner exactly, the blue space is shown to consist of a surplusage of blue, combined with the proportion of red and yellow, and the remainder of the blue necessary for whiteness. The other colours of the spectrum, according to Sir David Brewster, are secondary, or the result of combinations of red, yellow, and blue.

The means by which these three primary colours produce the tints of the spectrum may be more clearly understood by reference to *fig.* 130., wherein M N represents the prismatic spectrum with its usual tints. The curve M R N represents the varying intensity of the red spectrum, M Y N that of the yellow, and M B N that of the blue spectrum. The distance of each part of these curves respectively from M N is understood to be proportional to the intensity of the colour of that part, and the relative lengths of the perpendicular included within each curve represents the proportion of the intensities of the combined colours. Thus, at the point P, the three colours are mixed in the proportion of the lengths of the perpendiculars *p n*, *p′ n*, *p″ n*, the first representing the proportion of yellow, the second red, and the third blue; the red and yellow predominating, the colour at this point will be orange.

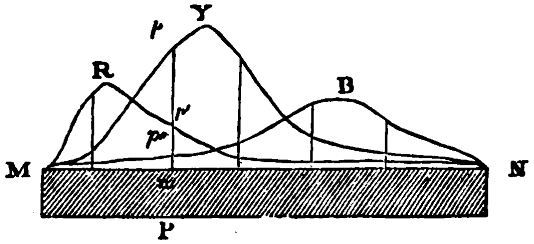

Fig. 130.

197. **Spectral lines.** — If the prismatic spectrum produced under certain conditions be examined by the aid of a telescope, it will be found to be crossed throughout its entire length by dark lines of various breadths. The total number of these lines is nearly seven hundred, and they are distributed over the spectrum without any apparent relation to the limits of its coloured spaces.

In *fig.* 131. M N represents a spectrum, M being its violet, and N its red extremity. The arrows to the left of the diagram represent the boundaries between the coloured spaces, these spaces being indicated by the letters R, O, Y, G, B, I, and V. The general distribution of the spectral lines is exhibited in the diagram.

It will be observed that in the distribution of these remarkable phenomena, there is no apparent regularity, either in their arrangement or in their intensity. In some places they are thickly crowded together, while in others they are separated by white spaces, more or less considerable. In some, the lines are extremely fine and scarcely visible; in others, they are of distinct breadth.

Among these numerous lines, seven were selected by their discoverer, Frauenhofer, as standards of reference or fixed points by which the position of the others could be designated. These seven are those marked on the right by the letters B′, C′, D′, E′, F′, G′, H′.

The first of these, B′, is in the middle of the red space; the second, third, and fourth, C′, D′, and E′, are nearer the boundaries

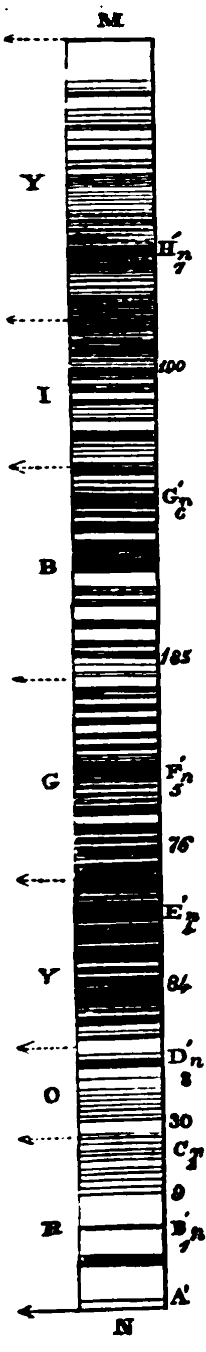

Fig. 131.

which separate the red and orange, the orange and yellow, and the yellow and green; the fifth, F, is near the middle of the green space, and the seventh near the middle of the violet space; while the sixth is near the boundary which separates the blue and indigo.

The numbers which appear in the diagram between each pair of these lines indicate the number of spectral lines which have been ascertained to exist between them. Thus, between B′ and C′ there are 9, between C′ and D′ 30, and so on; the entire number of lines between the first, B′, and the seventh, H′, being 574. The remainder of the spectral lines, between the extreme red and B′, and between the extreme violet and H′, amount to about 100, but they are more difficult of observation, and have not been so precisely ascertained.

A little above the extreme red, there is a well-defined dark line A′; and about half way between that line and the line B′, there is a dark band composed of seven or eight lines.

It was ascertained by Frauenhofer, that these lines are altogether independent either of the magnitude of the refracting angle, or of the matter of the prism; and that their number, order, and intensity are absolutely invariable, no matter what prism be used, provided only the light come through, directly or indirectly, from the sun.

198. The best method of observing these interesting phenomena is by means of telescopes and a prism, represented in *fig.* 132. Let a narrow slit be made in a window-shutter or a screen, so as to admit a broad thin beam of the sun's light. This slit is represented in section at right angles to its length at *o*. The beam of light is received on a prism. After being refracted by the prism, it is received by a small telescope, which plays upon a graduated arc, on which is a second telescope to indicate the original direction of the ray *o p*. The angle under the two telescopes will indicate the refraction which the ray has suffered by the prism. The prisms used in these observations should be of the purest and finest flint glass, perfectly free from threads and striæ. The prism ought to be placed at a distance of fifteen or twenty feet from the telescope.

By turning the telescope or the prism, the successive rays of the spectrum are made to pass through the telescope, so that the spectrum may be viewed successively from one extremity to the other. The telescopes suited to these observations should magnify from eight to ten times.

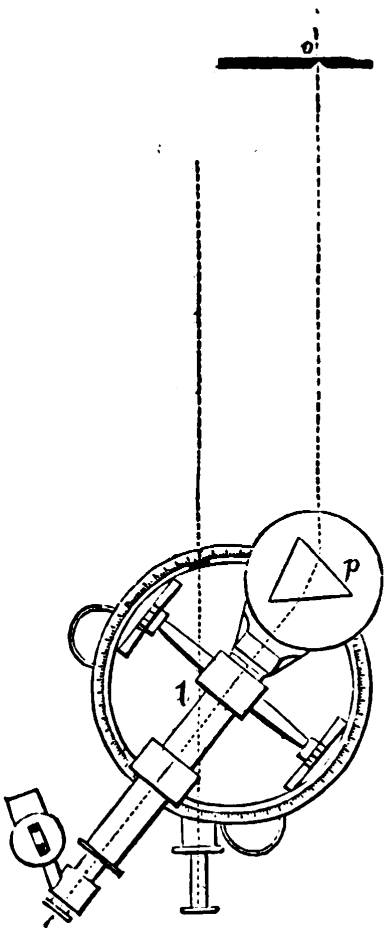

Fig. 132.

By these means the spectra produced not only by solar light but also by various artificial lights, have been observed. The electric light gives the spectral lines bright instead of dark, one of the most remarkable for its brilliancy passing through the green space. The flame of a lamp, whether produced by gas, oil, or spirits, also gives the spectral lines bright. Two of these are especially distinguishable in the red and orange spaces.

The moon and planets have the same dark lines as the sun, but less easily distinguishable, especially near the extremities of the spectrum. The spectra produced by the light of the fixed stars are marked with dark lines, but little different in their number, intensity, and disposition from those exhibited in the solar spectrum. It is remarkable that the spectra produced by different fixed stars differ from each other.

199. **Researches of Sir D. Brewster.**—Sir David Brewster made a considerable number of interesting experiments on the spectral lines produced by coloured stars, as well as those of the solar spectrum produced by various transparent media, a detailed account of which will be found in the "Edinburgh Transactions," vol. vii., the "Reports of the British Association for 1842 and 1847," and the "Proceedings of the French Institute for 1850." Our limits preclude us from entering into these details.

200. **Use of the spectral lines as standards of refrangibility.**—The invariable position which Frauenhofer's lines are found to have in the solar spectrum has rendered them eminently useful for establishing standards of refrangibility of the component parts of solar light. From what has been stated respecting the gradual variation of the tints composing the solar spectrum, it may be easily understood that much uncertainty will attend any methods of defining a particular ray to which a certain index of refraction

is imputed. Thus the middle of the red or the middle of the green space is necessarily an indefinite term, so long as the limits of these spaces admit of no exact definition.

The seven lines B′, C′, D′, &c., which have been already noticed, have been accordingly adopted as points invariable in their position, of which the indices of refraction once determined may always serve as standards of reference. The indices accordingly which have been given in table, p. 141., are those which belong to these points, n_1 being the index of refraction at B′, n_2 that of the rays at C′, n_3 at D′; and so on.

201. **Relative intensity of light in different parts of the spectrum.**—Frauenhofer also ascertained by photometric observations the relative intensity of the light in different parts of the spectrum.

The result of these observations is denoted by the curve marked "Luminous intensity," in *fig.* 133.; the perpendicular distance of each point of this curve from the edge of the spectrum being proportional to the brilliancy of the light produced by a flint glass prism. It appears from this that the most intense illumination corresponds to a point about the middle of the yellow space.

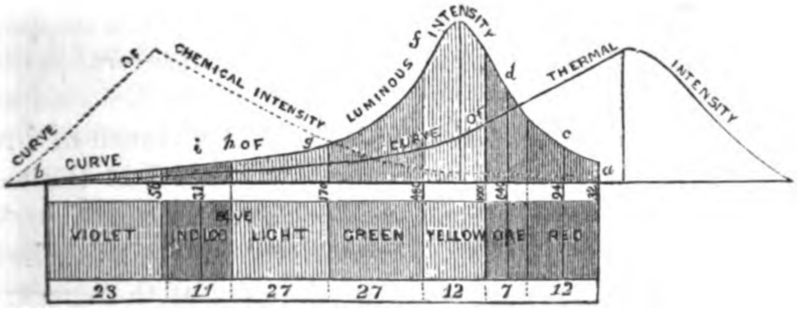

Fig. 133.

In the following table are given the numerical intensities of the other points, the light of the point of greatest intensity being expressed by 1000:—

At the red extremity	000
At B′	32
At C′	94
At D′	640
At E	480
At F	170
At G	31
At H	5·6
At violet extremity	000

Frauenhofer found that the most intense part of the spectrum divided its entire length in the proportion of 1 to 3½, being nearer the red than the violet extremity; and that the point of mean intensity is nearly in the middle of the blue space. As a great part, however, of the violet end is never seen except under extraordinary conditions, these results are not applicable in common experiments.

202. **Calorific analysis of the spectrum.** — The heating power of the light composing the different parts of the spectrum was examined first by the late Sir William Herschel, and later by Sir H. Anglefield, M. Berard, Sir H. Davy, MM. Seebeck, Wunsch, and, in fine, by M. Melloni, who has supplied a vast body of interesting experiments on this subject. The general result of these observations, the details of which would be inadmissible here, are as follows: —

The heating power, being nothing at the violet extremity, augments gradually as the thermometer is moved to the red extremity.

At this point, or near it, the heating power is a maximum; but the presence of thermal rays beyond the red extremity is manifested by the thermometer, which, though it declines on being moved beyond this extremity, continues to show a temperature greater than that of the surrounding air, to a considerable distance from the spectrum.

We are therefore compelled to admit the existence of invisible rays of light below the red extremity, which affect the thermometer, though they do not sensibly affect our organs.

The curve marked "Thermal intensity," in *fig.* 133., indicates the variation of the heating power of the rays of the spectrum in the same manner as the former curve represented the luminous intensity. The point of maximum thermal intensity is, according to some, at the red extremity, and, according to others, a little below it; but it is found that this depends in some degree upon the material composing the prism.

Sir John Herschel has shown, by some recent experiments, that by concentrating the invisible thermal rays they can be rendered visible, and that when visible their colour is lavender grey.*

203. **Seebeck's experiments.** — M. Seebeck has shown that the point of maximum calorific intensity varies with the material of the prism in the following way: —

Substance of the Prism.	Coloured Space in which the Heat is a Maximum.
Water	Yellow.
Alcohol	Yellow.
Oil of turpentine	Yellow.
Sulphuric acid concentrated	Orange.
Solution of sal ammoniac	Orange.
Solution of corrosive sublimate	Orange.
Crown glass	Middle of the red.
Plate glass	Middle of the red.
Flint glass	Beyond the red.

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* Philosophical Transactions, 1840.

The observations on alcohol and oil of turpentine were made by M. Wunsch.

204. **Herschel's experiments.**—More recently still Sir John Herschel has made an interesting series of experimental researches on the thermal properties of the spectrum, by trying the varying effects of its power when thrown upon a sheet of the thinnest writing paper smoked or blackened by Indian ink, on one side, and soaked on the other with rectified spirits of wine, the effect of which is to make it uniformly black. The spectrum being shown upon the wetted side of a sheet of paper thus prepared, the calorific intensity of its different parts is manifested by the varying degree of its bleaching power upon the paper produced by the evaporation of the alcohol.

The result of observations made in this way was, that the thermal influence extended over the whole length of the spectrum, but at a point α considerably beyond the limit of the extreme red, the heating power is a maximum, having gradually increased in ascending from the lowest limit to this point The calorific intensity then diminishes slightly for a short space, and, again increasing, attains a second maximum β. It then diminishes until it ceases altogether, after which it again increases until it attains another maximum γ, after which it again diminishes, vanishes and reappears, and increases until it attains a fourth maximum δ. After this a fifth maximum ϵ is more faintly indicated.

Supposing the length of the visible spectrum to consist of 57 equal parts, 43 of which are above and 14 below a certain line D drawn through the yellow space, the positions of the several calorific maxima will, according to Sir John Herschel, be as follows:—

$\alpha = 18{\cdot}2$ parts from the line D.
$\beta = 26{\cdot}7$ „
$\gamma = 35{\cdot}7$ „
$\delta = 45{\cdot}1$ „
$\epsilon = 55$ „

The thermic spectrum thus extending the whole length of the luminous spectrum beyond the line D.

With a crown glass prism and lens, "the insulation of γ," says Sir John Herschel, "was much less sensible, and the separation of α and β hardly to be perceived. This would go to point out the flint glass as the origin of the spots, and to that idea I rather incline."

With a prism of pure water, and also with one of a saturated solution of muriate of lime, the spot γ was greatly enfeebled, and δ invisible.

Green glasses cut off nearly the whole thermic spectrum.

205. **Chemical analysis of the spectrum.** — The action of light in changing the colour of certain substances has long been known; but one of the most remarkable of this class of objects has lately acquired increased interest from its application in the art called Daguerreotype.

If the chemical substance called muriate of silver be exposed to solar light, it will be blackened. Now, in order to ascertain whether this effect is due collectively to all the rays composing solar light, or is caused by the action of some rather than other rays, it is only necessary to expose it successively to all the rays composing the prismatic spectrum.

If this be done, it will be found that the least refrangible rays near the red extremity do not produce this effect in any sensible degree, while the more refrangible rays at the violet extremity produce it in a very great degree; in a word, by ascertaining and indicating the intensity of this chemical action in the same manner as the intensities of the illuminating and heating powers have been already expressed, we shall be enabled to determine the curve of chemical intensity indicated in *fig.* 133., from which it appears that this action is at its maximum near the boundaries between the violet and the indigo.

Since the importance of the art of Photography has been developed, a great number of experimental researches of more or less interest have been made upon the chemical effects of the various component parts of solar light. The necessary limits of this work precludes even an examination of these; but it may be stated that M. Edmund Becquerel has made experimental researches of considerable importance upon the effects of the spectrum on iodide of silver, chloride of gold, chromic acid, guaiacum, tournesol, and sulpho-cyanurate of iron; varying the experiments by transmitting the light through transparent screens uncoloured and variously coloured, composed of different materials, such as quinine, creosote, blue and yellow glass, &c. The details of these experiments will be found in the "Annales de Chimie et de Physique," for 1843.

206. **Herschel's experiments.** — But the most important experimental researches made on this subject are those of Sir John Herschel. He produced a visible spectrum, the total length of which expressed in 30ths of an inch was 53·92, and the position of the line, which he called D, was at a distance of 40·62 of these parts from the upper or violet extremity, and consequently 13·30 from the lower, or red extremity; — distinguishing the distance above this line D by +, and those below it by —. The following are the results obtained.

I. *Nitrate of silver.* — Paper washed with a solution, specific gravity 1·132. The colour of the spectrum impressed upon this paper by the chemical rays was pale *brown*, inclining to *pink*, and the most intense part was about the middle of the blue ray. The total length was 85 parts, and it terminated at the line D. The point of maximum intensity was 23 parts on the violet side of D.

II. *Nitrate of silver with muriate of soda.*—The paper was first washed with the nitrate, specific gravity 1·132, then with the muriate, 1 salt + 19 water, and again with the nitrate, specific gravity 1·096. The spectrum impressed upon this paper is more variously coloured than any other. The tint is a pretty light *red* at −7·6, beginning to pass into *green* at −3·8, through a kind of livid mixed tint. The best *green*, however, which is of a sombre and dull character, is developed a little above (+) D, and covers a breadth of about 4 parts. From that point, with a barely perceptible tinge of dark *blue*, it becomes rapidly an intense black, which, at +80, dies away into a *purplish brown*, and terminates the spectrum at +90·23, the whole length of the chemical spectrum, or the discoloured impression, being +97·83 parts.

III. *Nitrate of silver, as before, with hydro-bromate of potash, instead of muriate of soda.*—The spectrum impressed upon paper thus prepared is a most extraordinary one. *The instant the rays fall upon it* the action begins over its whole length, and the intensity is the same everywhere but just at the extremities, where it gradually dies away. It extends, too, all the way to the extremity of the visible *red* rays. Its tint is a *greyish black*. At the red extremity a contrary or oxydising action now commences, producing *whiteness* on the paper, and extends to −22·67. Hence the extent of the chemical beyond the luminous spectrum is −9·37. The most refrangible extremity of the darkest portion is +90·50. The total length of the darkened portion is 105·55; and the whole length of the paper visibly affected, 116·77.*

207. **Chromatic aberration.**—It appears from what has been explained, that the focal length of a lens, other things being the same, depends on and increases with the index of refraction. But it also appears that the index of refraction is different for the different component parts of light. It follows, therefore, that the focal length of a lens will vary according to the light which falls upon it.

Supposing the radii of the surfaces to be expressed as before by r and r', the focal length being F, we shall have, according to what has been already proved,

$$F = \frac{r\,r'}{(n-1)\,(r-r')},$$

---

* Phil. Trans. 1840, Part I. p. 26.

Since n is greatest for the violet and least for the red rays, it follows that the value of F corresponding to the violet will be less than its value corresponding to the red, and that it will have intermediate values for all the intermediate colours.

If a succession of objects, therefore, having the successive colours of the spectrum, be placed before a lens A C (*fig.* 134.), at the same distance from it, their images will be produced at different distances from the lens, that of the violet object being nearest and that of the red most distant from the lens, the images of the objects of intermediate colours being at intermediate distances, as shown in the figure.

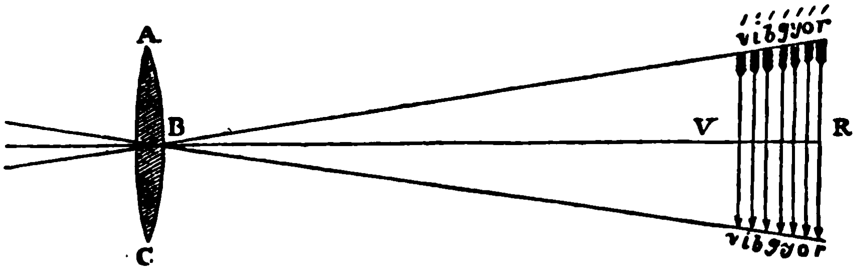

Fig. 134.

Now if, instead of a series of objects thus coloured, a white object, or the sun itself, be placed before the lens, the various component parts of the light which it transmits will be brought by the lens to different foci corresponding to their various degrees of refrangibility, and the lens will accordingly produce, not one white image, but an infinite number of coloured images included between the extreme positions V and R. Each ray will form an image, having a position and colour corresponding to its degree of refrangibility, and the space included between V and R will be a truncated cone filled with images, which increase in magnitude from V to R, and which, beginning with a violet colour at V, pass through all the tints of the spectrum; the last image at R having a red colour corresponding to the red of the extreme light of the spectrum.

A white screen held at R would exhibit a well-defined red image of the object, if it did not also receive upon it the pencils of rays forming all the other images between R and V, such pencils diverging from the various points of such images. Thus, a pencil which is brought to an exact focus upon the image $o\,o'$, would form upon a screen placed at $r\,r'$, not a point, but a small spot of orange light. In like manner, a pencil whose focus lies upon the image $y\,y'$ would form upon a screen placed at R a small spot of yellow light, greater in magnitude than the spot of orange light, because of the greater distance of its focus from the screen. In like manner, the points upon the image *g g′*, *b b′*, *i i′*, and *v v′*, would produce upon the screen at *r* luminous spots of green, blue, indigo, and violet light, increasing in magnitude in proportion to their respective distances from the screen.

208. We have assumed in the preceding examples that the lens is a converging lens; and, consequently, that the image of a distant object is real, and may be exhibited on a screen. If, however, the lens be a diverging lens, the effects will be the same, but the image, being imaginary, cannot be exhibited in the same manner.

Let the object, as before, be placed at such a distance from the lens A C (*fig.* 135.), that the pencils proceeding from it may be considered as parallel. After passing through the lens, they will diverge, as if they had proceeded from an object placed at a distance before the lens, equal to its focal length. Thus, if the object emit red light, the rays, after passing through the lens, will diverge as if they had proceeded from *r r′* at the distance B R, equal to the principal focal length corresponding to the index of refraction of red rays; and, in like manner, if the object transmit violet rays, the light, after passing through the lens, will diverge as if it had proceeded from points in an object placed at *v v′*, and for the intermediate colours it would diverge as if it had proceeded from intermediate points between R and V.

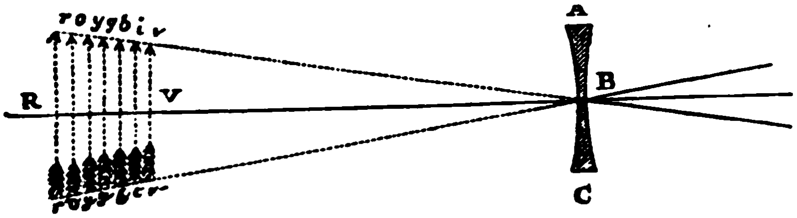

Fig. 135.

Thus, if, as before, the object be supposed to emit white solar light, the rays, after passing the lens, would diverge from points between R and V, varying according to their refrangibilities in the manner already expressed.

It appears, therefore, that in the case of a diverging, as well as in that of a converging lens, the violet image is formed nearest to, and the red most distant from, the lens; the difference being that the image is imaginary in the one case, and real in the other. This position, however, will be reversed whenever an imaginary image is formed by a converging, or a real one by a diverging lens. In these cases the red image will be nearest to, and the violet one most distant from, the lens, as is easily shown.

Let us suppose that the object is placed within the principal focus of a converging lens; in that case the image will be imaginary.

The pencils of rays proceeding from the various points of the object, after passing through the lens, will be still divergent, but less so than before. They will, in fact, diverge from points more distant from the lens than the object, and these will be the points of the imaginary image. Now, since the effect of the lens upon the violet rays will be greatest, and upon the red rays least, the divergence of the former will be most, and that of the latter least diminished, and, consequently, the red imaginary image will be nearest to, and the violent most distant from, the lens, the images of intermediate colours being at intermediate distances.

Thus the position of the series of coloured images relative to a converging lens is reversed according as the object is within or beyond the principal focus; if it be beyond that focus, the violet is nearest to, and the red most distant from, the lens, the images being real; if it be within the principal focus, on the contrary, the red is nearest to, and the violet most distant from, the lens, and the image is imaginary.

In whatever position an object may be placed with relation to a diverging lens, the violet image is nearest to, and the red most distant from, the lens, the images being imaginary.

If, however, a diverging lens receive rays, which are converging towards the points of a real image, it will either diminish their convergence or render them parallel or divergent, according as the real image towards which the rays converge is within, at, or beyond the principal focus of the lens. If it be within the principal focus, the rays, after passing through the lens, will be still convergent though less so than before, and a real image will be formed more distant from the lens than that to which the rays had been convergent. But the convergence of the violet rays being most, and that of the red rays least, diminished by the lens, the red image will be nearest to, and the violet image most remote from, the lens, the images of intermediate colours having intermediate positions.

If the real image towards which the rays converge be at the principal focus, the rays, after passing through the lens, will be parallel, and will form no image, either real or imaginary.

If the real image towards which the rays converge be beyond the principal focus, they will diverge after passing the lens, and the divergence of the violet rays will be greatest, and that of the red rays least. The points, therefore, from which the violet rays will diverge will be nearest to, and that from which the red rays diverge most distant from, the lens. A series of imaginary images will therefore result, the violet being nearest to, and the red most remote from, the lens.

By following out these principles, and applying them to all the particular cases which may arise, the student will find no difficulty in tracing the various positions of the images.

From all that has been explained, it follows that the images of objects produced by lenses, whether convergent or divergent, can only be single when the objects are illuminated by homogeneous light. But when they are, as all natural objects are, illuminated by light more or less compounded, a lens will produce as many distinct images as there are constituents in the compound light proceeding from the object. If the object be white, a series of images will be produced having all the tints of the prismatic spectrum. If it be coloured, the series will consist of the several coloured images which correspond to the constituents of the light proceeding from the object. If, in fine, the object be variously coloured, each part of it will have a distinct series of images corresponding with the constituents of its colour.

If the image produced by a lens be received upon a screen, it will, therefore, exhibit a confused representation of the object; the colours diffused over the internal parts of its area being those which, combined together, form white light, the general area of the image will not be coloured; but the coloured pencils thus mingled together, being none of them brought to their foci on the screen, except those of the extreme red light, a confusion will ensue. At the edges there will be coloured fringes, because at the edges the pencils diverging from the edges of the series of images do not overlay each other as they do at the central pencils; and, consequently, the colours necessary for the production of white light are not mingled in these pencils.

The consequence of all this is, that there will be formed upon the screen an image of the object, everywhere indistinct, and fringed with prismatic colours at its edges.

The degree of indistinctness and the breadth of the fringes will depend upon the length of the space V R; that is to say, upon the *dispersion* produced by the lens, and also upon the difference between the magnitudes of the extreme images $r\,r'$ and $v\,v'$, which latter depends upon the opening of the lens r B r', and on the dispersion V R conjointly.

The consequence of this is, the indistinctness of the image and the coloured fringes arising from this cause increase as the focal length of the lens diminishes, as its opening increases, and as the dispersive power of the material of which it is composed increases.

From all this it might be inferred that the optical utility of lenses would be utterly destroyed in the case of all objects save such as would emit or reflect homogeneous light. For if such a multitude of variously coloured images be formed at various distances from the lens, the effect which would be produced upon a card held at any distance whatever might be supposed to be a confused patch of coloured light, having no perceptible resemblance in form or colour to the object; and such would certainly be the case if the distances of the several images, one from another, were considerable. These distances, however, are so small, and the coloured images are so blended together, that the decomposition of their colours appears principally by coloured fringes produced upon their edges, and in general upon the outlines of their parts. Nevertheless, when these false lights and fringes are magnified, the general appearance of the object under observation would be so changed as to colour, and so indistinct as to outline, as to be rendered useless for all the purposes of scientific inquiry.

The indistinctness of the image thus produced, is called *chromatic aberration*, from the Greek word χρωμα (chroma) signifying *colour*.

The extent of the chromatic aberration produced by a lens measured by the interval between the red and violet images, is called the *dispersion* of the lens.

209. **Achromatic expedients.**—It appears, therefore, that no single lens can produce an image of an object free from coloured fringes; but by combining two or more lenses of which the dispersions are equal and opposite while their refractions are unequal, a refracting lens may be obtained free from dispersion, and which, therefore, will produce optical images of objects exempt from coloured fringes.

To make this more clear, let L and L′ be two lenses, whose form and material is such that the distances between the violet and red images of the sun which they would produce shall be equal, the violet being nearer to, and the red most distant from, the one lens, while the red is nearest to, and the violet most distant from, the other. It is clear that in this case the dispersion produced by the lens L will be equal and contrary to that produced by the lens L′, and that, therefore, they will destroy each other, just as positive and negative algebraical quantities destroy each other when equal.

If the lenses L and L′ in this case were made of the same material, the condition which imposes equality and opposition on their dispersions, would also impose equality and opposition on their refractions; and, consequently, the refractions would mutually neutralise each other as well as the dispersions, and the combination would be equivalent to a disc of plane glass.

But if the lenses L and L′ be made of different materials, they will necessarily produce unequal dispersions with the same refraction, or unequal refractions with the same dispersion. Thus, for example, if L L and L′ L′, *fig.* 136., be two lenses of different materials, so formed that the violet images v v′ of the same white object o o and o′o′, placed at equal distances from them at one side, shall be produced at equal distances from them on the other,

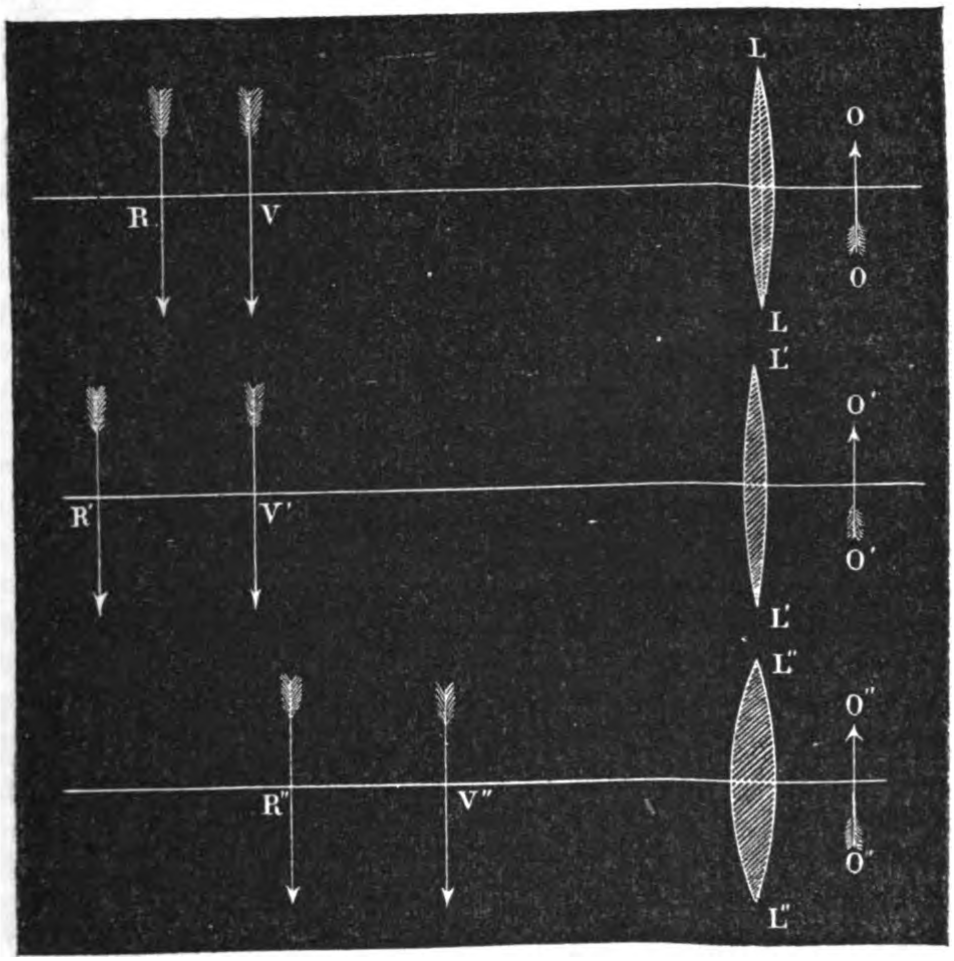

Fig. 136.

the red images R R′ will be produced at unequal distances from them, the lens whose material has the greatest dispersive power throwing, in that case, the red image R′ to a greater distance from the lens. Thus, while the two lenses produce the violet images at equal distances from them, they will produce all the other coloured images at unequal distances.

But since, with the same material, the increase of the refraction by increase of the convexity will be attended with a corresponding increase of dispersion, it is evident that by increasing the convexity of the lens L L, the coloured images will be formed nearer to it, and farther from each other; and, consequently, such a curvature L″L″ may be given to it, that the distance V″R″ between the violet and the red images shall be equal to the distance V′R′ between the violet and red images produced by the lens L′L′.

Thus, while L L and L′L′ are two lenses of equal refraction and unequal dispersion, L′L′ and L″L″ are two lenses of unequal refraction and equal dispersion.

In these examples we have selected cases in which the lenses compared together produce dispersions in the same direction, the violet image being, in each case, nearest to, and the red most distant from, the lens. Let us now consider the case in which the two lenses have opposite and equal dispersions.

210. **Achromatic compound lens.** — Let L′L′, *fig*. 137., be a diverging lens, and let it be supposed to receive rays proceeding from a white object, which, if not intercepted, would produce a real image of the object at a point O′, within the focal distance of the lens L′L′. In that case, the lens L′L′, according to what has been explained, will produce a series of coloured images of the object at a greater distance from the lens, the red image R′ being nearest, and the violet image V′ most distant from, the lens, the dispersion being R′V′. Now, this dispersion may be increased or diminished by increasing or diminishing the concavity, or the diverging power of the lens L′L′. It is evident, therefore, that such a form may be assigned to the lens L′L′ as will give the dispersion R′V′ any desired magnitude.

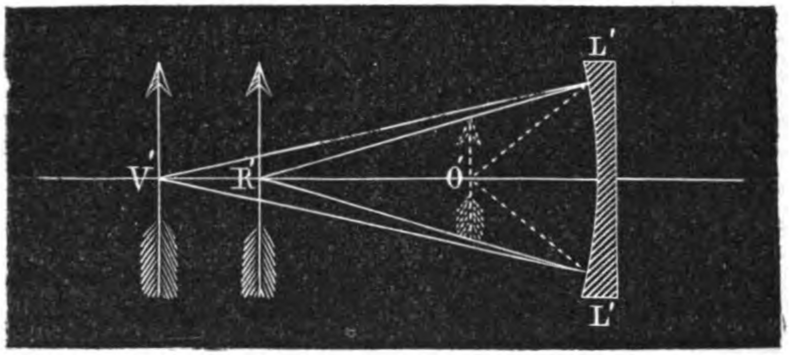

Fig. 137.

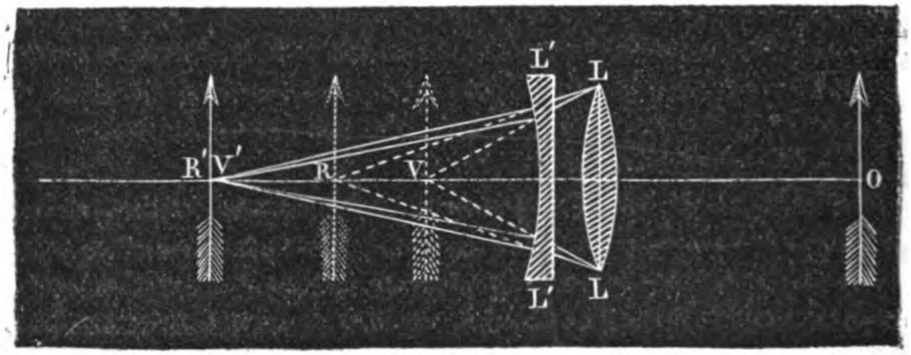

Fig. 138.

Let L L and L′L′, *fig*. 138., be two lenses made of different materials, the former being a convergent, and the latter a divergent, lens. Let O be a white object placed at such a distance from the lens L L, that its violet and red images would be formed at V and R, the distance V R being therefore its dispersion. But instead of allowing the rays transmitted through the lens L L to form this series of images, we will suppose them intercepted by the lens L′L′, and since the images would fall within its focal length, the effect of L′L′ will be to throw the images to a greater distance from it; but its effect upon the violet image V will be so much greater than its effect upon the red image R, that the distance of V from the lens will be more increased than that of R by a space exactly equal to V R, and, consequently, the two images will be made to coalesce, and the system will thus be rendered, for all practical purposes, achromatic. We say for all practical purposes, inasmuch as, although the conditions here supposed will produce the coincidence of the red and violet images, they will not rigorously produce the coincidence of all those of the intermediate colours. Nevertheless, the general effect will be the production of an image sensibly exempt from chromatic confusion.

211. **Mathematical formulæ.** — Such are the general principles upon which compound lenses, exempt from chromatic aberration, are produced. Such lenses are said to be *achromatic*, and the principles upon which they are constructed constitute the theory of *achromatism*.

Having explained these general principles, it may be satisfactory, in a question of such importance, to supply also the more rigorous mathematical details of its solution.

To put the question under its most simple form, let it be required to find what form must be given to two lenses composed of media having different refracting powers, so as to render the focal length of the compound lens for light of any one refrangibility, equal to its focal length for light of any other refrangibility.

Let F' and F'' be the focal lengths of the two lenses for light, of which the indices of refraction are n' and n'' for the media composing the lenses respectively.

Let f' and f'' be their focal lengths for light of which the indices of refraction are m' and m''.

Let F be the focal length of the compound lens.

The converging power of the compound lens on each kind of light will be equal to the sum of the converging powers of the two lenses separately on the same kind of light. The converging power of the compound lens, therefore, on the light whose indices of refraction are n' and n'', will be

$$\frac{1}{F'}+\frac{1}{F''};$$

and in like manner its converging powers on the light whose indices of refraction are m' and m'', is

$$\frac{1}{f'}+\frac{1}{f''}.$$

But since, by the supposition, these two converging powers must be rendered equal, we shall have

$$\frac{1}{\mathrm{F}'}+\frac{1}{\mathrm{F}''}=\frac{1}{f'}+\frac{1}{f''}.$$

The question is, then, to assign such magnitudes to the radii of the surfaces of the lenses as will make them fulfil this condition.

Let R_1 and R_2 be the radii of the surfaces of the first, and r_1 and r_2 those of the surfaces of the second lens. We shall then have, by the formulæ given in (147.),

$$\frac{1}{\mathrm{F}'}=\frac{(n'-1)\,(\mathrm{R}_1-\mathrm{R}_2)}{\mathrm{R}_1\,\mathrm{R}_2},\quad \frac{1}{\mathrm{F}''}=\frac{(n''-1)\,(r_1-r_2)}{r_1\,r_2};$$

$$\frac{1}{f'}=\frac{(m'-1)\,(\mathrm{R}_1-\mathrm{R}_2)}{\mathrm{R}_1\,\mathrm{R}_2},\quad \frac{1}{f''}=\frac{(m''-1)\,(r_1-r_2)}{r\;r_2}.$$

But since

$$\frac{1}{\mathrm{F}'}+\frac{1}{\mathrm{F}''}=\frac{1}{f'}+\frac{1}{f''},$$

we shall have

$$\frac{1}{\mathrm{F}'}-\frac{1}{f'}=\frac{1}{f''}-\frac{1}{\mathrm{F}''}:$$

therefore

$$\frac{(n'-m')\,(\mathrm{R}_1-\mathrm{R}_2)}{\mathrm{R}_1\,\mathrm{R}_2}=-\frac{(n''-m'')\,(r_1-r_2)}{r_1\,r_2};$$

and consequently

$$\frac{n'-m'}{n''-m''}=-\frac{(r_1-r_2)\,\mathrm{R}_1\,\mathrm{R}_2}{(\mathrm{R}_1-\mathrm{R}_2)\,r_1\,r_2}.$$

The numbers expressed by $n'-m'$ and $n''-m''$ are the differences between the indices of the two lights having different refrangibilities, which are supposed to be transmitted through the lenses. These are the dispersive powers of the media composing the lenses for each of the two lights. If, then, the radii of the two lenses be so selected as to render the fraction expressed by the second member of the preceding equation equal to the ratio of the dispersive powers of the material of the lenses for the two sorts of light, they will be brought to the same focus by the compound lens.

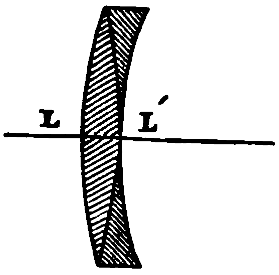

Fig. 139.

To simplify this, let us divest the preceding formula of its generality, and suppose that the first is a double convex lens L (*fig.* 139.), with equal radii, and that the second is a double concave lens L′, the surface of which, in contact with the first, has the same curvature with it, and consequently the same radius. Observing that when the convexities are turned in contrary directions, the radii have contrary signs, the preceding formula will now be reduced to

$$\frac{n'-m'}{n''-m''}=\frac{r_2-R_1}{2r_2}.$$

Now it is always possible so to select the radii as to fulfil this condition; and therefore a compound lens, composed of two lenses of different refracting media, can always be constructed which will bring to the same focus two lights of different refrangibilities.

Let us suppose that the double convex lens is composed of crown glass, for which

$$n'=1{\cdot}546566,\ m'=1{\cdot}525832,$$

and the double concave of flint glass, for which

$$n''=1{\cdot}671062,\ m''=1{\cdot}627749;$$

we shall therefore have

$$\frac{n'-m'}{n''-m''}=\frac{20734}{43313}=\frac{r_2-R_1}{2r_2};$$

from which we find that

$$r_2=23{\cdot}47\times R_1.$$

The radius of the second surface of the double concave lens must in this case, therefore, be 23½ times the radius of the double convex.

A compound lens, which produces such an effect, is called an *achromatic lens*.

212. **Structure of achromatic lenses.**—The materials which have been found most valuable for achromatic lenses, are flint and crown glass, which differ considerably in both their refracting and dispersing powers. The refracting and dispersing powers of these sorts of glass vary according to the proportions of their consti-

tuents, but they may always be rendered such as to fulfil the conditions necessary for an achromatic lens.

The forms of the lenses shown in *fig.* 139. are those of a double concave of flint, and a double convex of crown glass. It is neither necessary nor expedient, however, that these forms should be adhered to. The crown glass lens may be double-convex, with unequal convexities, or it may be plano-convex, or even meniscus. The flint glass lens may be in like manner double-concave, with unequal concavities, or it may be plano-concave, or concavo-convex. In the same way the curves of the surfaces may be indefinitely varied, the compound lens having still the same focal length.

The artist has therefore a wide latitude in the construction of achromatic lenses, of which the most eminent opticians have availed themselves with consummate skill and address, so as to efface, by the happy combination of curves, not only the spherical aberration, but also the chromatic aberration of the eye glass, and the spherical distortion of the final image in the compound microscope.

CHAP. VII.

THEORIES OF LIGHT.

213. THE optical phenomena attending ordinary reflection and refraction, which have formed the subjects of the preceding chapters, have been explained without reference to any hypothesis or theory. They have been deduced directly from experiments, the results of which are so simple and obvious, that the laws which prevail among them have been rendered evident without reference to theoretical considerations.

Other phenomena, however, will now have to be examined, in which the same simplicity does not prevail, and which do not admit of being explained or reduced to general laws without the occasional use of language derived from one or other of the theories respecting the nature of light which have been imagined by scientific inquirers.

214. **Two theories.** — Between the eye and any distant object, there intervenes a space of greater or less extent, and often, as in the case of the stars, so great as to be incapable of being clearly and adequately expressed by any standard or modulus of

magnitude with which we are familiar. Yet objects at these immense distances are rendered visible to us by some physical effects which they produce upon our organs of vision.

How then are we to conceive that an object placed at any distance, for example, say one hundred millions of miles, from the eye, can transmit over and through that space a mechanical effect to the retina? We answer that there are two, and only two, ways in which it is possible to conceive such an action to take place. These two are the following: —

215. **Corpuscular hypothesis.** — *First.* The distant object thus visible to us may emit particles of matter from its surface, which particles of matter may pass over the intervening space, may enter the pupil of the eye, may strike upon the nervous membrane, and so affect it as to produce vision.

216. **Undulatory theory.**—*Second.* There may be in the space between the distant visible object and the eye, *a medium possessing elasticity*, so as to be capable of receiving and transmitting pulsations or undulations like those imparted to the air by a sounding body. If this be admitted, the distant visible object may, without emitting any particles of matter from its surface, affect such a medium surrounding it with pulsations or undulations, in the same manner as a bell affects the air around it. These pulsations or undulations may pass along the space intervening between the visible object and the eye, in the same manner as the pulsations or undulations produced by a bell pass along the air between the bell and the ear. In this manner, the pulsations transmitted from the visible object, and propagated by the medium we have referred to, may reach the eye and affect the membrane which lines it, in the same manner exactly as the pulsations in the air affect the tympanum of the ear.

217. **Comparison of these theories.** — In the first there is an analogy between the eye and the organs of smelling. Odorous objects do actually emit material effluvia, which form part of their own substance. These effluvia reach the organ of smelling, and produce upon it a specific effect, which impresses the mind with a corresponding perception. According to the first supposition, a visible object at any distance would act in the same way, and would eject continual particles of light, which particles of light would move to the eye and produce vision, acting mechanically on its membrane in the same manner as the effluvia of a rose produce a sensible effect upon the organs of smelling.

The second method places the eye in analogy with the ear. So close is this analogy, that all the mathematical formulæ by which the effects of sound are expressed in acoustics, will, with very slight changes, be capable of expressing the effects of vision, according to the latter hypothesis.

It is evident, however, that as the first hypothesis requires us to admit that distant visible objects are continually ejecting matter from their surfaces to produce vision, so the second hypothesis as peremptorily requires the admission of the existence of some physical medium pervading the universe, — some subtle ethereal fluid endowed with a property of propagating the pulsations or undulations of distant visible objects, and transmitting them to the eye. This hypothetical fluid has been called the *luminiferous ether*.

The first of these two celebrated theories of light has been called the *corpuscular theory*, and the second the *undulatory theory*.

Newton, although he did not identify his investigations in optics with any hypothesis, but in the spirit of the inductive philosophy founded by Bacon based his conclusions on experiments and observations only, adopted nevertheless the nomenclature and language of the corpuscular theory, and probably, from veneration for his authority, English philosophers, until recently, have very generally given the preference to that theory.

The undulatory theory, on the other hand, was adopted by Huygens, and after him by most continental philosophers.

Optical researches within the last hundred years have been prosecuted with singular diligence and success. A vast variety of phenomena previously unknown have been accurately investigated, new laws have been developed, and the general result has been that the undulatory theory has prevailed over the corpuscular. It is perhaps not an unfair statement of the actual condition of these two celebrated hypotheses to say that while the corpuscular system is found sufficient to explain most of the common and obvious phenomena of optics, it totally fails in explaining many of the most remarkable effects brought to light by modern observations and experiments. On the other hand, the undulatory theory in general offers a satisfactory explanation for all. This circumstance has very properly and legitimately enlisted under that hypothesis almost all the leading scientific men of the present day.

Although the principal facts which we shall have now to explain are in fact independent of either of these two hypotheses, and incontestably true, whichever may be adopted, yet in their exposition it will be necessary to adopt the language of one or the other of these theories. We shall, for the reason just stated, use the nomenclature of the undulatory theory.

We are then to imagine light to consist of undulations propagated through the universal ether, in the same manner as the waves or undulations of sound are propagated through the air.

218. **Velocity of light.** — The first question, then, that arises is, what is the velocity with which these waves move? At what rate does light come from a distant star to the eye? Is it propagated instantaneously? Would a fire suddenly lighted at a point one hundred millions of miles from the eye be seen at the moment the light was produced? — or would an interval of time be necessary to allow the light to reach the eye? and if so, what would be the interval of time in relation to the distance of the luminous object?

In tracing the progress of human knowledge, we frequently have occasion to behold with surprise, and not without a due sense of humility, the important part which accident plays in the advancement of science. Often are we with diligent zeal in search of things, which, if found, would be of trifling or no value, when we stumble on inestimable treasures of truth. The frequency of this strongly impresses the mind with the persuasion, that there is in secret operation a power, whose will it is that knowledge and the human mind should be constantly progressive. It is in physics as in morals. We ignorantly seek that which is worthless, and find what is inestimable.

In the pursuit of knowledge we might well say that which we are taught to express in the pursuit of what is moral and good. We might say that the power which governs its progress knows better than we do "our necessities before we ask, and our ignorance in asking." We shall see a striking example of this in the narrative of the celebrated discovery of the motion of light.

219. **Determined by Jupiter's satellites.** — Soon after the invention of the telescope, and the consequent discovery of Jupiter's satellites, Roemer, an eminent Danish astronomer, engaged in a series of observations, the object of which was the discovery of the exact time of the revolution of one of these bodies around Jupiter. The mode in which he proposed to investigate this was, by observing the successive eclipses of the satellite, and noticing the time between them.

Let S (*fig.* 140.) represent the sun, and A B C D E F G H the successive relative positions of the earth. Let J be Jupiter projecting behind him his conical shadow, and let M N represent the orbit of one of his satellites. After each revolution the satellite will enter the shadow at M, and emerge from it at N.

Now if it were possible to observe accurately the moment at which the satellite would, after each revolution, either enter the shadow, or emerge from it, the interval of time between these events would enable us to calculate exactly the velocity and motion of the satellite. But by attentively watching the satellite we can note the time it enters the shadow, for at that moment it is deprived of the sun's light, and becomes invisible. We can also note the moment of its emergence, because then escaping from the edge of the shadow, it comes into the sun's light, and becomes visible. It was in this manner that Roemer proposed to ascertain the motion of the satellite. But in order to obtain the estimate with the greatest possible precision, he proposed to continue his observations for several months.

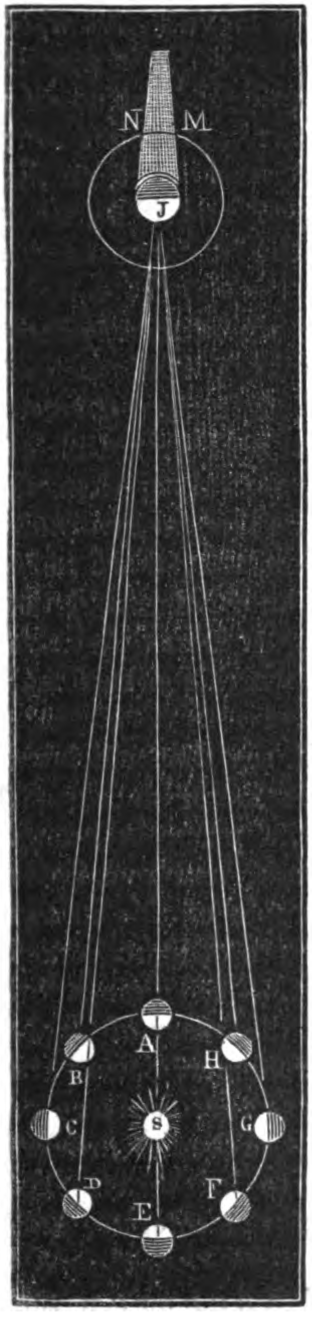

Fig. 140.

Let us, then, suppose that we have observed the time which has elapsed between two successive eclipses, and that this time is, for example, forty-three hours. We ought to expect that the eclipse would recur after the lapse of every successive period of forty-three hours.

Imagine a table to be computed in which we shall calculate and register beforehand the moment at which every successive eclipse of the satellite for twelve months to come shall occur; we shall then, as Roemer did, observe the moments at which the eclipses occur, and compare them with the moments registered in the table.

Let the earth be supposed at A, at the commencement of these observations, where it is nearest to Jupiter. When the earth has moved to B, which it will do in about six weeks, it will be found that the occurrence of the eclipse is *a little later* than the time registered in the table. When the earth arrives at C, which it will do at the end of three months, it will occur *still later* than the registered time. In fact, at C the eclipses will occur about eight minutes later than the registered time; at D they will be twelve minutes later; and at E sixteen minutes later.

By observations such as these, Roemer was struck with the fact that his prediction of the eclipses proved in every case to be wrong. It would at first occur to him that this discrepancy might arise from some errors of his observations; but if such were the case, it might be expected that the result would betray that kind of irregularity which is always the character of such errors. Thus it would be expected that the predicted time would sometimes be later, and sometimes earlier, than the observed time, and that it would be later and earlier to an irregular extent. On the contrary, it was observed during an interval of little more than six months which the earth took to move from A to E, that the observed time was continually later than the predicted time, and moreover, that the interval by which it was later, continually and regularly increased. This was an effect too regular and consistent to be supposed to arise from the casual errors of observation; it must have its origin in some physical cause of a regular kind.

The attention of Roemer being thus attracted to the question, he determined to pursue the investigation by continuing to observe the eclipses for another half year. Time accordingly rolled on, and the earth, transporting the astronomer with it, moved from E to F. On arriving at F, and comparing the observed with the predicted eclipse, it was found that the observed time was now only twelve minutes later than the predicted time. Soon after the expiration of the ninth month, when the earth arrived at G, the observed time was found to be only eight minutes later; at H it was only four minutes later; and, finally, when the earth returned to its first relative position with the planet, the observed time corresponded precisely with the predicted time.*

From this course of observation and inquiry it became apparent that the lateness of the eclipse depended altogether on the increased distance of the earth from Jupiter. The greater that distance, the later was the occurrence of the eclipse as apparent to the observers, and on calculating the change of distance it was found that the delay of the eclipse was exactly proportional to the increase of the earth's distance from the place where the eclipse occurred. Thus when the earth was at E, the eclipse was observed 16 minutes, or about 960 seconds later than when the earth was at A. The diameter of the orbit of the earth, A E, measuring about 190 millions of miles, it appeared that that distance produced a delay of 960 seconds, which was at the rate of 198000 miles per second. It appeared, then, that for every 198000 miles that the earth's distance from Jupiter was increased, the observation of the eclipse was delayed one second.

Such were the facts which presented themselves to Roemer. How were they to be explained? It would be absurd to suppose that the actual occurrence of the eclipses was delayed by the increased distance of the earth from Jupiter. These phenomena depend only on the motion of the satellite and the position of

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* The exact interval is 398 days, the synodic period of Jupiter.

Jupiter's shadow, and have nothing to do with, and can have no dependence on, the position or motion of the earth; yet, unquestionably the time at which they *appear* to occur to an observer upon the earth, has a dependence on the distance of the earth from Jupiter.

To solve this difficulty, the happy idea occurred to Roemer that the moment at which we see the extinction of the satellite by its entrance into the shadow is not, in any case, the very moment at which that event takes place, but some time afterwards, viz., such an interval as is sufficient for the light which left the satellite just before its extinction to reach the eye. Viewing the matter thus, it will be apparent that the more distant the earth is from the satellite, the longer will be the interval between the extinction of the satellite and the arrival of the last portion of light which left it, at the earth; but the moment of the extinction of the satellite is that of the commencement of the eclipse, and the moment of the arrival of the light at the earth is the moment the commencement of the eclipse is observed.

Thus Roemer, with the greatest felicity and success, explained the discrepancy between the calculated and the observed times of the eclipses; but he saw that these circumstances placed a great discovery at his hand. In short, it was apparent that light is propagated through space with a certain definite speed, and that the circumstances we have just explained supply the means of measuring that velocity.

We have shown that the eclipse of the satellite is delayed one second more for every 198000 miles that the earth's distance from Jupiter is increased, the reason of which obviously is, that light takes one second to move over that space; hence it is apparent that the velocity of light is at the rate, in round numbers, of 200000 miles per second.

We are then to remember that when light is propagated through space with the astonishing velocity of 200000 miles per second, there is no material substance which really has this progressive velocity; it belongs merely to the form of the pulsations, or undulations. The same observations, exactly, are applicable to the transmission of the waves of sound through the air.

220. **Amplitude of waves.** — In order to submit the phenomena of light to a strict physical analysis, it is not enough to measure the motion of its waves. We require also to know their amplitude or breadth, just as, in the case of the waves of the sea, we should require to know not only the rate at which they are propagated over the surface of the water, but also the space which intervenes between the hollow or crest of each and the hollow or crest of the succeeding one.

For the solution of this problem we are indebted to Newton himself. To render intelligible the mode in which he solved it, let us imagine a flat plate of glass, such as DE (*fig.* 141.), placed

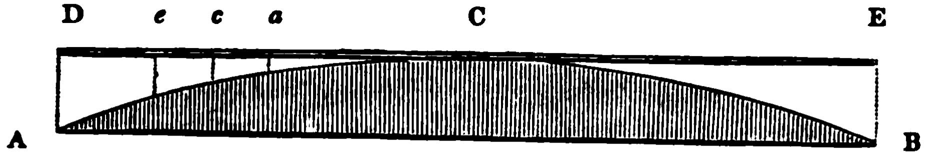

Fig. 141.

upon a convex lens of glass, the surface of which is represented by AB, but which must be supposed to have infinitely less curvature than that which appears in the figure.

The under surface of the flat plate will touch the vertex of the convexity at C, and the further any point on the under surface is from C, the greater will be the distance between the surfaces of the two glasses. Thus the distance between them at *a* is less than at *c*, and the distance at *c* is less than at *e*, and so on. The distance at the surfaces gradually increasing, in fact, from C outward.

If, looking down on the plate DE, we consider the point C as a centre, and a circle be described round it, at all points of that circle the surfaces of the glasses will have the same distances between them, and the greater that circle is, the greater will be the distance between the surfaces of glass.

Having the glasses thus arranged, Newton let a beam of light of some particular colour, produced by a prism, as red, for example, fall on the surface of the glass DE. He found that the effect produced was that a black spot appeared at the centre C, where the glasses touched; that immediately around this spot there appeared a circle of red light; that beyond that circle appeared a dark ring; that outside of that dark ring there was another circle of red light, still having the point C as its centre. Outside this second circle appeared another dark ring, beyond which there was another circle of red light, and so on, a series of circles of red light alternated with dark rings being formed, all having the point C as their common centre.

The distances between the surfaces of glass at which the successive circles of red light were found, were too minute to be directly measured, but they were easily calculated by measuring the diameters of the circles of light; and, knowing the diameter of the convex surface ACB, this was a simple problem in geometry easily solved, and admitting the greatest accuracy.

On making these calculations, Newton found that the distance between the glass surfaces where the second red circle was formed was double the distance corresponding to the first; that at the third red circle the distance was triple that of the first, and so on. It followed, of course, that wherever the dark rings were formed the distances between the glass surfaces were not an exact number of times the space corresponding to the first red circle.

Thus if we express the space between the glasses at the first red circle by 1, the space between them within that circle, toward the centre c, would be a fraction. The space corresponding to the first dark ring outside the first red circle would be expressed by 1 and a fraction; the space at the second red circle would be expressed by 2; the space at the second dark ring would be expressed by 2 and a fraction, and so on.

Newton was not slow to see that these phenomena were the direct manifestation of those effects which, in the corpuscular theory, whose nomenclature he used, corresponded to the amplitude of the waves of light in the undulatory theory. The space between the surfaces of glass at the first red ring was the amplitude of a single wave, the space at the second red circle the amplitude of two waves, and so on. Within the first red circle, the space between the glasses being less than the amplitude of a wave, the propagation of the undulation was stopped, and darkness ensued; in like manner, in the space corresponding to the second dark ring, the distance between the glasses being greater than the amplitude of one wave, but less than the amplitude of two, the propagation was again stopped, and darkness produced. But at the second red circle, the space being equal to the amplitude of two waves, the undulations were reflected and the red ring produced, and so on.

It was evident, then, that to measure the amplitude of the luminous waves, it was only necessary to calculate the distance between the glasses at the first red ring.

221. **Number of undulations in an inch.** — When light of other colours was thrown upon the glass, a similar system of luminous rings was produced, but it was found in each case that the first ring varied in its diameter according to the colour of the light, and consequently that the amplitude of the waves of lights of different colours is different. It appeared that the waves of red light were the largest; orange came next to them; then yellow, green, blue, indigo, and violet succeeded each other, the waves of each being less than those of the preceding. But the most astonishing part of this most celebrated investigation was the minuteness of these waves. It appeared that the waves of red light were so minute, that 40000 of them would be comprised within an inch, while the waves of violet light, forming the other extreme of the series, were so small, that 60000 spread over an

inch, and the waves of light of other colours were of intermediate magnitudes.

Thus was discovered the physical cause of the splendour and variety of colours, and a singular and mysterious alliance was developed between colour and sound. Lights are of various hues, according to the magnitude of the pulsations that produce them, vary exactly as musical sounds change their tone and pitch according to the magnitude of the aerial pulsations from which they result.

But this is not all. The alliance between sound and light does not terminate here. We have only spoken of the amplitude of the luminous waves, and have shown that it determines the tints of colours. What are we to say for the altitudes of the waves? Here, again, is another link of kindred between the eye and the ear. As the altitude of sonorous waves determines the loudness of the sounds, so the altitude of luminous waves determines the intensity or brightness of the colour.

There is one step more in the series of wondrous results which these memorable investigations have unfolded. As the perception of sound is produced by the tympanum of the ear vibrating in sympathetic accordance with the pulsations of the air produced by the sounding body, so the perception of light and colour is produced by similar pulsations of the membrane of the eye vibrating in accordance with ethereal pulsations propagated from the visible object. As in the case of the ear, the rigour of scientific investigation requires us to estimate the rate of the pulsation of the tympanum corresponding to each particular note, so in the case of light are we required to count the vibrations of the retina corresponding to every tint and colour. It may well be asked, in some spirit of incredulity, how the solution of such a problem could be hoped for; yet, as we shall now see, nothing can be more simple and obvious.

222. Let us suppose an object of any particular colour, a red star, for example, looked at from a distance. From the star to the eye there proceeds a continuous line of waves; these waves enter the pupil, and impinge upon the retina; for each wave which thus strikes the retina, there will be a separate pulsation of that membrane. Its rate of pulsation, or the number of pulsations which it makes per second, will therefore be known, if we can ascertain how many luminous waves enter the eye per second.

It has been already shown that light moves at the rate of about 200000 miles per second; it follows, therefore, that a length of ray amounting to 200000 miles must enter the pupil each second; the number of times, therefore, per second, which the retina will vibrate, will be the same as the number of the luminous waves contained in a ray 200000 miles long.

Let us take the case of red light. In 200000 miles there are in round numbers 1000,000000 feet, and therefore 12000,000000 inches. In each of these 12000,000000 of inches there are 40000 waves of red light. In the whole length of the ray, therefore, there are 480,000000,000000 waves. Since this ray, however, enters the eye in one second, and the retina must pulsate once for each of these waves, we arrive at the astounding conclusion, that when we behold a red object, the membrane of the eye trembles at the rate of 480,000000,000000 of times between every two ticks of a common clock!

In the same manner, the rate of pulsation of the retina corresponding to other tints of colours is determined; and it is found that when violet light is perceived, it trembles at the rate of 720,000000,000000 of times per second.

223. **Table of undulations.**—In the annexed table are given the magnitudes of the luminous waves of each colour, the number of them which measure an inch, and the number of undulations per second which strike the eye:—

Colours.	Length of Undulation in Parts of an Inch.	Number of Undulations in an Inch.	Number of Undulations per Second.
Extreme Red	0·0000266	37640	458,000000,000000
Red	0·0000256	39180	477,000000,000000
Orange	0·0000240	41610	506.000000,000000
Yellow	0·0000227	44000	535,000000,000000
Green	0·0000211	47460	577,000000,000000
Blue	0·0000196	51110	622,000000,000000
Indigo	0·0000185	54070	658,000000,000000
Violet	0·0000174	57490	699,000000,000000
Extreme Violet	0·0000167	59750	727,000000,000000

The preceding calculations are, as will be easily perceived, made only in round numbers, with a view of rendering the principles of the investigation intelligible. In the table the exact results of the physical investigations which have been carried on, on this subject, are given.

224. Whichever theory we adopt to explain the phenomena of light, we are led to conclusions that strike the mind with astonishment. According to the corpuscular theory, the molecules of light are supposed to be endowed with attractive and repulsive forces, to have poles to balance themselves about their centres of gravity, and to possess other physical properties which we can only ascribe to ponderable matter. In speaking of these properties, it is difficult to divest oneself of the idea of sensible magnitude, or by any strain of the imagination to conceive that particles to which they belong can be so amazingly small as those of light demonstrably are. If a molecule of light weighed a single grain, its momentum (by reason of the enormous velocity with

which it moves) would be such that its effect would be equal to that of a cannon-ball of one hundred and fifty pounds, projected with a velocity of one thousand feet per second. How inconceivably small must they therefore be, when millions of molecules, collected by lenses or mirrors, have never been found to produce the slightest effect on the most delicate apparatus contrived expressly for the purpose of rendering their materiality sensible!

If the corpuscular theory astonish us by the extreme minuteness and prodigious velocity of the luminous molecules, the numerical results deduced from the undulatory theory are not less overwhelming. The extreme smallness of the amplitude of the vibrations, and the almost inconceivable but still measurable rapidity with which they succeed each other; were computed by Dr. Young, and are exhibited in the above table.

225. **Researches of Young.**—That the sensation of light is produced by the vibrations of an extremely rare and subtle fluid, is an idea that was maintained by Descartes, Hooke, and some others; but it is to Huygens that the honour solely belongs of having reduced the hypothesis to a definite shape, and rendered it available to the purposes of mechanical explanation. Owing to the great success of Newton in applying the corpuscular theory to his splendid discoveries, the speculations of Huygens were long neglected; indeed, the theory remained in the same state in which it was left by him till it was taken up by our countryman, the late Dr. Young. By a train of mechanical reasoning, which in point of ingenuity has seldom been equalled, Dr. Young was conducted to some very remarkable numerical relations among some of the apparently most dissimilar phenomena of optics, to the general laws of diffraction, and to the two principles of colouration of crystallised substances.

226. **Malus, Arago, and others.**—Malus, so late as 1810, made the important discovery of the polarisation of light by reflection, and successfully explained the phenomenon by the hypothesis of an undulatory propagation. The theory subsequently received a great extension from the ingenious labours of Fresnel; and the still more recent researches of Arago, Poisson, Herschel, Airy, and others, have conferred on it so great a degree of probability, that it may almost be regarded as ranking in the class of demonstrated truths. "It is a theory," says Herschel, "which, if not founded in nature, is certainly one of the happiest fictions that the genius of man has yet invented to group together natural phenomena, as well as the most fortunate in the support it has received from all classes of new phenomena, which at their discovery seemed in irreconcileable opposition to it. It is, in fact, in all its applications and details, one succession of *felicities*; in-

asmuch as that we may almost be induced to say, if it be not true, it deserves to be."

227. **Relations of light and heat.**—Light and heat are so intimately related to each other, that philosophers have doubted whether they are identical principles, or merely co-existent in the luminous rays. They possess numerous properties in common; being reflected, refracted, and polarised according to the same laws, and even exhibit the same phenomena of interference. Most substances during combustion give out both light and heat; and all bodies, except the gases, when heated to a high temperature, become incandescent. Nevertheless, there are many circumstances in which they appear to differ.

A thin plate of transparent glass interposed between the face and a blazing fire intercepts no sensible portion of the light, but most sensibly diminishes the heat. Light and heat are therefore not intercepted alike by the same substances. Heat is also combined in different degrees with the different rays of the solar spectrum. A very remarkable discovery on this subject was made by Sir William Herschel, which would seem to establish the independence of the heating and illuminating effects of the solar rays. Having placed thermometers in the several prismatic colours of the solar spectrum, he found the heating power of the rays gradually increased from the violet (where it was least) to the extreme red, and that the maximum temperature existed some distance beyond the red, out of the visible part of the spectrum. The experiment was soon after repeated with great care by Berard, who confirmed Herschel's conclusions relative to the augmentation of the calorific power from the violet to the red, and even beyond the spectrum. This discovery of the inequality of the heating power of the different rays led to the inquiry whether the chemical action produced by light upon certain bodies was merely the effect of the heat accompanying it, or owing to some other cause. By a series of delicate experiments, Berard found that this action is not only independent of the heating power, but follows entirely a different law; its intensity being greater in the violet ray, where the heating power is the least, and least in the red ray, where the heating power is the greatest. We are thus led to the conclusion that the solar rays possess at least three distinct powers—those of heating, illuminating, and effecting chemical combinations and decompositions; and these powers are distributed among the different refrangible rays in such a manner as to show their complete independence of each other.

CHAP. VIII.

INTERFERENCE AND INFLECTION.

In all cases where systems of undulation are propagated along the surface of a fluid or through an elastic medium, phenomena are produced by the intersection of systems of waves, by which, at certain points, the undulations obliterate each other.

Such effects are called *interferences*, one system of waves being said to *interfere* with another when such reciprocal obliterations take place.

An instructive class of interesting optical phenomena are explained upon this principle.

228. **Fresnel's experiments.** — In order to exhibit the phenomena of the interference of light in such a manner as to develop the laws which govern it, and to supply numerical estimates of the data and constants of the undulatory theory, it is necessary to contrive means by which two pencils of light, whether homogeneous or compound, of the same intensity, shall intersect each other at a very oblique angle and at a considerable distance from their foci. Fresnel, to whose experimental researches in this department of physics science is largely indebted, accomplished this object by reflection and refraction in the following manner.

Let M C, M′ C, *fig.* 142., be two plane reflectors inclined to each other at a very obtuse angle. Let F be a focus of light produced by transmitting the light through a converging lens of short focus, or by reflecting it from a concave speculum. The rays diverging from F are received upon the two plane reflectors M C and M′ C. An image of F will be formed by the reflector M C at P just as far behind the plane of M C as F is before it; and, in like manner, another image of F will be produced by the reflector M′ C at P′ just as far behind the plane of M′ C as F is before it. It follows, therefore, that those rays which proceed from F, and are incident upon M C, will, after reflection, diverge as if they had originally proceeded

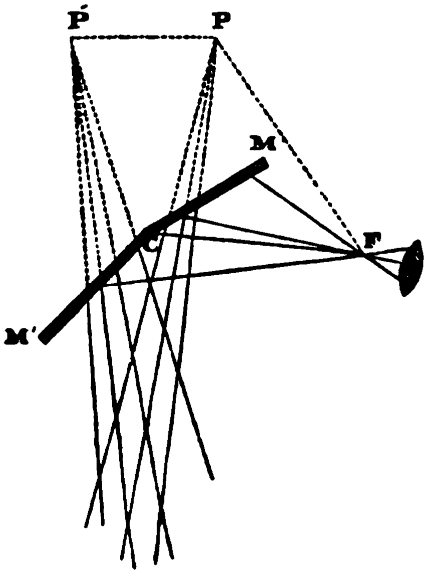

Fig. 142.

from P, and those rays which are incident upon M′ C will, after reflection, diverge as if they had originally proceeded from P′. Therefore the pencils after reflection will be optically equivalent to two pencils radiating from P and P′. Thus we shall have a single pencil radiating from the point F converted into two pencils intersecting each other at a very oblique angle, and proceeding from the distant foci P and P′.

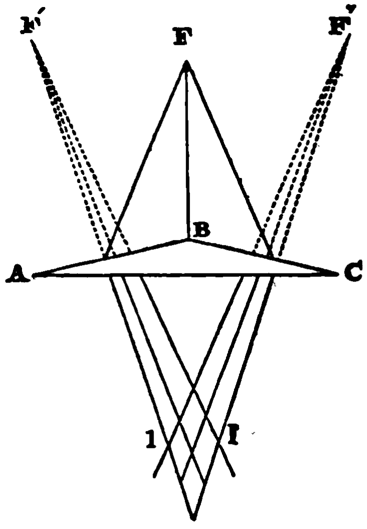

Fig. 143

Let A B C, *fig.* 143., be a prism, with a very obtuse angle at B, and let F be a radiant point produced as before by a converging lens or concave reflector. The rays diverging from F, and incident on the surface A B, will be refracted as if they proceeded from F′; and, in like manner, the rays proceeding from F and incident upon B C will be refracted as if they proceeded from F″. Thus we shall have two pencils as before, the rays of which will intersect each other obliquely at the points I, these pencils consisting of light of the same quality and intensity.

229. **Interference of homogeneous light.** — If two pencils of homogeneous light thus obtained be made to diverge from two points F and F′, *fig.* 144., and if the rays of these pencils intersect at very oblique angles below the line A B, which is drawn parallel to the line F F′, which joins the foci of the two pencils, the following effects will ensue: —

If a line C *o* O be drawn from the middle point of F F′ perpendicular to it, every point on this line *o* O will be illuminated; in fact, an illuminated line will be formed from *o* to O, as indicated by the dotted line in the figure. On either side of this illuminated line *o* O will be found a dark curved line 1 1 and 1′ 1′, so that any object held in either of these lines would be deprived of light. Outside these two dark curved lines will be found two other curved lines, 2 2 and 2′ 2′, which will be lines of light, so that any object held at any point of either of them will be illuminated. Beyond these again will be found two

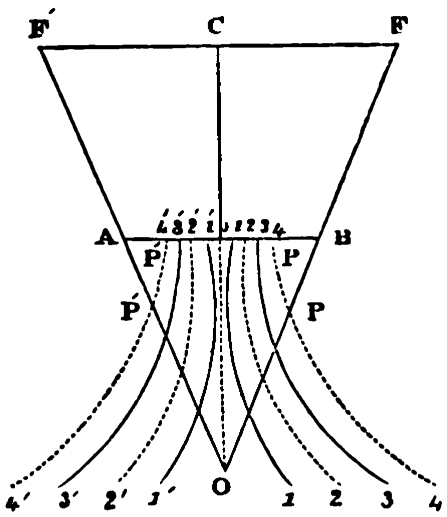

Fig. 144.

other dark curved lines, 3 3 and 3′ 3′, so that any object held in them will be in shadow or darkness; beyond these again will be two curved lines of light, as before, 4 4 and 4′ 4′, so that any object held in either of these will be illuminated. Thus there succeed each other a series of curved lines of light and darkness, the light lines having the colour and qualities of the light of the two pencils. The series of the illuminated curves of light and darkness at each side of the central line *o* o are symmetrically arranged, those on the one side having corresponding forms, positions, and distances to those on the other side.

The curves formed by these light and dark lines are those known in geometry as the species of conic section called the hyperbola, the points F and F′ being their common foci. Now, it is a well known property of this curve that the difference between the distances of every point in it from the two foci is the same. Thus, if lines be drawn from F and F′ to any point in any one of these curves, their difference will be the same as that of lines drawn from F and F′ to any other point in the same curve. Thus, for example, if P and P be two points upon the curve 4 4, then the differences between the distances of P and P from F and F′ will be equal; and, in like manner, if P′ and P′ be two points on the curve 4′ 4′, the differences between their distances from F and F′ will be equal.

It will presently be seen that this property gives rise to important consequences.

If an opaque screen be interposed between the line A B and either of the foci, F′ for example, all these curves of bright and dark lines vanish, and a uniform illumination will be produced throughout the space below the line A B. This illumination, however, will be found to have only half the intensity of the bright curves which were previously formed.

Now, since by the interposition of the screen no light has been diffused below the line A B which was not there before, but, on the contrary, all the light proceeding from the focus F′, which was there before, is now excluded, it follows that the effect of the rays which, proceeding from the focus F′, intersect those proceeding from the focus F, is to deprive the spaces marked by the dark curves 1 1, 3 3, 1′ 1′, and 3′ 3′ of light, and to increase in a twofold proportion the light in the spaces marked *o* o, 2 2, 4 4, 2′ 2′, and 4′ 4′.

Thus it appears that at the intersections of the rays proceeding from F, which take place upon the dark curves, the one light extinguishes the other; and that at the intersections which take place upon the bright curves, the lights add their mutual intensities, and an intensity is produced equal to their sum; for since they are equal to each other, this intensity is double the intensity of either.

Now it will be evident, by reference to what has been established relating to undulations*, that this fact is merely a consequence of the interference of the waves of light. The foci F and F′ may be considered as the centres round which two systems of luminous undulations are propagated. These systems, encountering each other, intersect below the line A B. At those points where the waves meet under corresponding phases, that is to say, where the crest of one wave coincides with the crest of another, or the depression of one with the depression of another, they produce waves of double the height or double the depression of either. But at those points where they meet under contrary phases, that is, where the crest of one wave coincides with the depression of the other, or *vice versâ*, then the waves obliterate each other, and no undulation takes place at such point. In the former case, the light at the point of intersection has double the intensity which it would have if the light from one focus alone was received; in the other case, the lights extinguish each other, and there is darkness.

Now it will be easy to show that the bright curves indicated by the dotted lines in the figure correspond to points where the systems of waves intersect under the first condition above mentioned, and that the dark curves correspond to those points where they intersect under the second condition.

The middle line *o* o, which is a line of light, is at all its points equally distant from F and F′. Thus two lines F O and F′O drawn from the focus to the same point in it are always equal; consequently the undulations which meet at any point such as *o* on this line, must necessarily meet under similar phases; for since the waves are of equal lengths, and since the distance F O is equal to the distance F′O, the same number of waves and parts of a wave must occupy the two distances, and consequently the waves must arrive at O under corresponding phases.

The distance of any point of the first dark curve 1 1 from the focus F′ exceeds its distance from the focus F by half an undulation. If, therefore, the crest of a wave proceeding from F′ arrive at any point on this curve, the depression of a wave proceeding from F must arrive at the same point at the same time; and the same will be true of all points in the dark curve 1 1. The same observation will also be applicable to the curve 1′ 1′, only that in this case the distance of any point from F exceeds its distance from F′ by half an undulation. Thus it appears that the waves propagated from the centres F and F′ always intersect on the dark curves 1 1 and 1′ 1′ under contrary phases, and consequently obliterate each other's effects, and produce darkness.

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* See Treatise on "Sound," Hand Book.

The distance of any point in the bright curve 2 2 from F′ exceeds the distance of the same point from F by the length of a complete undulation; consequently, if the crest of a wave proceeding from F arrive at any point in such line, the crest of the preceding wave proceeding from F must arrive at it at the same time; and the same will be true for every point, so that throughout this bright line 2 2 the intersecting waves increase each other's effect. The same will be true of the line 2′ 2′. Hence the illumination produced along these two bright curves will be equal to the sum of the illuminations proceeding from the two foci.

In the same manner, it appears that the distance of any point on the dark curve 3 3 from F′ exceeds the distance of the same point from F by the length of an undulation and a half, and the same consequences as in the case of the first curve follow; so that the waves intersecting on the dark curves 3 3 and 3′ 3′, meet under opposite phases, and obliterate each other.

It is evident, therefore, that the several hyperbolic curves formed by the successive light and dark lines on either side of the central bright line *o* o derive their character from the multiple of only half a wave's length, which expresses the difference between the distance of their successive points from the two centres of undulation F and F′, which are the common foci of all the curves; and this multiple is in such case the length of the transverse axis of the hyperbola, of which the point C is the centre.

The spaces intervening between the bright and dark curves correspond to points where waves intersect under phases which are neither perfectly coincident nor perfectly opposite, and where consequently they only partially efface each other. Hence the light gradually diminishes in these spaces between the bright and the dark curves. The difference between the distances of these intermediate points from the foci F and F′ exceeds a complete number of half undulations by a quantity which is less than half an undulation.

230. **Interference affected by refrangibility.** — In what has been here stated, it has been assumed that the light proceeding from the points F and F′ is homogeneous light. Now there are, as has been shown, various species of homogeneous light, differing from each other in refrangibility and colour; and it is necessary to explain in what respects each variety of refrangibility and colour affects the phenomena of the bright and dark curves just explained. We find, accordingly, that by causing pencils of homogeneous light of different colours and refrangibilities to intersect as above described,

the bright and dark curves formed by their interference retain the character of the hyperbola, but that while their general disposition on either side of the central line *o* o is the same, they are at different distances from each other; that is to say, the distance of the first bright curve 2 2 from the central line *o* o, as well as the distance of any two corresponding curves from each other, are different for different species of homogeneous light. In general, the more refrangible the light is, the nearer are the bright curves to each other. Thus the distance between one bright curve and another for violet or blue colour is less than the distance between the corresponding bright lines for red or orange colour.

231. **Undulations computed from interference.** — By an exact measurement of the dark and bright hyperbolic curves produced by each species of homogeneous light, aided by their known geometrical properties, Fresnel was enabled to deduce the lengths of the undulations of the ether which correspond to each species of homogeneous light. The following are the results of his observations and calculations: —

Colour of homogeneous Light.	Length of Wave in ten-millionth Parts of an Inch.	Number of Undulations to an Inch.
Extreme violet	160	62500
Mean violet	167	59880
Violet bordering on dark blue	173	57803
Dark blue	177	56497
Dark blue bordering on light blue	180	55555
Light blue	187	53476
Light blue bordering on green	194	51546
Green	205	48780
Green bordering on yellow	209	47847
Yellow	217	46083
Yellow bordering on orange	225	44445
Orange	230	43480
Orange bordering on red	235	42553
Red	244	40983
Extreme red	254	39370

232. **Interference of compound light.** — Since the distances between the bright and dark curves are different for each species of homogeneous light, it follows that if the light which radiates from F and F′ be white solar light which is composed of all the colours of the spectrum, we shall have all the systems of bright and dark curves which would be separately produced by each of the component parts of the solar lights superposed, and a mixture of colours will consequently ensue which will produce rows of fringes, the colours of which will be determined by the prismatic tints which will be thus mingled together.

A complete analysis of the combination of the colours which would produce these fringes in the case of solar light would be extremely complicated. Some idea, however, may be formed of the manner in which the combination of colours is produced from *fig.* 145., in which the relative breadths and distances of the light and dark curves produced by the three homogeneous lights, red, green, and violet, are represented. The series of red fringes with their alternate dark spaces are represented by R R, the series of green stripes are represented by G G, and that of violet stripes by V V. If these be considered, instead of being placed, as in the figure, one above the other, to be superposed, the effects which would be produced by a line proceeding from the two foci F and F′ composed of these three colours may be inferred.

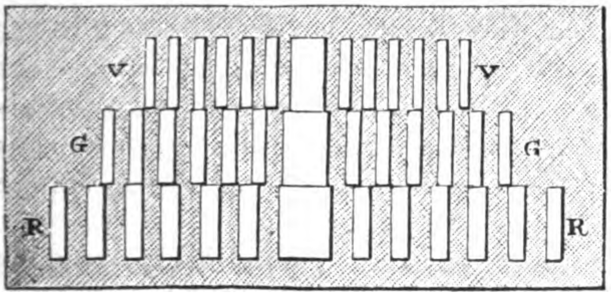

Fig. 145.

233. **Inflection or diffraction.**—If the rays of light diverging from a focus F (*fig.* 146.), be incident upon an opaque object A B, all those rays of the pencil which are included within the angle A F B will be intercepted, so that a screen held at C D will receive none of those rays.

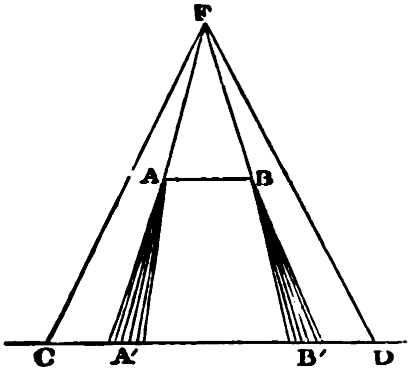

Fig. 146.

If the lines F A and F B be continued to A′ and B′, they will include upon the screen those spaces which would have been illuminated by the rays proceeding from F, which are intercepted by the opaque body A B. All the rays of the pencil included in the angles A F C and B F D will proceed uninterruptedly, and will fall upon the screen. If these rays underwent no change of direction, they would illuminate those portions of the screen included between C and A′ and D and B′. There would thus be an exact and well-defined shadow of the object A B formed upon the screen at A′ B′, and the remainder of the screen would be illuminated in the same manner as it would have been if the opaque body A B had not been present.

It is found, however, by experiment, that no such exact and well-defined shadow of the opaque object would be formed upon the screen. The outline of the space which would limit an exact and geometrical shadow of A B being determined, it is found that within this space light will enter, and that outside this space the illumination is not the same as it would have been if the object A B had not been interposed.

From this it is inferred that the rays of light which pass the edges of the opaque object do not proceed in the same straight lines A A′ and B B′, in which they would have proceeded if the opaque object was not present. In a word, the appearance of the edge of the shadow is not a well-defined line separating the illuminated from the dark part of the screen, but a line of gradually decreasing brilliancy from the illuminated part of the screen to that in which the shadow becomes decided.

This effect produced by the edges of an opaque body upon the light passing in contact with them, by which the rays are bent out of their course, either inwards or outwards, is called *inflection* or *diffraction*.

This phenomenon is a consequence of the general property of undulation.* When the system of waves propagated round F as a centre encounters the obstacles A B, subsidiary systems of undulation will be formed round A and B respectively as centres, and will be propagated from those points independently of and simultaneously with the original system of waves whose centre is F, and which will also proceed towards C A′ and D B′. In a certain space round the lines A A′ and B B′, along which the rays grazing the edge of the opaque body would have proceeded, the two systems of undulation will intersect each other and produce the phenomena of interference.

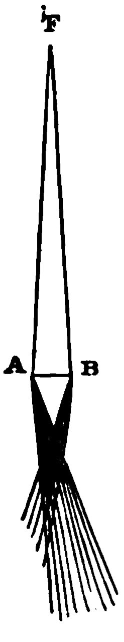

Fig. 147.

234. Combined effects of inflection and interference. — If the opaque body A B be very small, and the distance of the focus F from it be considerable, the two pencils formed by inflection, of which A and B are the foci, will intersect each other as represented in *fig.* 147., and in this case all the phenomena of interference already described will ensue. Thus, if the light be homogeneous, a bright line of light will be formed under the centre of the opaque object A B, outside which will be dark lines, and then bright and dark lines alternately. If the arrangement of these lines be examined, they will be found to be hyperbolic, as exhibited in *fig.* 144., and to vary in their relative distance with the quality of the light which radiates from the focus F. If the light radiating from such focus be compound solar light, then a series of coloured fringes will be formed, as already explained.

235. Examples of the effects of inflection and interference. — The variety of optical phenomena produced by light passing the edges of small opaque objects, or small openings made in opaque plates, is infinite. The principles, however, on which all these appearances are explained, are the same.

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* See Treatise on "Sound," Hand Book.

The following experiments form examples of the variety of which these phenomena are susceptible:—

I. If a small sphere formed of any opaque substance be suspended in a dark room, and a pencil of homogeneous light be allowed to fall upon it, so that its shadow may be received upon a screen, it will be found that a bright spot will appear in the middle of the shadow, outside which will be a dark circle, beyond which there will be a bright circle, and beyond that a dark circle, and so on; the circles corresponding successively to the interference of the rays, by which their brilliancy is either doubled or extinguished, and the colour of the bright circles corresponding to that of the light.

If the light which falls on the sphere in this case be compound solar light, the central spot on the screen will be white, and will be surrounded by a series of coloured fringes, produced by the superposition of the coloured rings which would be produced separately by each compound of the solar light.

II. If a fine wire or sewing-needle be held close to one eye, the other eye being closed, and be looked at so as to be projected upon the light of a window, or a white screen, several needles will be seen.

III. If the eye be directed in a dark room to a narrow slit in the window-shutter by which light is admitted, several slits will be seen separated by dark bands.

IV. If a piece of card, having a narrow incision made in it, be held between the eye and a candle, a series of slits will be seen parallel to each other, exhibiting the colours of the spectrum. The same appearance may be produced with increased effect by looking through the slit at the sun light admitted through an opening in the window-shutter.

236. **Thin transparent plates.**—It has been already shown that when light passes from any transparent medium to another of different density, a part of it is reflected from their common surface, and a part only transmitted. Thus, when light passing through air is incident upon the surface of glass, a certain part of it is reflected from such surface, but the greater part enters it. When that portion which penetrates the glass arrives at the second surface, which separates the glass from the air, on the other side a like effect ensues, a portion of the light is reflected from the second surface, the greater part, however, penetrating it, and passing into the air. There are, therefore, two systems of reflected rays, one

reflected from the first surface of the glass, and the other by the second surface.

The first system of reflected rays is thrown back immediately into the air; the second system is thrown back into the glass, and must pass through the first surface of the glass before it returns into the air.

If the two surfaces which thus successively reflect a portion of the light which passes through the transparent medium be very close together, and if they be not precisely parallel, the reflected rays will intersect each other, and produce the phenomena of interference.

237. **Iridescence of fish-scales, soap-bubbles, mother-of-pearl, feathers, &c.** — Hence arise the curious and beautiful appearances of iridescence which are observable whenever transparent substances are exhibited in sufficiently thin plates or laminæ, the prismatic colours observable in the scales of fishes, in spirit of wine spread in thin films on dark surfaces, in oil thinly diffused over the surface of water, and the thin laminæ of crystals and soap-bubbles, and bubbles of glass blown to extreme tenuity, in the laminæ of mother-of-peal, and in the wings of insects and feathers of birds.

CHAP. IX.

DOUBLE REFRACTION.

238. **Transparent media resolved into two classes.** — Transparent substances consist of two classes, which present optical phenomena depending on certain physical properties inherent in the constitution of each class of media respectively. The phenomena, both optical and physical, suggest in the first class the supposition that they consist of molecules which are uniform in their form and reciprocal effects, so that the forces which they exercise one upon the other are the same in every direction. To this class belong every species of aeriform fluid, all liquids, and certain transparent solids, such as glass, when properly annealed.

239. **Single refracting media.** — In all these substances the constituent molecules appear to be so arranged, that we might conceive them to be spherules of matter, from the centres of which forces emanate which are equal in every direction.

240. **Double refracting media.**—The second class of substances, which includes crystallised minerals, generally exhibits phenomena which lead to the supposition that their constituent molecules are not spherules, or, at least, that they do not exercise like forces in all directions round their centres. The phenomenon of crystallisation, already explained*, itself suggests this supposition; for when a substance passes from the liquid to the solid state, and undergoes the process of crystallisation, the particles affect a particular arrangement with reference to one another, so as to present themselves towards each other in certain directions, as if they had sides which mutually attracted or repelled each other.

241. **Uncrystallised medium.**—To render more clearly intelligible the effects produced by crystallised substances on light transmitted through them, we shall first briefly recapitulate the effects produced on rays of light by an ordinary transparent uncrystallised medium, such as air, water, or glass.

Let us suppose such a substance reduced to the form of a sphere, which, if it be gas or liquid, may be done by enclosing it in a thin globe of glass; and if it be a solid, it may be reduced to the spherical form in the lathe. Let E N Q S, *fig.* 148., represent a section of this transparent sphere, and E P Q O another section at right angles to it.

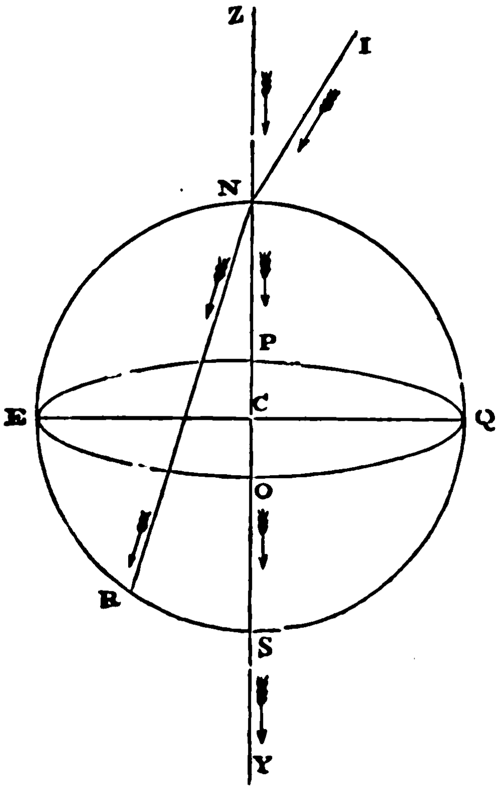

Fig. 148.

Let Z N and I N represent two rays of homogeneous light incident at N, one in a direction which, being continued, would pass through the centre C of the sphere, and the other, I N, in a direction oblique to the former.

If the sphere be composed of non-crystallised transparent matter, the ray Z N will pass through it, pursuing the original direction; and consequently, after passing the centre C, will emerge from the lowest point S in the direction S Y, so that its course shall be in no wise changed by the transparent medium through which it has passed; but the ray I N, which falls obliquely at the point N, will, according to the law of refraction already explained, be deflected from its course towards the diameter N C S, and will follow a direction such as N R, which makes an angle with N S, less than that which I N makes with N Z.

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* Hand Book "Mechanics" (60).

The laws which govern in this case the refracted ray are as follows: —

I. If the incident ray be perpendicular to the surface at the point of incidence, its direction will not be changed in passing through the transparent medium.

II. If the incident ray form an angle, such as I N Z, with the perpendicular N Z at the point of incidence, then the refracted ray N R will form an angle with the same perpendicular N Z, or with its production N S, the plane of which will coincide with the plane of the angle of incidence Z N I.

III. If the angle of incidence I N Z be varied, the angle of refraction R N S will be also varied, but in such a manner that the ratio of the sine of the angle of incidence I N Z to that of the angle of refraction R N S shall always be the same, so long as the transparent medium into which the ray passes is the same.

IV. If while the incident rays Z N and I N preserve their position, the sphere be turned round its centre C, so as to bring successively every part of its surface to coincide with the point of incidence N, the refracted ray N R will still maintain the same direction and position, and the ray Z N will still pass through the centre of the sphere C, no matter what position may be given to the sphere, so long as the position of its centre C remain unchanged.

Thus the direction and position of the incident rays I N and Z N, and of the refracted rays N R and N S, will remain fixed, although the transparent sphere which they penetrate may be changed in an infinite variety of ways, so as to bring all its points in succession to coincide with the point of incidence N of the rays.

Such are the phenomena which are produced when the rays I N and Z N are incident upon a sphere composed of uncrystallised transparent substance. The same phenomena will always prevail in the case even of certain crystallised substances; but in the case of other crystallised media, different and far more complicated phenomena are developed, which we shall now proceed to explain.

242. **Crystallised media.** — Let a sphere be formed of one of the class of crystals of which Iceland spar or the crystallised carbonate of lime is a specimen, and let this sphere be submitted to the same experiments as have been described in the former case. When the rays I N and Z N, *fig.* 149., penetrate the sphere at N, they will each of them be resolved into two rays, one of which, in the figure, is indicated by the uniform line, and the other by

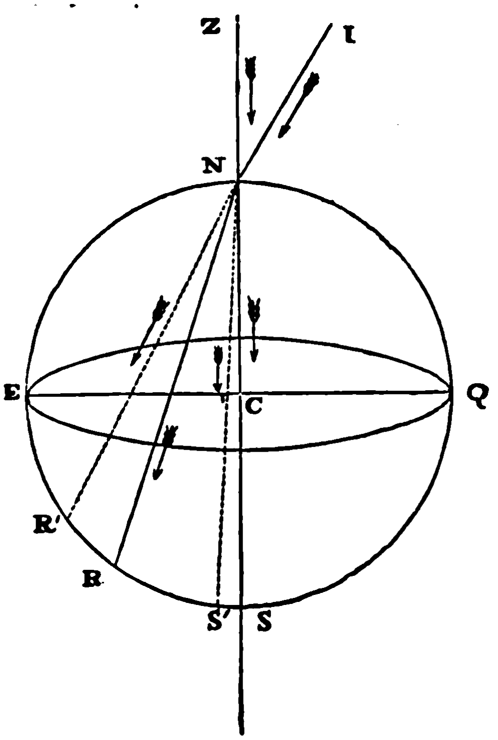

Fig. 149.

the dotted line. The rays indicated by the uniform lines N S and N R, will conform to the laws of refraction which prevail in uncrystallised media; that is to say, the ray N S will pass through the centre of the sphere C, preserving the direction of the incident ray Z N, which strikes the surface of the sphere at N in a perpendicular direction, and the ray N R will be in the plane of the angle of incidence I N Z. Also, if the ray I N be made to fall at N, so as to form any other angle of incidence, the ray N R will vary its inclination to the perpendicular N S, in conformity with the law of refraction, which establishes a constant ratio between the sines of the angles of incidence and refraction.

But none of these characters are found to attend the other rays N S′ and N R′ into which the original incident rays are resolved by the crystal. The ray N S′, although proceeding from the ray Z N, which is incident perpendicularly at the point N, does not penetrate the medium in the same direction, but makes a certain angle S′ N S with the perpendicular. Thus, in the case of this ray there is an acute angle of refraction corresponding to perpendicular incidence. In the case of the ray N R′ it is found that it deviates on the one side or the other of the plane of the angle of incidence I N Z, and thus this ray violates that general law of common refraction which declares that the plane of the angle of refraction coincides with the plane of the angle of incidence.

If the angle formed by the incident ray I N with the perpendicular Z N be varied, the angle which the refracted ray N R makes with the perpendicular N S will be also varied, but not according to the law of sines which prevails in the case of ordinary refraction.

243. **The ordinary and extraordinary rays.** — Thus it appears that in such crystallised media the incident ray is resolved into two rays, one of which conforms to the laws of common refraction, and the other violates them, and is regulated by other and different conditions. The two rays into which the incident ray is thus resolved are called the *ordinary* and *extraordinary* rays;

that which conforms to the laws of common refraction being called the *ordinary*, and that which violates them the *extraordinary* ray.

If the sphere be now supposed to be moved, as before, round its centre C, so as to bring successively all the points of its surface to coincide with the point of incidence N, it will be found that the ordinary rays N S and N R will preserve their direction and position fixed in all positions which the sphere shall assume; but that the direction and position of the extraordinary rays N S′ and N R′ will vary with every change of position of the sphere. They will sometimes approach to, and sometimes recede from, the ordinary rays; and they will sometimes deviate on one side, and sometimes on the other, of the plane of the angle of incidence; but in all cases there will be a maximum deviation from the ordinary ray, which will not be exceeded.

244. **Axis of double refraction.** — By varying the position of the sphere so as to bring the various points of its surface to coincide with the point of incidence N, a point will be found upon it at which the extraordinary ray N S′ will coincide with the ordinary ray N S. As this point approaches the point N, the angle S′N S under the ordinary and extraordinary ray will be observed continually to diminish; an effect which will indicate the change of position necessary to bring the desired point to coincide with the point of incidence N.

This point of the sphere then possesses a distinctive character, in virtue of which the incident ray Z N is not, as at all other points, resolved into two rays, but passes through the sphere in the direction N C S, exactly as it would pass through a sphere composed of an uncrystallised substance.

The diameter of the sphere which possesses this property is called its *optical axis*, or the *axis of double refraction*, being the only line in the sphere along which a ray of ordinary light can pass without being decomposed into two.

245. **Laws of double refraction.** — Having thus determined this optical axis of the sphere, let us next examine the conditions which affect a ray of light, such as I N, which falls obliquely at the extremity of such optical axis.

Let N C S, *fig.* 150., be the optical axis of the sphere. The ray Z N will then, as has just been explained, pass through the centre C to the point S, without double refraction, as it would through an ordinary medium. The ray I N, which falls obliquely at N, will, however, be doubly refracted, and will be resolved into the ordinary ray N R, and the extraordinary ray N R′. But this extraordinary ray N R′ will, in this case, conform to one of the laws of ordinary refraction, for it will invariably lie in the plane of

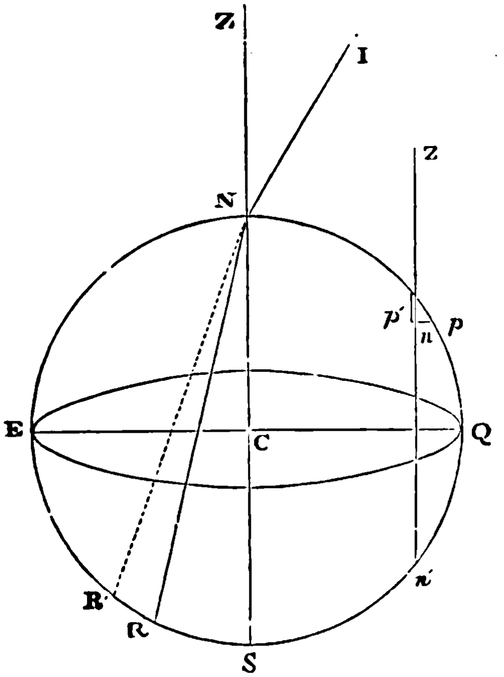

Fig. 150.

the angle of incidence INZ; and so long as the angle of incidence shall not be varied, the direction of this extraordinary ray will remain the same. This may be proved by causing the sphere to revolve round the axis NS. While it so revolves, the extraordinary ray NR′ will remain fixed in its direction, being always in the plane of the angle of incidence, and forming always the same angle of refraction with the axis NS.

If the incident ray IN be varied in its inclination, so as to form, as before, a greater angle with ZN, the extraordinary ray NR′ will also vary its inclination to the axis NS and to the ordinary ray NR. But, although it will remain during such variation always in the plane of the angle of incidence, it will not conform to the invariable ratio of sines which constitutes the law of ordinary refraction.

If we suppose the incident ray IN gradually to approach ZN so that the angle of incidence continually diminishes, then the two rays NR and NR′ will at the same time approach the axis NS and each other; and when the incident ray coincides with ZN, the ordinary and extraordinary rays NR and NR′ will coalesce with the axis NS.

As, on the other hand, the inclination of the ray IN to ZN is gradually increased, the ordinary and extraordinary rays NR and NR′ will also gradually recede from the axis NS, so that their angles of refraction will continually increase, and they will also recede from each other.

246. **Positive and negative crystals.** — In the case represented in the figure, the angle of refraction of the extraordinary ray NR′ is greater than that of the ordinary ray NR, so that the latter is more deflected by the refraction of the crystal than the former. This, however, is not always the case.

In some crystals the angle of refraction of the extraordinary ray is more than that of the ordinary ray, and, consequently, the former is less deflected towards the perpendicular than the latter. Crystals are accordingly resolved into two classes, based upon this distinction; those in which the extraordinary ray is less deflected than the ordinary ray being called *negative crystals*, and those in which it is more deflected *positive crystals*. It is evident that in the former case the index of ordinary refraction is greater, and in the latter less, than the index of extraordinary refraction.

It must be observed that while the incident ray varies its obliquity to Z N, increasing gradually from 0 to 90°, and while the index of ordinary refraction throughout this variation remains constant, the index of extraordinary refraction varies with every change of obliquity. In the case of positive crystals this index increases, in the case of negative crystals it diminishes, with the angle of incidence; while in all it is equal to the index of ordinary refraction when the ray of I N coincides with Z N. It increases and becomes a maximum when I N is at right angles to Z N in positive crystals; it diminishes and becomes a minimum when I N is at right angles to Z N in negative crystals.

It is easy to show that all lines passing through the crystal which are parallel to the line N S possess also the property which characterises such axis; that is to say, a ray which is incident perpendicularly in the direction of such lines will penetrate the crystal without double refraction. This we may prove by cutting a portion of the crystal in a direction perpendicular to the line N S.

Thus, at the point P, let a surface *p p'* be formed, which shall be perpendicular to N S. Then a ray *z n*, falling perpendicularly on such surface *p p'* will penetrate the crystal in the direction *n n'* without double refraction.

247. **Axis of double refraction coincides with crystallographic axis.** — Thus it appears that the lines passing through the crystal parallel to N S are axes of double refraction as well as the line N S. On comparing the direction of the line N S with the direction of the planes of cleavage of the crystal, it is found that this line has a direction which is symmetrical with respect to all these planes, and that it is in fact the direction of the crystallographic axis; that is to say, a line the direction of which bears the same relation to all the faces of the crystal.

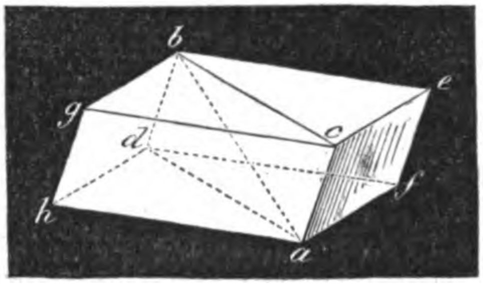

Fig. 151.

248. **Iceland spar.** — Thus in the case of Iceland spar, the primitive form of whose molecules is that of such a rhomboid as is represented in *fig.* 151., the crystallographic axis is the diagonal *a b* joining the obtuse angles of the rhomb. The rhomb itself is a solid bounded by six equal and similar parallelograms, whose obtuse angles *g b e* and

g c e are each 101° 55′, and whose acute angles *b e c* and *b g c* are accordingly each 78° 5′.

The inclination of the faces of the rhomb, which meet at *a c*, to each other is 105° 5′, consequently the inclination of those which meet at *f e* is 74° 55′. The crystallographic axis *a b* is equally inclined, not only to the three faces of the rhomb, which meet at *a* and *b* respectively, but also to its three edges. The angles which this axis makes with the three edges of the rhomb forming the angle *a* are equal to each other, their common magnitude being 60° 44′ 46″.

It is evident from this measurement, that the line *a b* is symmetrically placed with respect to all the elements which determine the primitive form of the crystal, and we thus find accordingly a distinct relation established between the optical and mineralogical characters of this substance, so that whenever the direction of its crystallographic axis is required to be ascertained, it can be done without any mechanical experiment or measurement, by merely determining that direction in which a ray of light incident perpendicularly on a surface of the crystal will pass through it without double refraction. What has been here stated with regard to Iceland spar will, *mutatis mutandis*, be applicable to a numerous class of crystallised substances, which are distinguished by the denomination of crystals having a single axis of double refraction. In all such crystals the crystallographic axis coincides with the optical axis.

249. **Uni-axial crystals.**—In attempting to explain the complicated phenomena of double refraction and other effects related to them, much convenience and clearness will be obtained by the adaptation of a nomenclature indicating the position of the axis of double refraction in certain sections of the crystal analogous to the well-known circles used in geography and astronomy for expressing the relative position of points on the earth and in the heavens. We shall therefore call the extremities of the axis N and S the *poles* of the crystal, and a section of the crystal E P Q S intersecting this axis at right angles the *equator*. We shall also call all sections of the crystal made by the planes passing through the axis *meridians*.

These terms being understood, it will follow that whenever the plane of the angle of incidence coincides with the plane of a meridian, the angles of refraction, both of the extraordinary and ordinary rays, will be in the plane of the same meridian; but the ratio of the sine of the angle of incidence to the sine of the angle of extraordinary refraction will not in this case be constant.

If the plane of the angle of incidence intersect the crystal at right angles to the optical axis N S, and be consequently parallel to the line coincident with the plane of the equator, the angle of extraordinary refraction will have its plane coincident with that of the angle of incidence, thus fulfilling one of the laws of ordinary refraction, as is the case when the plane of the angle of incidence coincides with the plane of the meridian. But in this case the second law of refraction, which establishes a constant ratio between the sines of the angles of incidence and refraction, is also fulfilled by the extraordinary ray, so that when the angle of incidence coincides with, or is parallel to, the plane of the equator, the extraordinary refraction fulfils all the conditions of ordinary refraction, although the extraordinary ray does not coincide with the ordinary ray; the constant index of refraction of the one being greater or less than the constant index of refraction of the other, according as the crystal is positive or negative.

There are therefore two systems of planes which intersect crystals, one system having the axis of the crystal as their common line of intersection, and the other having directions parallel to each other and perpendicular to this axis. In the former, one of the laws of ordinary refraction is fulfilled, and in the latter both of them. In the former, the plane of the angle of extraordinary refraction coincides with the plane of the angle of incidence, but the ratio of the sines is not constant; in the latter, the planes also coincide, and the ratio of the sines is constant, but not the same as that of the ordinary ray.

250. **Table of uni-axial crystals.**—The following is a table of the crystals which have a single axis of double refraction, according to Mons. Pouillet, "Elemens de Physique," vol. ii. p. 365. 1853: —

Table of Crystals with a single Axis.

NEGATIVE.

Carbonate of lime (Iceland spar).
Carbonate of lime and magnesia.
Carbonate of lime and iron.
Tourmaline.
Rubellite.
Corundum.
Sapphire.
Ruby.
Emerald.
Hydro-chlorate of lime.
Hydro-chlorate of strontian.
Sub-phosphate of potash.
Sulphate of nickel and copper.
Cinnabar.
Mellite.
Molybdate of lead.
Beryl.
Phosphate of lime (apatite).
Idocrase (of Vesuvius).
Wernerite.
Mica (of Kariat).
Phosphate of lead.
Arseniated phosphate of lead.
Hydrate of strontian.
Arseniate of potash.
Octo-hedrite.
Prussiate of potash.
Phosphate of lime.
Arseniate of lead.
Arseniate of copper.
Nepheline.

POSITIVE.

Zircon.
Quartz.
Oxide of iron.
Tungstate of zinc.
Stannite.
Boracite.
Apophyllite.
Sulphate of potash and iron.
Super acetate of copper and lime.
Hydrate of magnesia.
Ice.
Hypo-sulphate of lime.
Dioptase.
Ruby silver.

Besides the above, Sir David Brewster gives the following: —

NEGATIVE.

Carbonate of zinc.
Nitrate of soda.
Levyne.
Alum stone.
Gmelinite.
Chlorate of soda.
Cyanide of mercury.
Paranthine or scapolite.
Meionite.
Somervillite.
Edingtonite.
Phosphate of ammonia and magnesia.
Muriate of lime.
Muriate of strontian.
Hypo-sulphate of lime.
Mica with amianthus.
Hornerite or nacrite.
Mica from Kariat.

POSITIVE.

Oxide of tin.
Tungstate of lime.
Apophyllite of uton.
Oxaliverite.
Titanite.
Murio-carbonate of lead.
Tortoise-shell.

251. **Biaxial crystals.** — There is another class of crystals which present optical phenomena still more complicated. Let us suppose, as before, one of these formed into a sphere, and let its various points, as before, be brought to coincide with the point of incidence N of two rays, one of which, Z N, *fig.* 150., is directed to the centre of the sphere, and the other I N forming any angle with the latter. By bringing the various points of the spherical surface to coincide with the point N, it will be found that two points, and two only, upon it, possess the property of transmitting the ray Z N, which falls perpendicularly upon the surface, through the object, without double refraction. The diameters passing through these two points have each of them the character of an axis of double refraction; and the crystals characterised by this property are accordingly called crystals with two axes of double refraction.

In this class of crystals it is found that neither of the rays into which the incident ray is resolved conforms to the laws of ordinary refraction; that both deviate from the plane of the angle of incidence, and that neither of them fulfils the second law, which determines the constant ratio between the sines of incidence and refraction. Both rays, therefore, are extraordinary rays.

There are, however, two planes in which the angle of incidence may be placed, in one of which one of the two rays and in the other the other will conform to both the laws of ordinary refraction, so that in these planes one or other of the two extraordinary rays becomes an ordinary ray. The position of these planes is determined by the following conditions: —

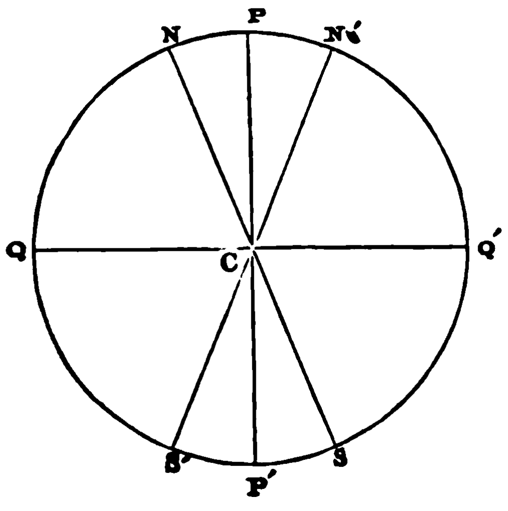

Fig. 152.

Let N S and N' S', *fig.* 152., be the two axes of double refraction. Let P P' be a line which divides into equal parts the angle N C N' formed by these two angles, and let Q Q' be a line which divides into equal parts the other angle N′ C S formed by the same axis.

If a plane pass through C perpendicular to P C, any ray incident upon the crystal in that plane will be resolved into two rays, one of which will conform to the laws of ordinary refraction; and if a plane be drawn perpendicular to the line Q Q′, any ray incident upon the crystal in that plane will be resolved into two, one of which will also conform to the laws of ordinary refraction, and the ray which thus becomes an ordinary ray in the one plane will be different from that which becomes an ordinary ray in the other plane.

252. The following list of crystals having two axes of double refraction, with the magnitude of the angle included between such axes, is given by M. Pouillet in the work already cited: —

Table of Crystals with two Axes.

Names of Substances.	Angles of Axes.	
	°	′
Sulphate of nickel (certain samples)	3	0
Sulpho-carbonate of lead	,,	,,
Carbonate of strontian	6	56
Carbonate of barytes	,,	,,
Nitrate of potash	5	20
Mica (certain samples)	6	0
Talc	7	24
Pearl	11	28
Hydrate of barytes	13	18
Mica (certain samples)	14	0
Arragonite	18	18
Prussiate of potash	19	24
Mica (certain samples)	25	0
Cymophane	27	51
Anhydrite	28	7
Borax	28	42
Mica (several samples examined by M. Biot)	30	0
	31	0
	32	0
	34	0
	37	0
Apophyllite	35	8
Sulphate of magnesia	37	24
Sulphate of barytes	37	40
Spermaceti (about)	37	42
Borax (native)	38	48
Nitrate of zinc	40	0
Stilbite	41	42
Sulphate of nickel	42	4
Carbonate of ammonia	43	24
Sulphate of zinc	44	28
Anhydrite (examined by M. Biot)	44	21
Mica	45	0
Lepidolite	45	0
Benzoate of ammonia	45	8
Sulphate of soda and magnesia	46	49
Sulphate of ammonia	49	42
Brazilian topaz	49 to 50	0
Sugar	50	0
Sulphate of strontian	50	0
Sulpho-hydrochlorate of magnesia and iron	51	16
Sulphate of magnesia and ammonia	51	22
Phosphate of soda	55	20
Comptonite	56	6
Sulphate of lime	60	0
Oxynitrate of silver	62	16
Iolite	62	50
Feldspar	63	0
Aberdeen topaz	65	0
Sulphate of potash	67	0
Carbonate of soda	70	1
Acetate of lead	70	25
Citric acid	70	29
Tartrate of potash	71	20
Tartaric acid	79	0
Tartrate of potash and soda	80	0
Carbonate of potash	80	30
Cyanite	81	48
Chlorate of potash	82	0
Epidote	84	19
Hydrochlorate of copper	84	30
Peridote	87	56
Succinic acid	90	0
Sulphate of iron	90	0

The researches of Sir David Brewster, published in the "Philosophical Transactions of London and Edinburgh," have led to the discovery of various other properties of double and multiple refracting crystals, which are too complicated for admission into a work so elementary as the present; the reader is therefore referred to the above collections, where their details will be found.

253. If a visible object be placed behind a double refracting crystal, the pencil of rays proceeding from each point in it will be resolved into two pencils, and will emerge from the crystal as if they had proceeded from two different objects in directions corresponding to the respective directions of the two pencils.

An eye, therefore, placed before the crystal, so as to receive these emerging pencils, will see two different images of the object, corresponding to the two systems of pencils. If the crystal be one having a single axis of double refraction, then one of these images will be that produced by the pencils consisting of ordinary rays, and the other will be that produced by pencils consisting of extraordinary rays.

254. The one is called the ordinary, the other the extraordinary image. Thus, if P, *fig.* 153., be such an object, and A B C D be a double refracting crystal, such as Iceland spar, the pencils which proceed from P and are incident upon the surface B C will be divided into two systems of pencils, the axis of the ordinary system passing perpendicularly through the crystal in the direction I O, and emerging on the other side in the same direction, so as to meet the eye at Y. The extraordinary pencils will follow the direction I E through the crystal, and will emerge parallel to the ordinary pencil in the direction E Y′, so as to reach the eye at Y′. An eye placed therefore at any point, in looking towards the crystal, will perceive two images of the point P in juxtaposition in the direction of the rays Y′ E and Y O.

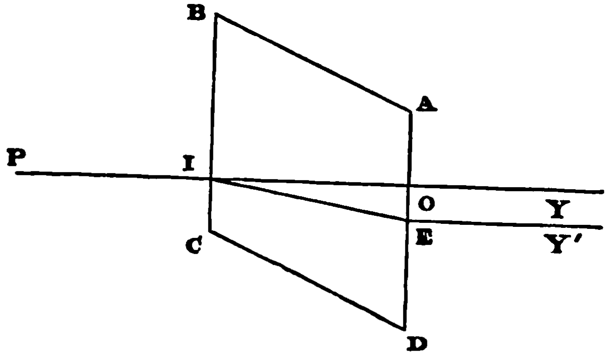

Fig. 153.

255. It is evident that the thicker the crystal is, the more widely separated will be these two images. A crystal of Iceland spar three inches thick, will be sufficient to produce a distinct separation of the two images of a spherical object having a diameter of one third of an inch.

If while the object and the eye remain fixed, the crystal be turned round the line P Y, joining the eye and the object as an axis, the extraordinary image will appear to revolve round the ordinary image, showing that in this case the extraordinary pencil

I E revolves round the ordinary pencil I O, so as to move in the surface of a cone. This effect is in conformity with what has been already explained.

If, after passing through a crystal A B C D, *fig.* 154., the rays be

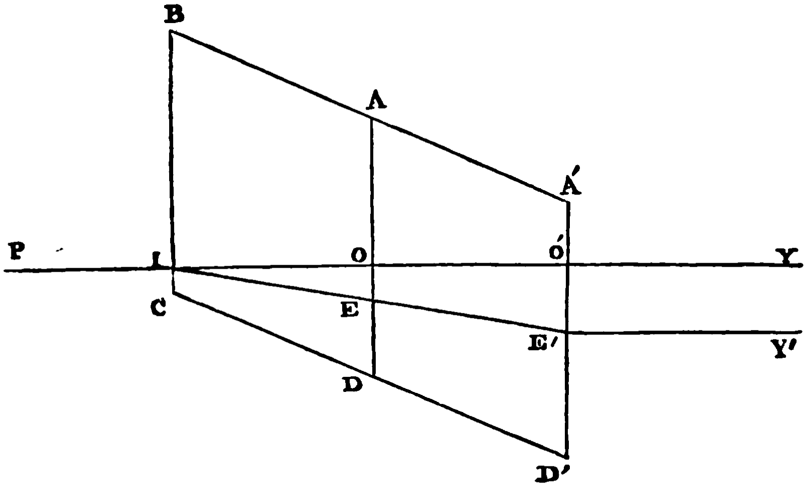

Fig. 154.

received by another crystal A A' D' D, whose sides and axes have a position similar to those of the first, the two crystals being in contact at the surface A D, the ordinary and extraordinary rays will pass through the second crystal, following the same direction as those which they followed in the first crystal, the lines O O' and E E' being the continuation of the lines I O and I E.

256. **Case in which two similar crystals neutralise each other.** — If the two crystals in this case have the same thickness, then the effect will be that the rays E' Y' and O' Y emerging from the second will be separated by a space twice as great as that by which they were separated in passing through the first crystal.

If the second crystal, instead of having been placed upon the first crystal so that its corresponding sides shall have the same direction, be placed upon it so that they shall have contrary directions, as represented in *fig.* 155., then the second crystal will have the effect of causing the reunion of the two pencils separated by the first crystal, and the ordinary and extraordinary rays will accordingly emerge from the same point O of the second crystal in the same direction, so that an eye placed at Y will see but one image of the object P. In this case the ordinary ray follows the direction P I O O' Y, and the extraordinary ray follows the direction P I E O' Y. Thus the separation of the rays takes place only in passing through the crystals, the reunion being established at the point of the emergence O' from the second crystal.

257. **Four images.** — If we suppose the second crystal A A' D' D (*fig.* 154.), to be turned round the line P I O Y as an axis, the mo-

ment it moves from the position represented in *fig.* 154., the ordinary and extraordinary rays I O and I E incident upon it from the first crystal will be each doubly refracted so as to be resolved into four rays, and thus an eye placed at Y would see four images of the point P. As the second crystal is gradually turned round, these four images assume a series of different positions with relation to each other, and also have different degrees of brilliancy.

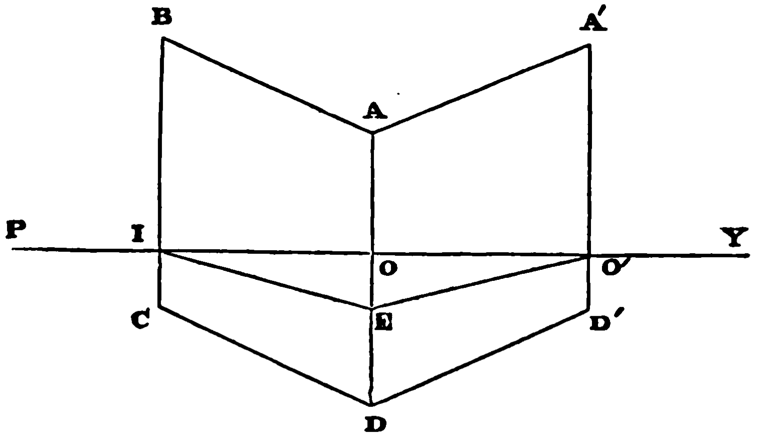

Fig. 155.

After the crystal has made half a revolution, and assumed the position represented in *fig.* 155., all these four images unite in one. In the position intermediate between these two, that is to say, when the second crystal has made a quarter of a revolution round the line P I O Y, then the four images will be reduced to two, which, however, will have a different position relative to the line A D from that which the image produced in the position represented in *fig.* 154. has.

258. The successive positions assumed by the four images during the half revolution of the second crystal between the position represented in *fig.* 154., and that represented in *fig.* 155., are given in *fig.* 156., where B represents the position of the images corresponding to *fig.* 154., and K to *fig.* 155.; F represents their

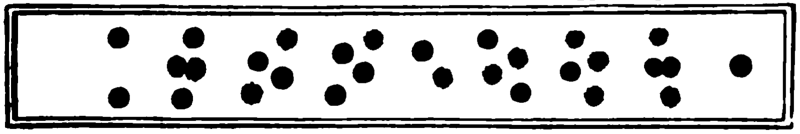

Fig. 156.

position when the second crystal has made one fourth of a revolution; C, D, and E, represent three successive positions of the images in three equally distant stages of the first quarter of a revolution; and G, H, and I represent their respective positions in three equally distant stages of the second quarter of a revolution.

The relative brilliancies of the images are indicated by the shading of the dots, the dark dots being understood to represent greater brilliancy than the shaded ones.

259. **Axes of bi-axial crystals.** — In uni-axial crystals the axis has the same position, whatever be the colour of the light, but in bi-axial crystals the position of the axes is different for different coloured lights. Sir John Herschel found that in tartrate of potash and soda their inclination for violet light was 56°, and for red light 76°. In other crystals, such as nitre, their inclination for violet was greater than for red, but in all cases the axes for all coloured light in the same crystal are in the same plane. Sir David Brewster found that glauberite had two axes for red light, inclined at an angle of 50°, and only one for violet light. The same eminent philosopher found that in the case of analcine there were several planes along which there was no double refraction, however various the angle of incidence might be, so that that substance might be considered as having an infinite number of axes of double refraction.

260. **Double refracting structure produced by artificial processes.** — The property of double refraction may in some cases be imparted by artificial processes to substances which do not naturally possess it. If a cylinder of glass be brought to a red heat, and held upon a plate of metal until it becomes cold, it will acquire the double refracting property, the axis of the cylinder being a single positive axis of double refraction. This axis differs, however, from the positive axis of crystallisation, because in this case it is a single line, while in the crystal the lines parallel to it are equally axes of double refraction. Sir David Brewster says that if, instead of heating the cylinder, it had been immersed in a vessel of boiling water, it would have acquired the same double refracting virtue when the heat had reached its axis, but that the property would not be permanent, disappearing when the cylinder should become uniformly heated. Also, if the cylinder were uniformly heated in boiling oil, or at a fire so as not to soften the glass, and had been placed in a cold fluid, it would acquire a temporary double refracting virtue when the cooling had reached the axis; but in this case the axis would be a negative one, instead of a positive, as in the former case.

According to him some other analogous structures may be produced by pressure, and by the induration of soft solids, such as animal jellies, isinglass, &c.

If the cylinder in the preceding explanations is not a regular one, but have its section perpendicular to the axis everywhere an ellipse in place of a circle, it will have two axes of double refraction.

In like manner, if we use rectangular plates of glass instead of cylinders, as in the preceding experiment, we shall have plates with two planes of double refraction, a positive structure being on one side of each plane, and a negative one on the other.

If we use perfect spheres there will be axes of double refraction along every diameter, and, consequently, an infinite number of them.

The crystalline lenses of almost all animals, whether they are lenses, spheres, or spheroids, have one or more axes of double refraction.

CHAP. X.

POLARISATION OF LIGHT.

261. WHEN a ray of light has been reflected from the surface of a body under certain special conditions, or transmitted through certain transparent crystals, it undergoes a remarkable change in its properties, so that it will no longer be subject to the same effects of reflection and refraction as before. The effect thus produced upon it has been called *polarisation*, and the ray or rays of light thus affected are said to be *polarised*.

The name *poles* is given in physics in general to the sides or ends of any body which enjoy or have acquired any contrary properties. Thus, the opposite ends or sides of a magnet have contrary properties, inasmuch as each attracts what the other repels. The opposite ends of an electric or galvanic arrangement are, for like reasons, denominated poles.

Following the common rule of analogy in nomenclature, a ray of light which has been submitted to reflection or transmission under the special conditions referred to, has been called *polarised light;* inasmuch as it is found that the sides of the ray which lie at right angles to each other, possess contrary physical properties, while those of a ray of common or unpolarised light possess the same physical properties.

To illustrate the relative physical condition of common light and polarised light, we may compare a ray of common light to a round rod or wire of uniform polish and uniformly white, while a ray of polarised light may be compared to a similar wire, two of whose opposite sides are rough and black, while the other opposite sides at right angles to these are polished and white. Thus, if

ABCD, *fig.* 157., be a section of the former, the entire circumference ABCD is white and polished, and if A′ B′ C′ D′ (*fig.* 158.) be

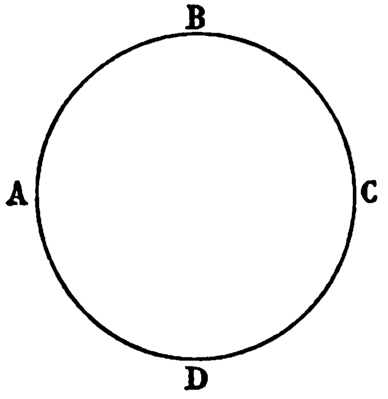

Fig. 157.

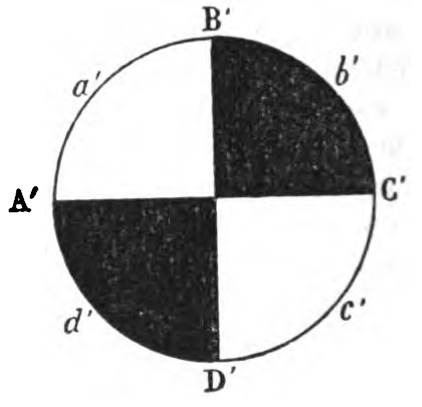

Fig. 158.

a section of the latter, *a′* and *c′* will be white and polished, while *b′* and *d′* will be black and rough.

A group of physical properties, very numerous and complicated, characterise the polarised state of light, the discussion and exposition of which constitute the subject of an extensive and important section of optics.

Let a plate of glass be blackened on one side, so that when used as a reflector no light will be reflected from its posterior surface. Such a plate will therefore reflect light only from one surface, which will be its anterior surface. This precaution is necessary in the cases now to be examined, in order to prevent the effects which would ensue from the combination of the rays, which would otherwise be reflected from both the anterior and posterior surfaces of the glass.

Let such a plate, so prepared, be presented to the polarised ray at an angle of incidence of 54° 35′, so that the plate shall make with the ray an angle of 35° 25′; and let it be turned round the ray, so as to be presented successively on every side of it, still forming, however, the same angle with it. During this process, it will be observed that there is a certain direction of the plane of the angle of incidence at which no reflection will take place; the ray will be absorbed or extinguished, so to speak, by the reflecting surface. The plane of incidence will have this direction in two opposite positions of the reflector.

Let the line *b′ d′*, *fig.* 158., represent this position of the plane of incidence: then *b′* and *d′* will be the two opposite sides of the ray, at which the reflector being presented will cause the extinction of the light. Now as the reflector is carried round from either of these positions respectively, so that the plane of the angle of incidence shall turn round the axis of the ray, reflection will begin to take place, and will increase in intensity until the plane of the angle of incidence takes a position, such as *a′ c′*, at right angles to *b′ d′*, when the intensity of the reflection will be a maximum. After passing this position, the intensity of the reflection will again diminish, and will continue to decrease until the plane of the angle of incidence shall again coincide with the diameter *b′ d′*. It is evident, therefore, that different sides of such a ray have different properties. Thus, the sides *a′* and *c′* have a susceptibility of being reflected, of which the sides *b′* and *d′* are deprived; and the susceptibility of reflection diminishes gradually in going round the ray from either *a′* or *c′* towards *b′* or *d′*, when it altogether ceases.

A plane passing through the axis of the ray, and coinciding with the diameter *a′ c′*, is called the plane of polarisation. It is evident, therefore, from what has been explained, that when the reflector is so presented to the ray that the plane of the angle of incidence shall coincide with the plane of polarisation, reflection will take place with the greatest intensity, and that when the plane of the angle of incidence is at right angles to the plane of polarisation, no reflection takes place, and the ray is extinguished.

262. **Angle of polarisation.** — If, instead of glass, any other reflecting surface be used, like effects would be produced; only that the angle at which it would be necessary to present the reflecting surface to the ray would be different, each species of reflector having its own particular angle. This angle is, for reasons which will be hereafter explained, called the *angle of polarisation.*

263. **Polariscopes.** — Instruments called *Polariscopes*, adapted for the experimental illustration of the phenomena of polarisation, have been constructed in various forms. One of the most convenient for the purposes of elementary explanation consists of several detached pieces, which are represented in *fig.* 159. A B is a brass tube like that of a telescope, along the axis of which the polarised pencil to be submitted to examination is transmitted. C is a short tube capable of being inserted, after the manner of telescopic tubes, in the main tube at A. This tube C carries a plane

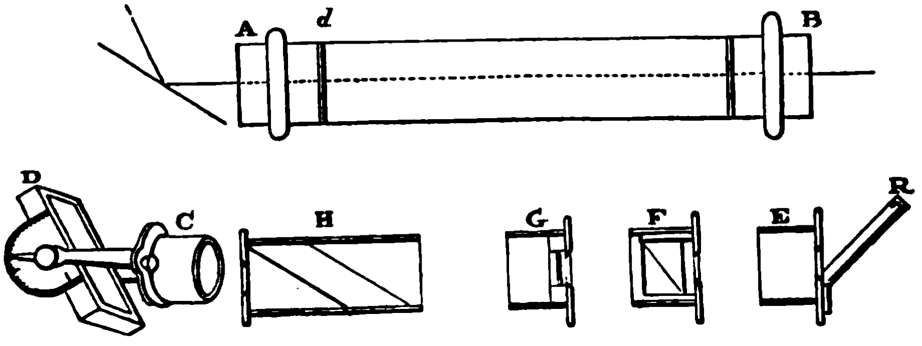

Fig. 159.

reflector D of the blackened glass already described, which is capable of being turned on pivots, and is supplied with a double scale and index, by which the angle it makes with the axis of the tube can be regulated at pleasure. By turning the tube C round its axis, the plane of the reflector D may be presented successively on every side of the axis of the main tube.

A diaphragm is fixed in the tube at *d*, having a circular hole in its centre, to limit the magnitude of the transmitted pencil. The pieces E, F, G, and H, are severally capable of being inserted in the end B of the tube, and of being turned round in the same manner as already described with relation to the piece C inserted at the end A. The short tube E carries a plane reflector R, similar to that already described, which is capable of being adjusted at any desired angle with the axis of the tube. The tube F contains a double refracting prism; the tube G contains a thin disc of tourmaline with parallel faces, so cut that the optic axis is parallel to these faces. In fine, the tube H contains a bundle of plates of glass, with parallel surfaces placed in contact with each other, and inclined obliquely to the axis of the tube.

All these pieces severally inserted in the tube A B can be turned round its axis, so that the reflector R, or the prism, or the tourmaline G, or the included plates H, may be severally presented in succession on all sides of the ray transmitted along the axis of the tube A B.

264. **Polarisation by reflection.** — Let the tube C, *fig.* 159., carrying the reflector D, be inserted in the main tube A, and let a plate of blackened glass be inserted in the frame D, as already described. Let the apparatus be so adjusted that when a ray of light falling upon the plate D at an angle of incidence equal to 54° 35′ is reflected, the reflected ray will pass along the axis of the tube A B. Such a ray will be polarised, and the plane of its polarisation will coincide with the plane of the angle of incidence upon the plate D.

To prove this, let the tube E carrying the reflector R be inserted in the end B of the main tube, and let the reflector R be adjusted so that the ray which passes along the axis of the tube shall fall upon it at the same angle of incidence, 54° 35′ as represented in *fig.* 160.

If the tube E be so placed that the plane of the angle of incidence upon the reflector R shall coincide with the plane of the angle of incidence on the reflector D, then the ray coming along the axis of the tube will be reflected from R with the greatest possible intensity. If the tube E be then turned round within the tube B, so as to present the reflector R successively on different sides of the ray which passes along the axis of the tube, it will be found that when the reflector E assumes such a position that the plane of the angle of incidence upon it is at right angles to the plane of the angle of incidence upon the reflector D, no reflection will take place, and the ray will be extinguished. It follows, therefore, from this, first, that the ray passing along the axis of the tube is polarised; and, secondly, that its plane of polarisation coincides with the plane of the angle of incidence of the original ray from the reflector D.

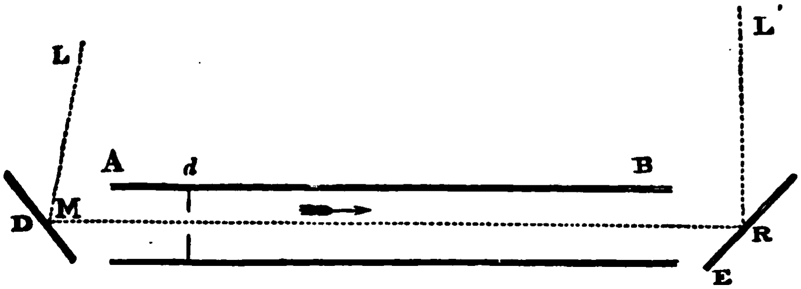

Fig. 160.

If, instead of a blackened glass, any other reflecting surface were placed in the frame D, the same effects would ensue; but the angle of incidence upon such surface which would produce polarisation, would be different for different surfaces.

265. **Method of determining the polarising angle for different reflecting surfaces.**—It was discovered by Sir David Brewster by observation, and afterwards confirmed by theory, that the polarising angle for any reflecting surface is that angle of incidence which, being added to the corresponding angle of refraction, supposing the ray to enter the medium, would make up the sum of 90°. Thus, if A B C D, *fig.* 161., be a transparent medium bounded by parallel surfaces A B and C D, and if P I be a ray of light incident upon it at such an angle of incidence P I F that the angle of refraction R I F′ corresponding to it shall, when added to F I P, make 90°, then the angle P I F will be the polarising angle, and a ray incident at such angle and reflected from I in the direction I P′ will be polarised.

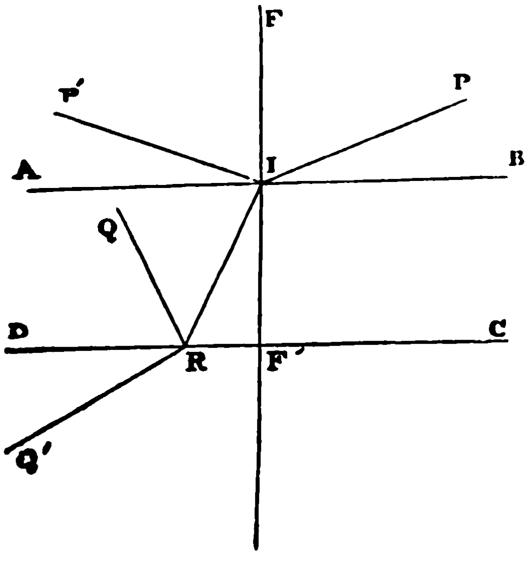

Fig. 161.

It is easy to show that in this case the directions of the reflected ray IP′ and the refracted ray IR are at right angles; for we have

$$FIP + PIB = 90^\circ.$$

And since PIB is equal to P′IA, we shall have

$$FIP + P'IA = 90^\circ.$$

But since $FIP + RIF' = 90^\circ$, it follows that

$$P'IA = RIF'.$$

If to both of these we add the angle AIR, we shall have the angle P′IR equal to the angle AIF′; but since AIF′ is 90°, the angle P′IR will be also 90°.

The angle of polarisation is therefore determined by the condition that the reflected ray IP′ shall be at right angles to the direction it would have pursued, had it been refracted instead of reflected at I.

It is easy to show that when the ray IR emerges from the lower surface in the direction RQ′, parallel to PI, it will be at right angles to the direction it would have taken, if, instead of passing through the surface at R, it were reflected from it in the direction RQ; for since RQ′ and RD are respectively parallel to PI and BI, the angle DRQ′ is equal to the angle PIB, or, what is the same, to the angle P′IA, or, in fine, to the angle RIF′.

But the angle IRF′ is equal to the angle QRD, therefore the angles RIF′ and IRF′, taken together, are equal to the angle QRQ′; and since the former are equal to 90°, QRQ′ is a right angle. Hence it follows that the ray IR also falls upon the surface D at R at the angle of polarisation, since its directions reflected and refracted are at right angles.

It follows from what precedes, that the polarising angle corresponding to any surface separating two media is that angle whose trigonometrical tangent is equal to the index of refraction; for since the angle RIF′ is the complement of the angle FIP, the sine of FIP divided by the sine of RIF′ will be equal to the tangent of the angle FIP. Thus, whenever the index of refraction for any medium is known, the polarising angle for the surface of such medium can be determined; and whenever the polarising angle can be found by observation, the index of refraction may be inferred.

Since the indices of refraction for the different component parts of solar light are different, it follows that the polarising angle for each species of homogeneous light will also be different.

266. **Table showing the polarising angle of certain media.** — Sir David Brewster gives the following table: —

		Index of Refraction.	Maximum polarising Angle.	Difference between the greatest and least polarising Angles.
			° '	° '
Water	Red rays	1·330	55 4	0 15
	Mean rays	1·336	53 11	
	Violet rays	1·342	53 19	
Plate glass	Red rays	1·515	56 34	0 21
	Mean rays	1·525	56 45	
	Violet rays	1·535	56 55	
Oil of cassia	Red rays	1·597	57 57	1 24
	Mean rays	1·642	58 40	
	Violet rays	1·687	59 21	

267. **Effects of reflection on polarised light.**—If a ray of polarised light be incident upon any plane reflecting surface, the position of the plane of its polarisation will in general be changed after reflection, and will be turned more or less towards the plane of the angle of incidence. If the angle at which the ray is incident be equal to the polarising angle, then the plane of polarisation, whatever may be its position in the incident ray, will coincide with the common plane of incidence and reflection in the reflected ray, so that the effect of reflection will be to turn this plane round the axis of the ray through the angle formed by it with the plane of incidence.

If, however, the angle at which the ray is incident be not equal to the polarising angle, then the plane of polarisation will not be turned entirely round to coincide with the plane of the angle of incidence, but will be turned towards that plane, so that the angle formed by the plane of polarisation of the reflected ray with the plane of incidence will be less than the angle formed by the plane of the angle of polarisation of the incident ray with the same plane.

The angle through which the plane of polarisation is thus turned will depend upon the relation which the angle of incidence bears to the polarising angle.

If the ray be incident perpendicularly upon the surface, no change will take place in the position of the plane of polarisation, that of the reflected ray coinciding with that of the incident ray. If the angle of incidence be very small, then the plane of polarisation of the reflected ray will be slightly turned towards the plane of incidence, and it will be more and more turned towards it as the angle of incidence approaches to equality with the polarising angle. When they are equal, the plane of polarisation will coincide with the plane of the angle of incidence. When the angle of incidence exceeds the angle of polarisation, the plane of polarisation of the reflected ray will be turned from the plane of the angle of incidence, and on the other side of it; and it will continue to be turned from it more and more as the angle of incidence is increased, until it becomes a right angle. All these phenomena can be illustrated experimentally by means of the polariscopic apparatus already described, the plane of polarisation being always capable of being determined by the means already explained.

268. **Effects of ordinary refraction on polarised light.**—When a ray of polarised light enters any transparent medium, the plane of its polarisation is changed after refraction, and is turned from the plane of the angle of incidence more or less, according as the angle of incidence differs more or less from the polarising angle. The effect, therefore, of refraction on the plane of polarisation is contrary to that produced by reflection. The more nearly the angle of incidence approaches to equality with the polarising angle, the more nearly will the plane of polarisation in the refracted ray be turned to a direction at right angles to the plane of incidence; and if the angle of incidence be absolutely equal to the polarising angle, then the plane of polarisation of the refracted ray will be at right angles to the plane of incidence, whatever may have been its position in the incident ray. It follows, therefore, that if the plane of polarisation of the incident ray be at right angles to the plane of incidence, it will suffer no change by refraction; but the further it departs from this direction, the greater will be the change produced upon it by refraction.

269. **Composition of unpolarised light.**—It was first suggested by Sir D. Brewster, and since confirmed by theory, that a ray of ordinary or unpolarised light consists of two rays polarised in planes at right angles to each other, the absolute direction of these planes being arbitrary. When such a ray is perfectly polarised, these planes of polarisation are made to coincide, either or both being turned round the axis of the ray.

Polarised rays may, however, also be obtained from a ray of natural light, either by resolving the ray into the two pencils of which it consists, and exhibiting them separately polarised in planes at right angles to each other, or by extinguishing one of the two rays, and not the other.

270. **Polarisation by double refraction.**—A double refracting crystal supplies the means of obtaining polarised rays by the first method.

When a ray of common light is incident upon such a crystal in a plane passing through its axis, it will be divided, as has been already explained, into two rays, the ordinary and extraordinary, both of which will be found to be polarised if examined by the test already explained. The plane of polarisation of the ordinary ray will coincide with the plane of the angle of incidence, and the plane of polarisation of the extraordinary ray will be at right angles to it. Thus the double refracting crystal resolves the ray of common light into its two component polarised rays, exactly as a common prism resolves a ray of solar light into its component rays of different refrangibility.

271. **Partial polarisation.**—As a ray of light is completely polarised when the two planes of polarisation naturally at right angles are brought to absolute coincidence, and as it is completely unpolarised when these planes are at right angles, it is partially polarised when they are in any intermediate position; and it approaches more and more to the state of complete polarisation as the obliquity of the two planes of polarisation increases. Thus, when they form an angle of 45°, the ray may be considered as half polarised.

It was long contended that a pencil partially polarised consisted of rays completely polarised mixed with rays completely unpolarised in various proportions, according to the degree of partial polarisation of the pencil; but Sir David Brewster suggested, what has been since confirmed by theory, that partial polarisation must be otherwise understood, and that a pencil partially polarised contains in it no ray, either perfectly polarised or perfectly unpolarised, but consists of rays, each of which is imperfectly polarised, as just explained.

272. **Polarisation by successive refractions.**—It has been already shown that a ray of polarised light, when it enters a transparent medium, and is refracted by it, has its plane of polarisation turned from the plane of the angle of incidence through an angle greater or less in magnitude according to the relation which the angle of incidence bears to the polarising angle. Now, since a ray of natural light consists of two rays of light polarised in planes at right angles to each other, such a ray, when it enters a refracting medium, will have both planes of polarisation of its component rays turned towards a right angle with the plane of the angle of incidence.

If such a ray then be successively refracted by a series of media bounded by parallel planes, the planes of polarisation of its component rays will undergo a series of changes of direction, each having a tendency to turn them into a direction at right angles to the common plane of incidence and emergence.

Sir David Brewster found that the light of a wax candle placed at the distance of ten or twelve feet from a series of parallel plates of ground glass was polarised at angles of incidence which depended on the number of plates as exhibited in the following table:—

No. of Plates of Crown Glass.	Observed Angle at which the Pencil is polarised.		No of Plates of Crown Glass.	Observed Angle at which the Pencil is polarised.	
	°	′		°	′
8	79	11	27	57	10
12	74	0	31	53	28
16	69	4	35	50	5
21	63	21	41	45	35
24	60	8	47	41	41

He inferred from these experiments that if we divide the number 41·84 by any number of crown glass plates, we shall obtain the tangent of the angle at which a pencil of light may be polarised by this number. He also inferred that the power of polarising the refracted light increased with the angle of incidence between 0, or a minimum, at a perpendicular incidence, and the greatest possible, or a maximum, as the incidence approached 90°.

The apparatus represented at H, *fig.* 159., is adapted for the experimental demonstration of this. In the tube H is placed a series of five or more plates of glass resting with their surfaces one upon the other, and capable of being adjusted in the tube so as to form any desired angle with its axis.

If this piece H be inserted in the end A of the tube, and if the plates of glass be applied at the proper angle, it will be found that the light after passing through them is nearly polarised, and that its plane of polarisation is perpendicular to the common plane of the angles of incidence and refraction. In this case the more brilliant the pencil of light transmitted through the plates, the more numerous the plates must be in order to effect complete polarisation.

Strictly speaking, no number of plates can bring the planes of polarisation to absolute coincidence; but they may be said to approach so near to it, that the pencil will be to all appearance completely polarised with lights of ordinary intensity.

A pencil thus polarised by refraction will exhibit the same properties when submitted to reflection, or when incident upon a plate of tourmaline, as has been already described with respect to light polarised by reflection.

273. **Effect of tourmaline.** — Let a plate of tourmaline be cut with surfaces parallel to each other and to its optic axis. Such a plate being fixed in the piece G (*fig.* 159.), may be inserted in the end of the tube B, so as to receive the polarised rays transmitted along the axis of the tube perpendicular to its surface. When thus arranged, the tube G being turned within the tube B, so as to bring the optic axis of the tourmaline to coincide with the plane of polarisation of the ray, the ray will be totally intercepted. If the tube be then turned, so that the axis of the tourmaline shall form an increasing angle with the plane of polarisation, light will begin to be transmitted, and the intensity of the light so transmitted will gradually increase, until the axis of the tourmaline

is at right angles to the plane of polarisation, when its intensity will be a maximum. After it passes that, the tube G being slowly turned, the intensity will again diminish until the axis of the tourmaline again coincides with the plane of polarisation, when the light will be completely intercepted. The tourmaline supplies therefore a test of polarisation and a means of ascertaining the position of the plane of polarisation more convenient still than that which has been already explained by means of the reflecting surface R.

274. **Polarisation by absorption.** — Sir David Brewster showed that agate and some other crystals had the effect of intercepting one of the two polarised rays which constitute common light, and transmitting the other; and suggested this as a means of obtaining polarised light. Thus, if a ray of common light be transmitted through a plate of agate, one of the oppositely polarised beams will be converted into nebulous light in one position of the crystal, and the other in another position, so that one of the polarised beams will in each case be transmitted. The same effect may be produced by Iceland spar, Aragonite, or artificial salts, prepared in a peculiar manner, so as to produce a dispersion of one of the two polarised rays forming common light.

If common light be transmitted through a thin plate of tourmaline, one of the polarised rays which constitute it will in like manner be absorbed by the tourmaline, and the other transmitted; and when the tourmaline is applied in a position at right angles to this, the ray which was before transmitted is absorbed, and *vice versâ.*

275. **Polarisation by irregular reflection.** — When a pencil of light is directed obliquely on any imperfectly polished surface so as to be irregularly reflected from it, the rays thus reflected will be partially polarised, as may be ascertained by looking at the reflecting surface through the plate of tourmaline G (*fig.* 159.). On turning round the plate of tourmaline, it will be found that the brightness of the surface will vary according to the direction of the axis of the tourmaline, the positions of the axis which render its brilliancy greatest and least being at right angles to each other. That the polarisation in this case is imperfect is demonstrated by the fact that the tourmaline in no position produces a complete extinction of the light.

Since light is more or less polarised by successive refractions and by successive reflections, whether regular or irregular, it follows that light is almost never found without being more or less polarised. Thus the light of day proceeding from the solar rays, reflected and refracted by the atmosphere and the clouds, must always be more or less polarised, — an effect which may be verified by examining this light by one or other of the tests of polarisation, but more especially by the tourmaline already described.

276. **Interference of polarised pencils.** — If two pencils of light have their planes of polarisation parallel, they will exhibit the same phenomena of interference as have been already described for ordinary light. The production of bright and dark fringes, when the pencils are homogeneous, and the production of coloured fringes, when the pencils consist of compound light, will occur as in the case of unpolarised light.

But if the two pencils be polarised in planes at right angles to each other, none of the phenomena of interference will be exhibited. No matter under what circumstances the rays shall intersect, it can never happen that either ray will extinguish the other, or that the phenomena of dark and light or coloured fringes are produced.

When two pencils are polarised in planes forming with each other an oblique angle, they will produce fringes, but of inferior brilliancy to those exhibited when their planes of polarisation are parallel.

If two pencils are first polarised in planes at right angles to each other, and afterwards have their planes of polarisation rendered parallel, which may always be accomplished either by refraction or reflection, they will not recover the property of forming fringes of interference, of which they were deprived by rectangular polarisation. But if a pencil of common light be first completely polarised, and then be divided into two pencils polarised in rectangular planes, these two pencils, if their planes of polarisation be again rendered parallel, will acquire the property of interference, and will exhibit fringes.

All these phenomena admit of verification by the polariscopic apparatus already described.

277. **Compound solar light cannot be completely polarised by reflection, but may be nearly so.** — Since the polarising angle varies with the index of refraction, and since white solar light is a compound of rays having different indices of refraction, it follows that a pencil of solar light can never be completely polarised by a reflecting surface, for the angle which would polarise completely one of its constituents would be different from the angle which would polarise completely another. But since the difference between the polarising angles for the extreme rays in the case of glass is only 21′, and in the case of water still less, it follows that if the polarising reflector be adjusted at the polarising angle of the rays of mean refrangibility, the rays of extreme refrangibility will fall upon it at an angle differing very little

from their polarising angle, and, consequently, although they will not be completely, they will still be very nearly polarised.

278. Nevertheless, the absence of complete polarisation in this case is rendered extremely evident by the test of the plate of tourmaline already described.

If the reflector D, *fig.* 159., be adjusted to the polarising angle of the rays of mean refrangibility, and the plate of tourmaline G be applied to the end B of the tube, the rays corresponding to the middle of the spectrum only will be completely intercepted when the axis of the tourmaline is brought into the plane of polarisation. A portion of the extreme rays at both ends of the spectrum will be transmitted through the tourmaline, and will be perceivable as bright purple light proceeding from the mixture of the red and violet rays which are transmitted. If the plate D be then adjusted to the polarising angle of the violet rays, the red rays will be transmitted in considerable quantity, and the yellow less, so that the light transmitted will be a reddish orange; and if, on the other hand, the polarising plate D be adapted to the polarising angle of the red rays, the light transmitted will be a bluish green. If the polarising plate D be composed of any highly dispersive substance, such as cassia, diamond, chromate of lead, realgar, or specular iron, the colour of the unpolarised light transmitted from the tourmaline will be found to be extremely bright and beautiful.

279. **Effect of a double refracting crystal on polarised light.** — Let us suppose a pencil of polarised light R P, *fig.* 162.,

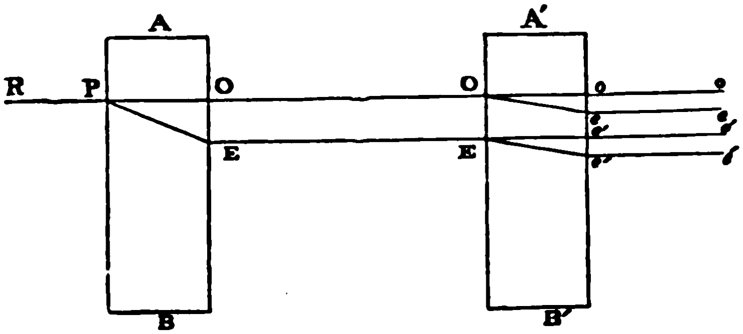

Fig. 162.

to be incident perpendicularly upon a plate A B, cut from a double refracting crystal, in such a manner that its surfaces are parallel to each other and to the optic axis of the crystal. The pencil R P, in passing through this plate, will be doubly refracted, the ordinary pencil proceeding in the direction P O O of the original pencil R P, and the extraordinary pencil taking another direction P E through the crystal, and emerging in the direction E E, parallel to that of the incident ray R P. These two pencils will be polarised in rectangular planes, the plane of polarisation of the ordinary pencil o o coinciding with the optical axis of the crystal, and the plane of polarisation of the extraordinary pencil E E being perpendicular to it.

To render this more clear, let the circle, *fig.* 163., represent a section of the incident ray R P, and let C P be the direction of the plane of primitive polarisation of the ray R P. Let C O be parallel to the optic axis of the crystal A B, and C E be perpendicular to it. It follows, therefore, that C O, *fig.* 163., will be the direction of the plane of polarisation of the ordinary pencil O O, *fig.* 162., and C E, *fig.* 163., will be the direction of the plane of polarisation of the extraordinary pencil, E E, *fig.* 162.

It follows from the principles of the undulatory theory (and this consequence is confirmed by observation) that the proportion in which the light of the original pencil R P is shared by the ordinary and extraordinary pencils O O and E E will be expressed by the squares of the cosines of the angles which the plane of primitive polarisation C P, *fig.* 163., makes with the planes of polarisation of the two pencils O O and E E, *fig.* 162., respectively.

If, therefore, the number of rays in the original pencil R P be expressed by the square of the radius, *fig.* 163., the number of rays in the ordinary pencil O O will be expressed by the square of C *m*, and the number of rays in the extraordinary pencil E E will be expressed by the square of C *n*. The changes incident to the relative intensities of the ordinary and extraordinary pencils produced by the plate A B, may then be easily inferred from the diagram, *fig.* 163.,

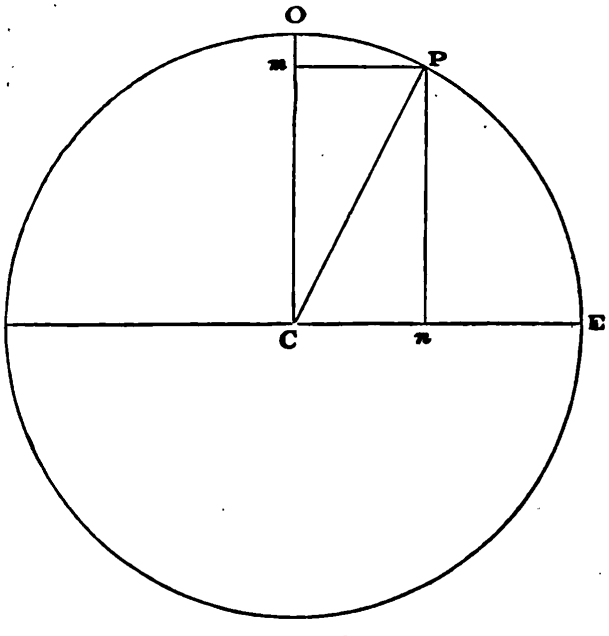

Fig. 163.

If the plane of polarisation of the original ray R P coincide with the axis of the crystal A B, then C P, *fig.* 163., will coincide with C O, and the number of rays in the pencil O O, *fig.* 162., will be expressed by the square of the radius C O, while the pencil E E will vanish; for, in this case, C *m* will become equal to C O, and C *n* will vanish.

As the plane of primitive polarisation C P makes an increasing angle with C O, C *m*, whose square represents the number of rays in the pencil O O, will decrease, and C *n*, whose square represents the number of rays in the pencil E E, will increase. The one pencil, therefore, will diminish and the other increase in intensity. When the plane of primitive polarisation C P makes an angle of 45° with the axis C O of the crystal, the line C P will bisect the angle O C E, and C *m* will become equal to C *n*. In this position, therefore, the ordinary and extraordinary pencils O O and E E, *fig.* 162., will become equally intense, or contain the same number of rays.

When the plane of primitive polarisation C P makes with the axis C O of the crystal A B a greater angle than 45°, C *m* becomes less than C *n*, and consequently the ordinary pencil O O, *fig.* 162., contains less rays than the extraordinary pencil E E; and as the angle included between C P and C O increases, the extraordinary pencil will become relatively more intense, and the ordinary pencil less so, until the plane of primitive polarisation C P makes a right angle with the axis C O of the crystal; in which case C P will coincide with C E, C *n* will become equal to C E, and C *m* will vanish.

Thus the ordinary pencil O O, *fig.* 162., will disappear, and all the rays of the incident pencil R P will pass into the emergent extraordinary pencil E E. A like succession of changes of intensity will take place if we suppose the axis of primitive polarisation C P to revolve through another quadrant; the rays of the extraordinary pencil gradually passing into the ordinary one, and the extraordinary one vanishing, and the ordinary pencil acquiring the same intensity as the incident pencil, when the plane of polarisation again coincides with the direction of the axis of the crystal.

It thus appears that in a complete revolution of the plane of primitive polarisation, or, what is the same, if that plane be fixed, in a complete revolution of the plate A B in its own plane, there will be two positions, 180° asunder, in which all the rays of the primitive pencil will pass into the ordinary pencil, and, consequently, in which the primitive pencil will undergo no change either in its intensity or its polarisation. Therefore, there will be two positions at right angles to these in which the primitive pencil again undergoes no change in intensity, but in which it is converted into the extraordinary pencil E E, its plane of polarisation being turned through 90°, and receiving a direction at right angles to that of the plane of primitive polarisation. In the intermediate positions between these four directions, the relative intensities of the ordinary and extraordinary pencils undergo constant change; that of the ordinary pencil being greater or less than that of the extraordinary pencil, according as the plane of primitive polarisation makes a less or greater angle than 45° with the axis of the crystal A B, and the intensities of the two pencils are equal in the four positions in which the axis of primitive polarisation is inclined at 45° to the axis of the crystal.

280. **Effects produced by a second double refracting crystal.**—If we now suppose the ordinary and extraordinary pencils O O and E E, *fig.* 162., to be incident perpendicularly upon another double refracting plate A B, cut with surfaces parallel to each other and to its optic axis, as before, they will each be again doubly refracted. The ordinary pencil O O will be divided into another ordinary pencil *o o* and an extraordinary pencil *e e*, while the extraordinary pencil E E will also be doubly refracted and resolved into two, an ordinary pencil *o′ o′*, and an extraordinary pencil *e′ e′*, all these four pencils emerging parallel to the primitive pencil R P.

To determine the proportion in which the rays of the original pencil R P are distributed among these four pencils, let C O, *fig.* 164., represent, as before, the direction of the optical axis of the plate A B, and therefore the plane of polarisation of the ordinary pencil O O; and consequently C E, perpendicular to C O, will represent the plane of polarisation of the extraordinary pencil E E. Let C *o* represent the direction of the optical axis of the plate A′ B′, and let C *e* be a line perpendicular to it.

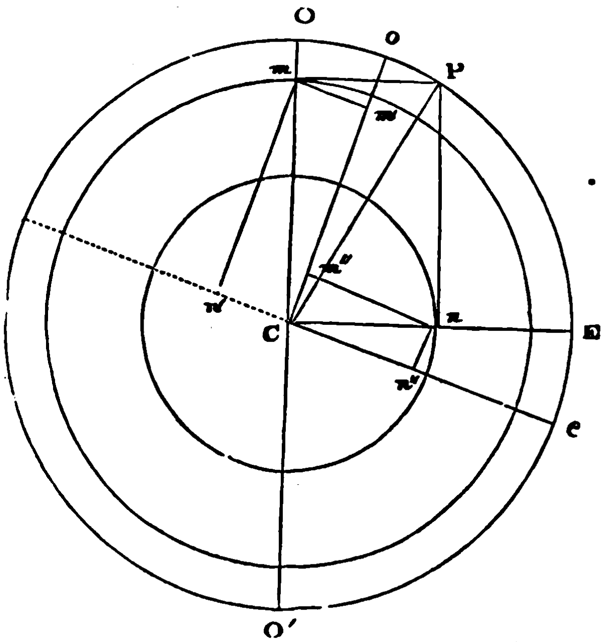

Fig. 164.

According to what has been already explained, the planes of polarisation of the ordinary pencils *o o* and *o′ o′* will coincide with C *o*, the optical axis of the plate A′ B′, while the planes of polarisation of the extraordinary pencils *e e* and *e′ e′* will coincide with the line C *e* perpendicular to C *o*.

If the square of the radius C P, *fig.* 164., express the number of rays in the original pencil R P, the square of C *m*, as already explained, will express the number of rays in the pencil O O, and the square of C *n* the number of rays in the pencil E E.

To obtain expressions for the intensities of the pencils into which these latter are resolved by the second crystal A′ B′, let circles be described with c as a centre, and c*m* and c*n* respectively as radii. From *m* draw *mn′* perpendicular to c*e* and *mm′* perpendicular to c*o*. Since, then, the square of c*m* expresses the number of rays in the pencil O O, the square of c*m′* will express the number of rays in the pencil *o o*, and the square of c*n′* will express the number of rays in the pencil *e e*.

In like manner, if from *n* we draw *nm″* and *nn″* at right angles respectively to c*o* and c*e*, the number of rays in the pencil *o′o′* will be expressed by the square of c*m″*, and the number of rays in the pencil *e′e′* will be expressed by the square of c*n″*. We shall therefore have the following analysis of the intensities of the emergent pencil of the ordinary and extraordinary rays produced by the first plate A B, and of the four pencils, ordinary and extraordinary, produced by the second plate A′B′.

Intensity of	original pencil R P	is expressed by - -	C P^2.
„	ordinary pencil O O	„ - - -	$\text{C}\, m^2$.
„	extraordinary pencil E E	„ - - -	$\text{C}\, n^2$.
„	ordinary pencil *o o*	„ - - -	$\text{C}\, m'^2$.
„	extraordinary pencil *e e*	„ - - -	$\text{C}\, n'^2$.
„	ordinary pencil *o′ o′*	„ - - -	$\text{C}\, m''^2$.
„	extraordinary pencil *e′ e′*	„ - - -	$\text{C}\, n''^2$.

If we suppose the plate A′B′ to be turned round its centre, so as to make its optical axis c*o*, *fig.* 164., revolve, making varying angles with the planes of polarisation of the rays O O and E E, a succession of changes will take place in the two pairs of ordinary and extraordinary pencils emerging from the plate A′B′, in all respects analogous to those which have been already described as having taken place in the pencils O O and E E emerging from the first plate A B.

This change can be easily inferred from *fig.* 164., where c*o* represents the direction of the optical axis of the crystal A′B′, and c O and c E the planes of polarisation of the pencils O O and E E.

Thus, if we suppose the crystal A B turned into such a position that its optical axis c*o* shall coincide with c O, then c*m′* will become equal to c*m*, and c*n′* will vanish; therefore the pencil *oo* will contain all the rays of the incident pencil O O, and will have the same plane of polarisation, while the pencil *ee* will vanish. At the same time that this takes place, c*e* will coincide with c E, and consequently c*n″* will become equal to c*n*, and c*m″* will vanish. Therefore the pencil *e′e′* will contain all the rays of the incident pencil E E. Thus it appears that in this case the second plate A B will make no change whatever, either on the intensities or the planes of polarisation of the two rays O O and E E that emerge from the first crystal A B. If the axis of the second crystal c*o* be turned round so as to make a gradually increasing angle with the axis c O of A B, then the lines c*n′* and c*m″* will gradually increase, and the lines c*m*′ and c*n*″ will gradually diminish. Therefore the intensities of the ordinary pencil *oo* will gradually diminish, and that of the extraordinary pencil *ee* will gradually increase; and, at the same time, the intensity of the extraordinary pencil *e*′*e*′ will gradually diminish, and that of the ordinary pencil *o*′*o*′ will gradually increase.

When the axis c*o* of the crystal A′B′ makes an angle of 45° with the axis CO of the crystal AB, then the four pencils will have equal intensities, for in such case c*o* will bisect the angle OCE, and the line c*e* will bisect the angle O′CE; and in this case it is evident that all the four lines c*m*′, c*n*′, c*m*″, and c*n*″ will be equal; and since their squares express the intensities of the four pencils, these intensities will be equal. When the angle formed by the axis c*o* of the plate A′B′, still increasing, forms an angle greater than 45° with the axis CO of the plate AB, then the line c*n*′ becomes greater than c*m*′, and consequently the pencil *ee* becomes more intense than the pencil *oo*. At the same time, the line c*n*″ will become less than c*m*″, and consequently the pencil *e*′*e*′ will become less intense than the pencil *o*′*o*′. These inequalities between the respective pencils will gradually increase with the gradually increasing angle formed by the axis of the plate A′B′ with the axis of the plate AB, until these axes form a right angle with each other, in which case the pencils *oo* and *e*′*e*′ will vanish, and the pencil *ee* will contain all the rays of the pencil OO, and the pencil *o*′*o*′ will contain all the rays of the pencil EE. Thus when the axis of the crystal A′B′ is applied at right angles to the axis of the crystal AB, no change is made in the intensities of the two pencils incident upon this second crystal; but if the planes of polarisation are respectively moved through a right angle, the ordinary pencil being converted into an extraordinary one, and the extraordinary pencil being converted into an ordinary pencil, it is clear that the same succession of changes will take place throughout each successive quadrant through which the optical axes of the plates are turned.

CHAP. XI.

CHROMATIC PHENOMENA OF POLARISED LIGHT.

281. **Chromatic phenomena explicable by undulatory hypothesis.** — The splendid prismatic colours arranged in the form of concentric rings, intersected by dark and bright rectangular crosses,

and occasionally by hyperbolic curves, are among the most remarkable and beautiful phenomena developed by modern experimental researches in optics. No triumph of theory can be more complete than the solution of these complicated appearances afforded by the undulatory hypothesis.

Any description, however, of these multitudinous and various appearances, much more any exposition of the mathematical solution of them supplied by the undulatory theory of light, would be incompatible with the objects and the necessary limits of this volume. While, however, we cannot enter into these details, we must not, on the other hand, pass over in absolute silence such phenomena.

282. **Effect produced by the transmission of polarised light through thin double refracting plates.** — To convey some idea of the principles on which these phenomena are based, let us suppose the plates A B and A′ B′ (*fig.* 162.), to be so thin that the separation of the pencils into which the primitive pencil R P is resolved will be inconsiderable. In such case, although the changes described in the last chapter will still be made in their planes of polarisation, the pencils will more or less overlay each other, so that the rays composing one will fall within the limits of, and be mixed with, the rays of the other.

It might therefore be inferred that the intensity or brilliancy of the pencils formed by each combination would be found by adding together the measures of their separate intensities. Thus, the two pencils *o o* and *o′ o′*, (*fig.* 164.), whose separate intensities are expressed by $c\,m'^2$, and $c\,m''^2$, would have their combined intensity expressed by

$$c\,m'^2 + c\,m''^2.$$

But it must be considered that polarised light is subject to interference when its planes of polarisation are parallel, which they are in the two cases here supposed, the planes of polarisation of the pencils *o o* and *o′ o′* being both parallel to the axis of the crystal A′ B′, and the planes of polarisation of the pencils *e e* and *e′ e′* being both perpendicular to it. If, therefore, the other conditions of interference be fulfilled, it will follow that the rays of these two pairs of pencils would alternately extinguish one another, or produce a brilliancy equal to the sum of their intensities, according to the phases under which the luminous undulations meet.

But it is easy to show that, provided one or both of the crystals A B and A′ B′ have a certain degree of thinness, the rays of the two pencils would fulfil the conditions which determine interference.

To prove this, it must be considered that the indices of ordinary and extraordinary refraction are different; therefore the velocities of the undulations in passing through the crystals will be different, if one be ordinarily and the other extraordinarily refracted; and if this difference be such as to produce by the undulation of the emergent pencils that relation which determines interference, that phenomenon must ensue. Now, on considering the refraction which the pencils *o o* and *o′ o′* have suffered, it will appear that the former has undergone ordinary refraction by both crystals, while the latter has suffered extraordinary refraction by the crystal A B, and ordinary refraction by the crystal A′ B′. Their velocities, therefore, through the crystal A B will be different; and if the thinness of the crystal be such that the undulations of the original rays are so related as to fulfil the conditions of interference, interference will ensue.

The same observations will be applicable to the pencils *e e* and *e′ e′*, the latter of which has suffered extraordinary refraction by both crystals, and the former ordinary refraction by A B, and extraordinary refraction by A′ B′.

283. **Coloured rings and crosses.** — If, therefore, the plates be reduced to such a degree of thinness as to produce the phenomena of interference, a series of bright and dark rings will be produced; but as such rings will depend on the indices of refraction, and as these indices differ for each species of homogeneous light, it will follow that a different system of rings would be produced by each species of homogeneous light of which the primitive pencil R P might be composed; and if such pencil be composed of compound solar light, then the resulting appearances are those which will be produced by the superposition of all the systems of rings which would be separately produced by each species of homogeneous light. The effect of the optical axes of the crystals, and of the revolution of either of them round its centre in its own plane, will be to produce dark or bright rectangular crosses corresponding to the planes of polarisation of the emergent pencils, these crosses intersecting the systems of coloured rings.

We have here adopted for illustration, for the sake of simplicity, the case of crystals having a single axis of double refraction. The appearances produced by crystals with two axes are analogous to these, though somewhat more complicated.

In these, two systems of rings, which sometimes assume the form of the curves called lemniscates, which have the form of the figure of 8, are produced, and the cross is often converted into hyperbolic curves, which in certain positions assume the form of a cross, the hyperbola passing into its asymptotes.

To give a complete analysis of these complicated and beautiful chromatic phenomena would be impossible within the space we can devote to them; enough, however, has been explained of the principles of polarisation to render their general theory intelligible; and we shall therefore now confine ourselves to a description of some of the most interesting of the phenomena produced by transmitting polarised light through double refracting media.

284. **Apparatus of Noremberg.** — The polariscopic apparatus of Noremberg, represented in *fig.* 165., supplies convenient means of observing and analysing the chromatic phenomena of polarised light.

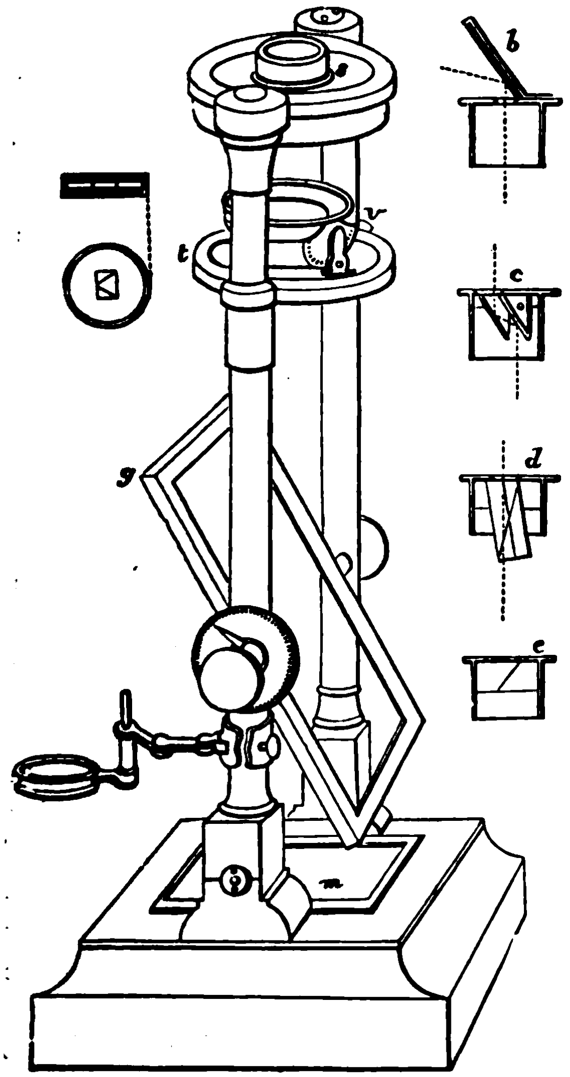

Fig. 165.

The polarising apparatus is mounted in the lower part of the instrument, and consists of the frame *g* containing the polarising plate, the horizontal reflector *m*, and other accessories. By means of these a pencil of light polarised in any required plane can be transmitted vertically upwards, so as to pass through the centre of the rings *v* and *s*.

The rings *v* and *s* are graduated, and a tube is inserted in each of them, having an index which plays on the divided scale as the tube is turned round its centre within the ring. Plane reflectors inclined at variable angles, plates of doubly refracting crystals, doubly refracting prisms, bundles of parallel plates of glass and other polariscopic tests, are set in short tubes capable of being fixed in one or other of the rings *v* and *s*. So the polarised pencil transmitted upwards along the axis of the apparatus may first be made to pass through the plate inserted in *v*, and may then be examined by an inclined reflector or tourmaline plate, a doubly refracting prism, or by any other polariscopic test which may be fixed in *s*. The position of the indices which move on the divided circles of *v* and *s* will indicate the position and changes of position of the planes of polarisation.

285. **Rock crystal.** — Let a plate of rock crystal, with surfaces cut parallel to its optic axes, the thickness of which does not exceed the 50th of an inch, be placed on the ring *v*; and let a doubly refracting prism, with a single axis of double refraction, be placed in *s*.

Let us first suppose that the axis of this prism coincides with the plane of polarisation of the pencil incident on the plate *v*, and let the axis of this plate be first placed in the plane of polarisation. In that case the incident ray will pass through both crystals without change, and an eye placed above the prism at *s* will see only the ordinary image of the object from which the pencil issues. If the axis of *v* be turned at right angles to the plane of polarisation, a single image only will be seen; but in this case it will be the extraordinary image; and the plane of its polarisation will be perpendicular to the plane of primitive polarisation. The images will in both cases be white.

In all intermediate positions of the axis of the plate *v*, two images will be seen, which will partly overlay each other, as represented in *fig.* 166. Those parts which are not superposed will have colours exactly complementary, and the superposed parts on which these colours are combined will be white.

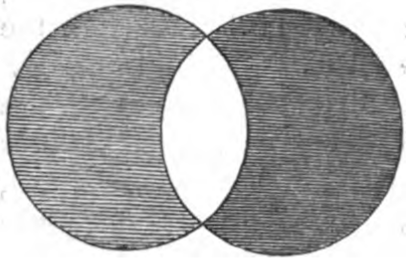

Fig. 166.

As the plate *v* is turned round its centre through 90°, from the position in which its axis coincides with the plane of primitive polarisation to the position in which it is at right angles to that plane, the two images pass through a series of tints of colour (always, however, complementary), and through various degrees of relative brightness, their most vivid colours being exhibited when the axis is at 45° with the plane of primitive polarisation.

The same changes take place in each successive quadrant through which the axis of *v* revolves.

If the axis of the prism *s* be placed at right angles with the plane of primitive polarisation, a like succession of appearances will be exhibited, the ordinary and extraordinary images, however, interchanging places.

If the axis of the prism *s* be placed at any oblique angle with the plane of primitive polarisation, a like succession of effects will be observed; but, in this case, the single images will be exhibited when the axis of the prism *s* coincides with, and is at right angles with that of the plate *v*; and the double coloured images appear in the intermediate positions, the images having the greatest splendour when the two axes intersect at an angle of 45°.

There is, therefore, in all cases, a single image in four positions in each revolution, these four positions being at right angles to each other; and intermediate between these, there are four other positions, also at right angles to each other, at which the complementary images attain their greatest brightness.

Plates of rock crystal more than the 50th of an inch in thickness produce like effects, but with less brilliant colours. In general, the colours vary with the thickness of the plate, the more brilliant tints being produced by the thinnest plates.

Different crystals exhibit striking differences in these chromatic phenomena. Thus Biot found that carbonate of lime cut parallel to the axis, required to be eighteen times thinner than rock crystal to produce the same tint. This circumstance renders it difficult to observe these phenomena with carbonate of lime.

286. Let a plate of Iceland spar less than an inch thick be cut with parallel surfaces at right angles to its optic axis. If this be placed between two plates of tourmaline cut parallel to their axis, a series of beautiful chromatic phenomena will be observed by looking through it at the clouds. If the axes of the tourmalines are placed at right angles, the crystal will exhibit a system of concentric rings of the most vivid colours, intersected by a dark cross, as represented in *fig.* 167.

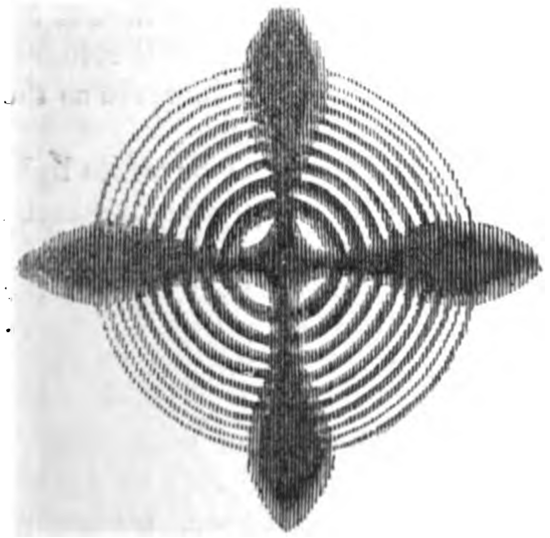

Fig. 167.

If the axis of one of the tourmalines be turned gradually round, making a decreasing angle with the axis of the other, the tints of the rings will undergo a series of changes, and the dark cross will show a space in the midst of each of its arms faintly luminous, as represented in *fig.* 168. These changes will proceed until the axis of the one tourmaline becomes parallel to the other, when the cross will become white, and all the tints of the rings will become complementary to those which they had in the first position, as represented in *fig.* 169.

If, instead of presenting the crystal to the white light of the heavens, a pencil of homogeneous light be transmitted through it, the rings, instead of showing various tints, will be alternately dark and of the colour of the homogeneous light; and the cross, in like manner, will be either dark or of the colour of the same light. The diameters of the successive rings will be different for each coloured light, being greater for the more refrangible colours; and the diameters of rings for the same colour will increase as the thickness of the crystal is diminished.

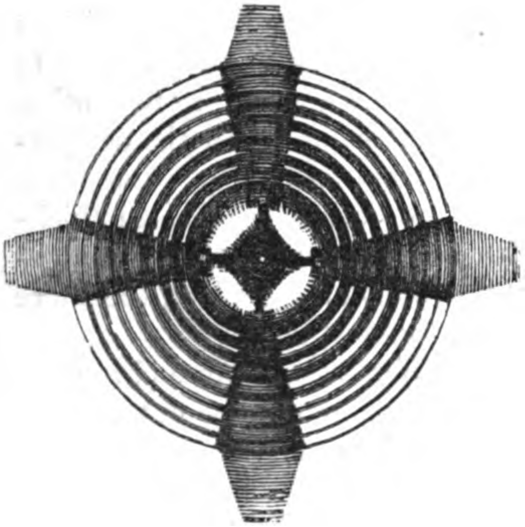

Fig. 168.

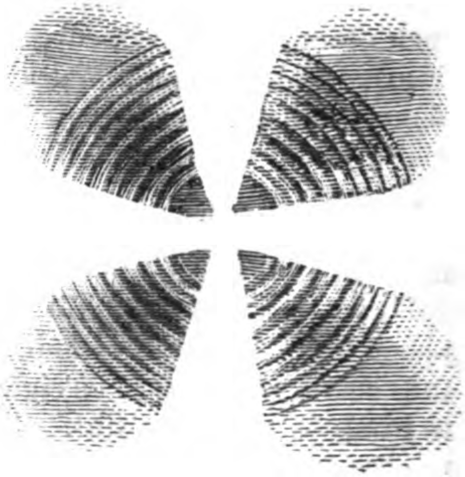

Fig. 169.

It is evident that the system of rays produced by white light results from the superposition of the several systems produced separately by the homogeneous coloured lights.

The white cross produced by white light, when the axes of the tourmalines are parallel, is in like manner produced by the superposition of all the coloured crosses produced by the homogeneous lights severally.

287. **Other uni-axial crystals.** — Phenomena analogous to these are produced by all crystals having a single axis of double refraction, such as rock crystal, tourmaline, zircon, nitrate of soda, mica, hyposulphate of lime, apophyllite, &c. In some cases, however, the effects are modified by conditions peculiar to the species of crystal under examination. Thus, in the case of rock crystal, the cross disappears, in consequence of the effect of circular polarisation, which we shall presently notice. In other crystals there appear to be different optic axes for lights of different refrangibilities, which produce modifications in the appearance of the rings and crosses.

Of all crystals the most convenient for the exhibition of these phenomena is Iceland spar.

288. **Bi-axial crystals.** — If a plate of nitrate of potash (a crystal having two axes), with parallel surfaces cut at right

angles to its optic axis, be placed in like manner between two plates of tourmaline cut parallel to their axes, a series of chromatic appearances will be observed, which are represented in *figs.* 170., 171., and 172.

If the axes of the tourmalines are placed at right angles, the crystal itself being properly placed between them, a dark cross (*fig.* 170.) will be seen intersecting a double system of coloured rings, the common centres of which correspond to the position of the two axes of the intermediate crystal.

If the crystal be turned gradually round its centre between the tourmaline plates without deranging the position of the latter, the cross will gradually assume the form of two hyperbolic curves, and the rings will change their position and tints as represented in *fig.* 171. When the crystal has been turned through half a quadrant, the appearance will be that represented in *fig.* 172., and after which it will assume a form like that of *fig.* 171., but more inclined to the horizontal position; and, in fine, when the crystal has been turned through a quadrant, the appearance will be that represented in *fig.* 170., the vertical arms of the cross, and the line joining the centres of the systems of concentric rings, being, however, horizontal.

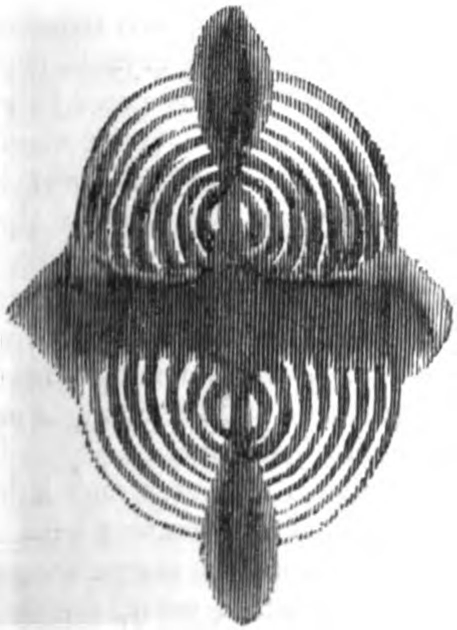

Fig. 170.

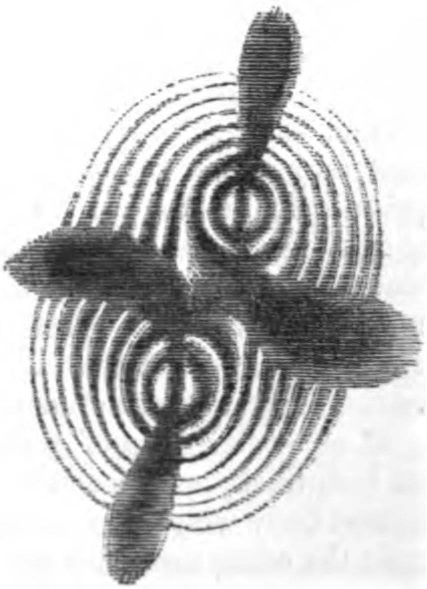

Fig. 171.

289. The carbonate of lead, another crystal with two axes, gives appearances analogous to those of nitrate of potash. These are represented in *fig.* 173.

290. **Coloured bands produced by an acute prism of rock crystal.**—If a piece of rock crystal be cut in the form of a prism

with a very acute angle, one surface forming the angle being parallel to the optic axis, and the other therefore slightly inclined to it, a pencil of polarised light transmitted through it will exhibit to the naked eye a series of alternated red and green fringes, provided the eye is placed at some distance from the crystal, and the thickness through which the light passes does not exceed the 50th of an inch. These coloured bands are more vivid when viewed through a plate of tourmaline, and it is easy to observe that they attain their greatest brightness when the axis of the prism is inclined at 45° to the plane of primitive polarisation.

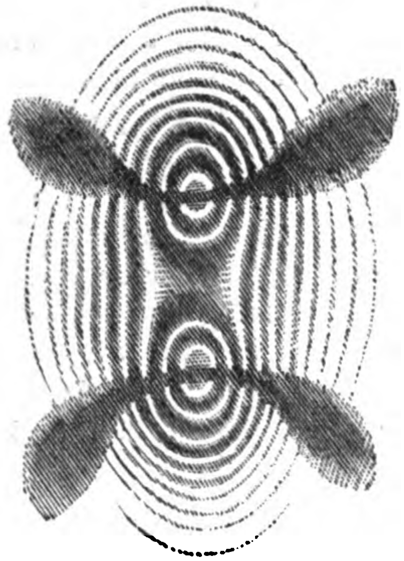

Fig. 172.

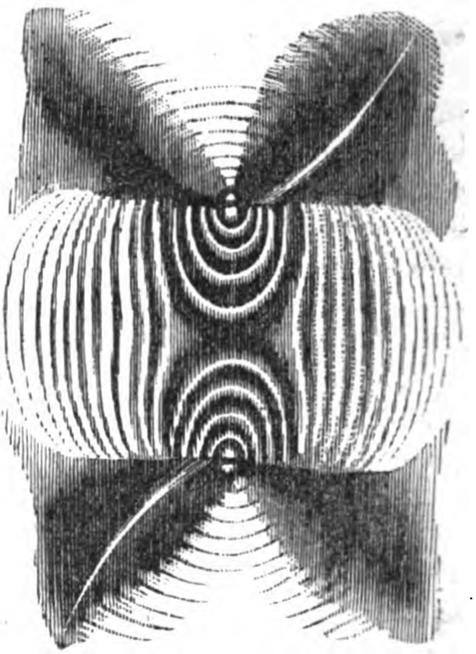

Fig. 173.

291. **Polarising structure artificially produced in glass and other media.**—A doubly refracting and polarising structure may be produced in glass and other transparent bodies by molecular changes in their structure consequent on sudden changes of temperature, and sometimes by mere mechanical pressure.

If a circular plate of glass, about an inch in diameter and half an inch thick, be exposed to a high temperature by contact with a heated body which is a good conductor, so that its temperature near the edges shall be higher than at the centre; or if, on the contrary, it be raised to a higher temperature at the centre than near the edges, it will exhibit the phenomena of rectangular crosses and coloured rings, like those produced by doubly refracting crystals.

If, in this case, the plate be oval, it will exhibit appearances indicating two axes of double refraction. When the plate is reduced to an uniform temperature, these appearances cease.

These phenomena are susceptible of infinite variation, according to the shape of the plate, which may be square, oblong, or of any other form. The disposition and form of the fringes and rings will vary with the form of the plate.

A permanent doubly refracting and polarising structure may be imparted to glass by raising it to a high temperature, and then cooling it rapidly, by placing it in contact with the cold surfaces of metals. The metallic surfaces, in this case, may be formed into an infinite variety of fancy patterns, which will have the effect of producing corresponding optical effects of great beauty.

CHAP. XII.

ROTATORY POLARISATION.

292. **Rotation of the plane of polarisation.** — When a polarised ray of homogeneous light passes through a transparent medium its plane of polarisation generally maintains a constant direction, being the same when it issues from the medium as it was when it entered it. Thus, for example, if a ray upon entering a medium have its plane of polarisation directed north and south, the plane will continue to have that direction while passing through and after issuing from the medium. Certain media, however, have been discovered which are endowed with the property of producing a continual change of direction of the plane of polarisation while the ray passes through them, imparting to it a uniform motion of rotation round the ray as an axis, so that if the ray be imagined to move through the medium with a uniform linear velocity, its plane of polarisation will revolve round it with a uniform angular velocity. Thus if we suppose that while the ray passes through a thickness of a hundredth of an inch of the medium its plane of polarisation turns through 1°, it will turn through 2° in passing through two hundredths of an inch, 3° in passing through three hundredths of an inch, and so on; so that the plane would make one complete revolution in passing through 360 hundredths of an inch, or 3·6 inches.

This phenomenon was called *circular polarisation*, but more recently the name *rotatory polarisation* has been given to it.

293. **Different media have different rotatory power.** — Transparent media which possess this property are endowed with it in different degrees; that is to say, a ray must pass through different thicknesses of them to produce a given change in its plane

of polarisation; or, what is the same, the thicknesses which would produce a complete revolution of that plane are different for different media.

294. **Right-handed and left-handed polarisation.** — It appears also that the direction of the rotation is not only different for different media, but sometimes for different specimens of the same medium.

When the rotation takes place in the direction of the motion of the hands of a watch, or of the thread of a right-handed screw, the medium is said to have right-handed polarisation, and if in the contrary direction, left-handed polarisation.

295. Different specimens of the same medium always have the same rotatory power, though the direction of the rotation may be different. Thus different specimens of rock crystal of equal thickness will always turn the plane of polarisation through the same angle; one may turn it to the right, while the other turns it to the left. If a polarised ray pass through two such plates placed in contact, its plane of polarisation will suffer no change, for it will be turned as much to the right by one as it is turned to the left by the other. If any number of plates of rock crystal of different thicknesses, some right-handed and some left-handed, be superposed, the polarised ray transmitted through them will be turned through an angle equal to the difference between the sum of all the angles through which it would be separately turned by the right-handed plates, and the sum of all those through which it would be turned by the left-handed plates. It will be turned to the right or to the left according as the sum of the thicknesses of the right-handed plates is greater or less than the sum of the thicknesses of the left-handed plates.

296. **Rotatory polarisation varies with refrangibility.**— The rotatory power of a medium varies with the refrangibility of the ray, and is found to be in the inverse proportion of the squares of the lengths of the luminous waves. Thus the degrees of rotatory polarisation produced by a given medium on homogeneous red light is less than that which the same medium would produce on homogeneous orange light, and the latter is less than it would produce on homogeneous yellow light, and so on.

297. **Results of Biot's experiments.**—In a series of experimental researches which supplied a large proportion of the discoveries made in this branch of physical optics, Biot ascertained the changes of direction produced by a plate of rock crystal having the thickness of a millimetre upon the planes of polarisation of different homogeneous rays extending from one extremity to the other of the spectrum. I have reduced his results to English measures, and have computed the several thicknesses of

the plates which would produce one complete revolution of the plane of polarisation. These are given in the following table, together with the lengths of the undulations in each case: —

	Length of an Undulation in 10,000000ths of an Inch.	Angular Deviation of the Plane of Polarisation corresponding to the Tenth of an Inch of Thickness.	Angle of Separation.	Thickness of a Plate which would produce one complete Revolution of the Plane of Polarisation.
		°	°	
Extreme red of Newton - -	253·9	44·45		0·81
Red of the glass used by M. Biot	247·3	46·77	2·32	0·77
Limit of the red and orange of the spectrum - - - - -	234·7	50·80	4·03	0·71
Limit of the orange and yellow -	224·8	56·57	5·77	0·64
Mean yellow - - - - -	216·5	60·96	4·39	0·59
Limit of the yellow and green -	209·5	65·18	4·22	0·55
Limit of the green and blue -	193·7	76·20	11·02	0·47
Limit of the blue and indigo -	180·7	87·55	11·35	0·41
Limit of the indigo and violet -	172·8	95·71	8·16	0·38
Extreme violet of Newton - -	159·8	111·91	16·20	0·32

298. **Rotatory polarisation of compound solar light.**—It appears from the numbers consigned to this table that if a ray of solar light polarised in the usual manner, so that the planes of polarisation of all its component parts shall coincide, they will, on entering the crystal separate one from another, and their angular divergence will be augmented with the thickness of the crystal through which they pass. Thus in passing through the tenth of an inch the plane of polarisation of the extreme violet is turned through 111·91°, while that of the extreme red is turned through only 44·45°. These two planes, therefore, after the original ray passes through a plate having the tenth of an inch thickness, will make with each other an angle of 67·46°.

The plate of crystal used by Biot in his experiments had the thickness of a millimetre, or about the twenty-fifth of an inch, and consequently the divergence which it imparted to the planes of polarisation of the extreme rays was only two-fifths of 67·46°, that is 27°.

299. **Polarising property of amethyst.**—In experimenting on different species of quartz, Sir David Brewster ascertained that one of them, amethyst, was characterised by the singular property of imparting alternately right and left handed polarisation to a ray passing through it, from which he inferred that this variety actually consists of alternate strata of right and left handed quartz, whose planes are parallel to the axis of double refraction of the prism.

300. **Other media.**—Quartz, though the most remarkable of the solid media having the property of rotatory polarisation, is not the only one. Sir John Herschel ascertained that camphor in

the solid state has the property; and Sir David Brewster discovered it in certain specimens of unannealed glass. Professor Dove found it in compressed glass.

301. **Rotatory polarisation of liquids.**—Biot showed that this property belongs to a great number of liquids, and those solids in which it cannot be otherwise traced exhibit it in a marked degree when they are in a state of solution.

To determine the rotatory polarisation of liquids they are included in brass tubes tinned on the inside surfaces and having their ends closed by plates of plane glass. When a polarised ray is transmitted along the axis of such a tube, its plane of polarisation will be changed if the liquid have the rotatory property, and the angle through which it is turned will be always proportional to the length of the column of liquid in the tube.

In the case of solutions having different degrees of concentration it is found that the extent through which the plane of polarisation is turned increases with their strength, and instruments have been constructed upon this principle by which the strength of solutions is determined by their power of rotatory polarisation.

302. **Physical properties detected by it.**—Biot has shown that by this means differences in the composition of bodies can be detected which altogether escape the most subtle chemical analysis. For example, it is known that sugar can be produced from various vegetable productions, such as the sugar cane, the grape and most sorts of fruit, beet, carrots, and other roots. Now the sugars produced from these several substances present to the chemist no distinguishing characteristics. Submitted to analysis, they give precisely the same constituents. Not so, however, when submitted to the test of polarised light. If, for example, sugar made from the grape be dissolved in water, the solution will be found to have left-handed polarisation, while the sugar produced from the sugar-cane has right-handed polarisation.

Biot has also shown that the rotatory polarisation of liquids, even in the case of the most concentrated solutions, is much less than that of rock crystal. Thus, for example, the concentrated solution of the sugar of the sugar-cane has a rotatory polarisation, not more than the thirty-sixth part of that of the rock crystal. In experiments on such liquids sensible effects, therefore, can only be produced by transmitting the polarised light through columns from 8 to 12 inches in length.

303. **Saccharimeters.**—In France, where a duty is levied upon the fabrication of sugar made from beet, instruments called *saccharimeters* have been constructed upon this principle, to determine the strength of the syrup, in the same manner as the hy-

drometer is used in England by the excise officer to determine the strength of spirits.

304. **Biot's rotatory polarising apparatus.**—This apparatus, which is a modified and improved form of that with which Biot made the experimental researches above mentioned, consists of three principal parts: 1st, the part *dd′* (*fig.* 174.) by which the light is polarised; 2nd, *oo* the part which supports the substance on which the experiments are made; and, 3rd, the analysing part *z* by which the state of the ray is ascertained, after it has passed through the transparent substance.

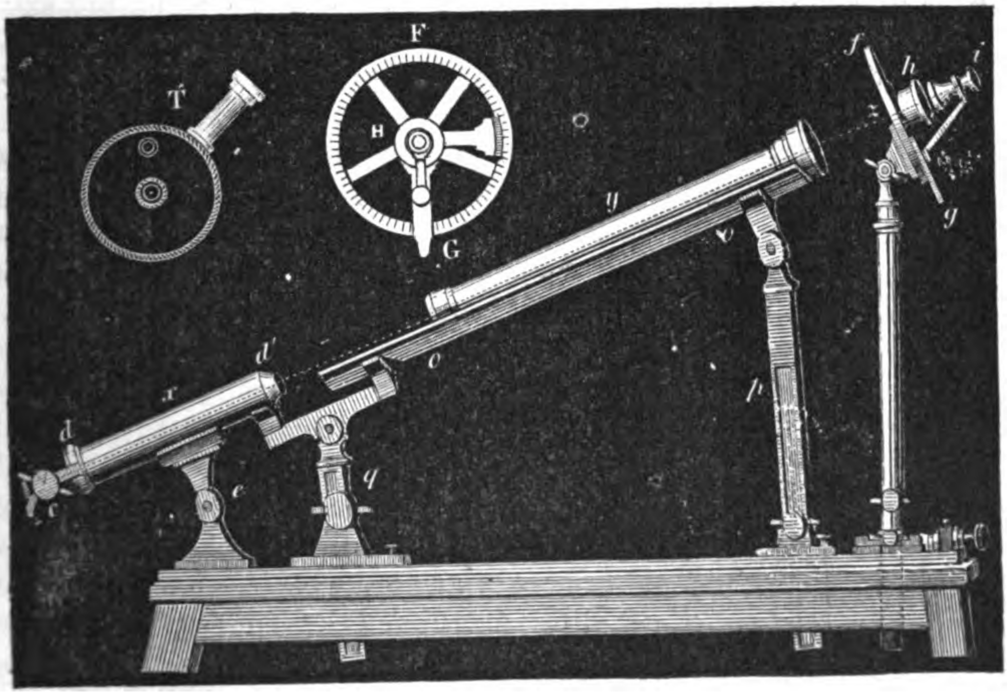

Fig. 174.

The entire apparatus is fixed upon a base of cast-iron, which is screwed down upon a strong table of wood.

The polariser *dd′* consists of a tube terminated at each end by a circular diaphragm, and having attached to it at *d* a frame carrying a square plate of blackened glass *c*, mounted so as to be inclined at any desired angle to the axis of the tube *dd′*, and to be turned round that axis in the same manner as in the polariscopes already described. If this plate be inclined to the axis of the tube at the polarising angle proper to glass, that is, at 35° 25′, it will polarise any ray incident upon it at the same angle, and will reflect that ray along the axis of the tube *dd′*.

In the stand by which the tube *dd′* is supported, a cradle-joint *e* is provided, by which the inclination of the axis of the tube can be varied at pleasure.

The apparatus for supporting the substance under experiment consists of an angular bed or gutter, *o o*, supported on two rods *p* and *q*, which are provided with cradle-joints and other adjustments, by which the inclination and elevation of *o o* can be adjusted so as to bring the object in the direction of the axis of the tube *d d'*.

The liquid solution under experiment being contained in a tube such as already described, this tube *y* is placed in the angular groove of the support *o o*, and the apparatus is so arranged by means of the joints and other movable adjustments provided in the supports of the rods *p* and *q*, that the direction of its axis shall coincide with that of the axis of the tube *x*. When this has been accomplished, the polarised ray reflected by *c* along the axis of *x* will be transmitted through the solution along the axis of *y*.

The analysing apparatus *z* consists of a divided circle, *f*, having an index *g* movable on its graduated limb, as shown in the front view presented at F G. This index *g* is connected with a double refracting prism, *h*, mounted in the centre of the circle, and a small telescope is placed at *i* in front of the prism by which the ray emerging from it can be more easily examined. When the index *g* is turned round, the prism turns round with it, and the usual effect upon the polarised ray is produced, being extinguished at two opposite positions of the index, *g*, and being brightest at the two positions at right angles to these. This apparatus, therefore, will always determine the direction of the plane of polarisation of the ray which emerges from the tube, *y*, while the position of the polarising plate *c* will determine its direction before the ray enters *y*, and the difference between these two directions will give the rotatory power of the substance under experiment.

When it is desired to ascertain the effect of the change of temperature of the medium under experiment, the tube containing the liquid is immersed in a cylindrical heater filled with a heated liquid, represented in its transverse section at T. To place this heater, the angular groove *o o* is removed, and the heater will rest in its supports. A thermometer is immersed in the liquid contained in the heater, by which its temperature, which is the same as the temperature of the liquid under experiment, can be ascertained.

It is obvious that if the object on which the experiment is made be solid, it can be easily placed in a convenient position in the angular groove *o o*, or in various other ways between the polariser and the analyser.

In all experiments made with this apparatus it is necessary to operate with homogeneous light; for the different constituent parts of solar or any other compound light, being susceptible of different degrees of rotatory polarisation, the planes of polarisation of such component parts would, after passing through the liquid, be inclined to each other, so that the position of the index *g*, which would extinguish some component parts, would not extinguish others. The consequence of this would be that, by turning the handle, certain component parts of the incident light would be extinguished, while others would remain visible. The visible colours being, in the case of solar light, always complementary to those which are extinguished.

To obviate this inconvenience, a disc of plane glass, coloured red by means of the oxide of copper, is set in the end *d′* of the tube *x*; this medium has the property of transmitting red light, which is almost perfectly homogeneous.

305. **Magnetic rotatory polarisation.** — In November 1845, Professor Faraday presented a memoir to the Royal Society, in which he announced the discovery of the action of magnetism on polarised light; thus, for the first time, establishing a connection between two physical influences, before regarded as distinct and independent, namely, the forces which impart undulation to the luminiferous ether, and those which call into play the phenomena of electricity and magnetism.

Our limits prevent us from entering into the details of Professor Faraday's important researches on this subject, for which we must refer to his published work.*

If a plate, about two inches square and half an inch thick, of the sort of heavy glass called, from its constituent parts, the silicated borate of lead, be laid upon the poles of an electro-magnet having the horse-shoe form, a polarised ray, transmitted through it in the direction of its length, will suffer no change so long as the soft iron of which the magnet is formed continues in its unmagnetised state; but the moment that an electric current, transmitted through the coils surrounding the horse-shoe (*fig.* 175.), renders the soft iron magnetic, the direction of the plane of polarisation will be changed, and its new direction will be found in the usual way by turning the analyser, as in Biot's apparatus above described. The effect of the current, when produced or discontinued, is instantaneous. If the analyser be so placed that the observer sees the polarised light transmitted through the plate before the current is established, it will be extinguished instantly upon making the connections which transmit the current, and will instantly reappear on breaking these connections.

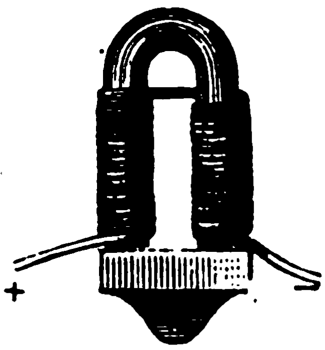

Fig. 175.

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* Experimental Researches in Electricity, vol. iii. p. 1.

If, by means of the commutator, the direction of the current is reversed, the direction of the rotatory polarisation will also be reversed, as is proved by the reappearance of the light being produced, in the one case by turning the index of the analyser to the right, and in the other by turning it to the left.

The voltaic current used in this case was produced by five pairs of Grove's batteries, and the electro-magnets had a power such that the poles would singly sustain a weight of from 28 lbs. to 56 lbs. A person looking for the phenomenon for the first time would not perceive it if a feeble magnet were used.

The same phenomena were produced, though in a more feeble degree, by a good permanent horse-shoe magnet, without the intervention of any voltaic current.

It was also found, as in all other cases of rotatory polarisation, that the angle through which the plane of polarisation was turned, was proportional to the length of the glass through which the ray passed; and that, *cæteris paribus*, the angle of rotation was proportional to the intensity of the magnetic force.

It was ascertained that many other transparent media, besides the particular species of glass above mentioned, acquire a similar property under the influence of magnetism. Transparent media, which, without the intervention of magnetic force, have a rotatory power, suffer a modification of that power from the action of magnetism. If the natural power of the medium and the effect of the magnetic influence be both right-handed or be both left-handed, the magnetism increases the rotatory power; but if they have contrary powers, it diminishes it.

Transparent media, differing from each other in all other properties, chemical, physical, and mechanical, whether they be solid or liquid, acids, alkalies, oils, water, alcohol, and ether, were all found to receive the rotatory power, and in all of them the direction of rotation was changed with the change of direction of the current. The species of glass, however, above mentioned, was found to be by far the best medium for exhibiting the phenomena.

Some further notice on the subject of magnetic polarisation will be found in Hand Book, "Voltaic Electricity," Chap. XI.

CHAP. XIII.

THE EYE.

306. Of all the organs of sense, that to which we are most largely indebted is unquestionably THE EYE. It opens to us the widest and most varied range of observation. The pleasures and advantages we derive from it, directly and indirectly, have neither cessation nor bounds. It guides our steps through the world we inhabit. It invests us with a space-penetrating power to which there seems to be no practical limit.

Although this organ, strictly speaking, is cognisant only of light and colours, yet, from an habitual comparison of combinations and tints of colour, and variations of light and shade, with the forms of bodies, as ascertained by the sense of touch, we are enabled, with the greatest facility, promptitude, and precision, to recognise by the sight, the forms, magnitudes, motions, distances, and positions, not only of the objects which surround us, and which we can approach, but also of those which are inaccessible.

This vast range of observation, however, great as it is, forms but a small part of the powers conferred by the eye. We have, besides, the inestimable advantages which arise from the ability it bestows upon us to acquire knowledge through the study of books. It enables us to converse with and derive instruction from the most learned, wise, and virtuous of our own and all former ages; and although those who have the misfortune to be deprived of this sense can, to some small extent, replace it by the ear, aided by the eye of another, yet this, and all other expedients contrived for their relief, supply results infinitely small and insignificant compared with those which are obtained by the organ itself.

The eye, considered in itself, apart from its uses, is an interesting and instructive object. It affords beyond comparison, the most beautiful example of design, structure, and contrivance that is to be found in the animal economy. Nowhere do we find so remarkable an adaptation of means to an end, of means consisting of the most profound combination of scientific principles, and an end manifesting the operation of a will directed by boundless beneficence.

307. **Structure of the eye.**—In the human race the organ of vision consists of two hollow spheres, each about an inch in diameter, filled with certain transparent liquids, and deposited in cavities of suitable magnitude and form in the upper part of the

front of the skull, on each side of the nose. These cavities are lined with soft matter, serving as a cushion for the protection of the eyeballs, which can move freely in them, the surfaces being lubricated by fluids secreted in surrounding glands. The organs are further protected from external injury by the projecting bones of the forehead above, forming the brows, the bones of the temples on the outside, those of the cheeks below, and those of the nose on the inside.

308. **The motor muscles.**—The eyeball is moved in the socket so as to be capable of being turned within certain limits in various directions by muscles inserted at different points of its surface. These are shown in *fig.* 176., where the external bones of the temple are supposed to be removed, in order to render visible the muscular mechanism. The muscle 1 raises the eyelid, and is consequently in constant action while we are awake. During sleep, this muscle being in repose and relaxed, the eyelid falls and protects the organ from the action of light. The muscle 4 turns the eye upwards, and 5 downwards, 6 outwards, and a corresponding one on the inside, not seen in the figure, turns it inwards. There are two others, 2 and 10, called *oblique muscles*, upon the effects of which anatomists are not agreed, but which are supposed to turn the eye round its axis.

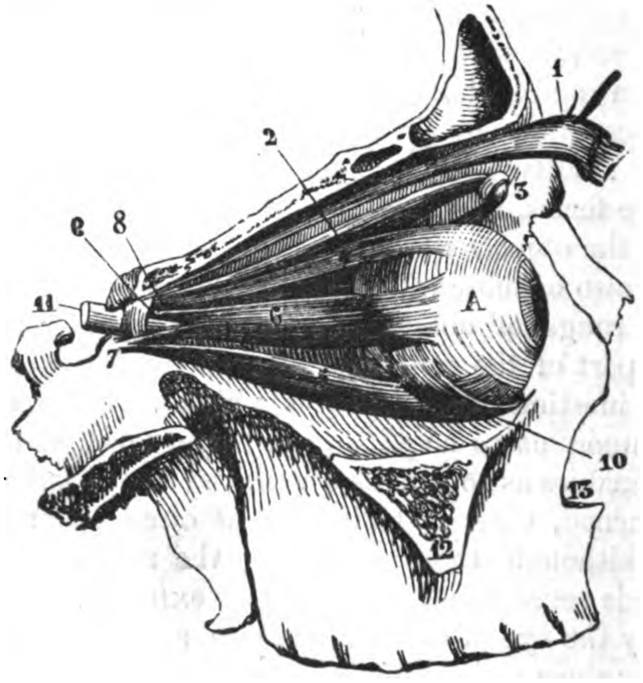

Fig. 176.

309. **Coats and humours. — Optic nerve.**—The form of the eyeball is nearly spherical, and the transparent liquids called

humours, which fill its internal cavities, are inclosed in a triple membranous envelope.

The external coat, called the *sclerotica*, upon which the maintenance of the form of the eye chiefly depends, is a strong, opaque, tough structure, composed of bundles of strong white fibres, interlacing each other in all directions. This membrane covers about four fifths of the external surface of the eyeball, leaving, however, two circular openings; a large one in front, which is covered by a transparent convex piece of nearly uniform thickness, called the *cornea*, and a smaller one behind, which is the embouchure of the nerve called the *optic nerve*, which, proceeding backwards and upwards, and, passing through foramina in the bones of the skull, terminates in the brain. It is by this nerve that the impressions made by external objects on the organ of vision are transmitted to the brain. It is represented at 11, in *fig.* 176., cut off at a point where it passes through the bones of the skull behind the eye.

310. **Cornea.** — The cornea is closely united at its edge with the corresponding edge of the circular opening in the sclerotica. It is slightly elliptical in its form, its horizontal being rather longer than its vertical diameter. Its external surface is more convex than that of the sclerotica, so that it forms a segment of a sphere smaller than that of the general surface of the eyeball. It therefore projects outwards in front of the eye, rendering that axis of the eye which passes through its centre a little longer than the diameter, which is at right angles to it. The cornea being of nearly uniform thickness, the concavity of its inner surface corresponds with the convexity of its outer, and gives the whole the character and form of a watch-glass, or a concavo-convex lens, whose surfaces have equal radii.

311. **Optic axis.** — In looking at an open eye, that part of the sclerotica which is uncovered is what is popularly called the white of the eye, and the cornea covers the coloured part.

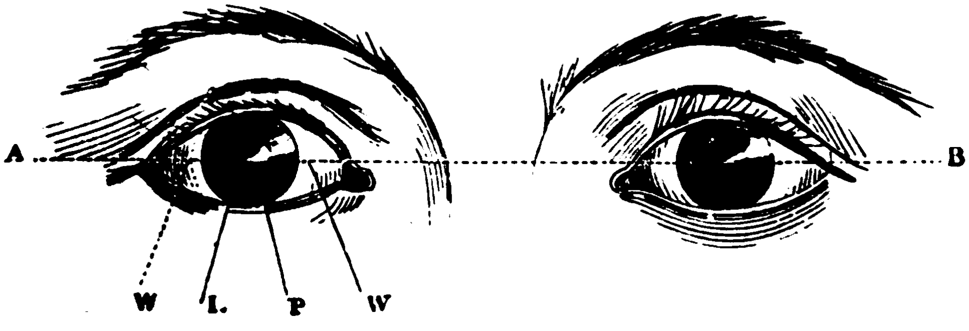

Fig. 177.

A front view of the eyes and surrounding parts is shown in *fig.* 177. a section of them, made by a horizontal plane through the line **A B** passing through the centre of the front of the eyeballs, being shown in *fig*. 178.

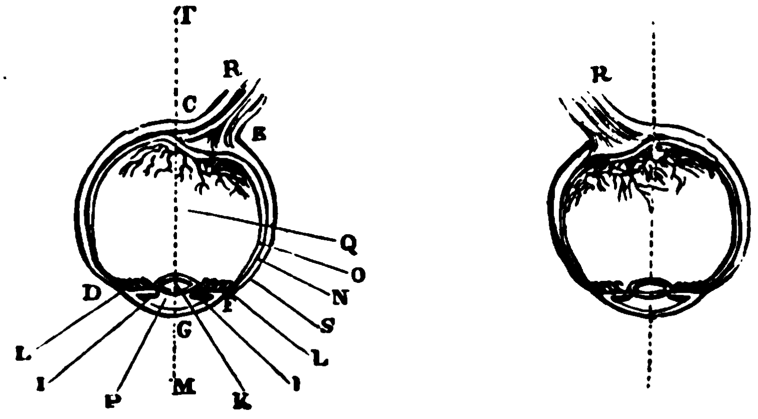

Fig. 178.

The sclerotica is shown at **C D F E**, and the cornea at **D G F**.

A line **M T**, drawn through the centre of the cornea and the centre of the eyeball is called the *optic axis*, and the embouchure **C E** of the optic nerve lies at the distance of about the tenth of an inch from this axis, between it and the nose. The optic nerves **R**, therefore, issuing from the two eyeballs at the corners, beside and

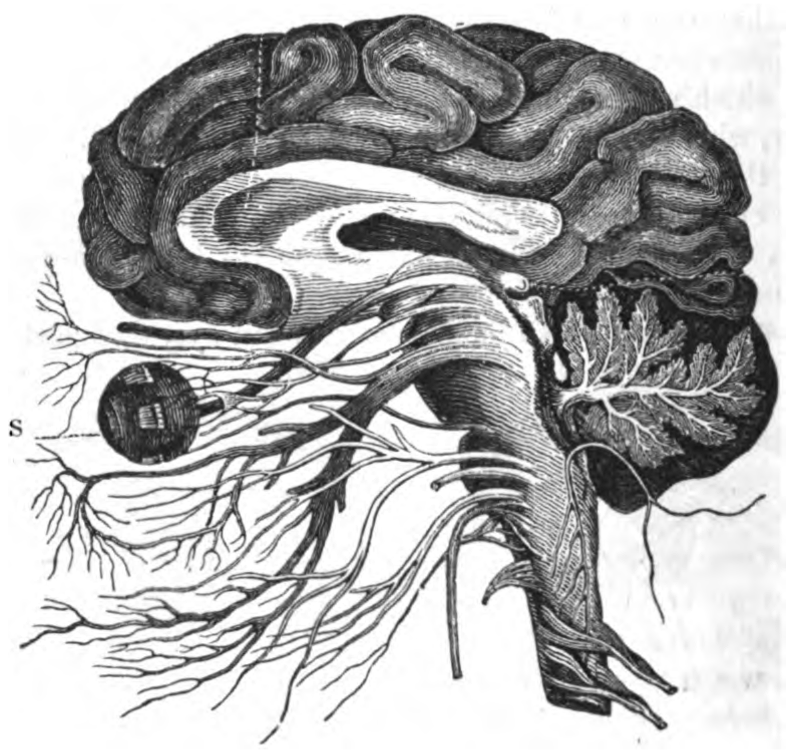

Fig. 179.

behind the nose, proceed in a converging direction to the brain, as shown in *fig.* 178.

312. **Connection with the brain.** — The manner in which the globe of the eye is connected with the brain by the optic nerve, is shown in *fig.* 179., where s is the eyeball, the end of the optic nerve entering its posterior part, and receding backwards from thence to the brain. The other nerves here represented as terminating in the eyeball are those which govern the motion of the several muscles shown in *fig.* 176., which direct the movements of the eye.

Within the sclerotica, and in contact with it, is the second coat, called the *choroid* N, (*fig.* 178.), which is a dark-coloured vascular membrane, having openings before and behind corresponding with the cornea and optic nerve, similar exactly to those of the sclerotica.

313. **Retina.** — Within this choroid is the third membranous coating (*fig.* 178.), called the *retina*, which is, in fact, the continuation of the fibres of the optic nerve spreading over the chief part of the internal surface of the eyeball.

The retina is a delicate, pulpy, and perfectly transparent membrane. It is spread over all the posterior and lateral parts of the surface, terminating near the margin of the frontal opening covered by the cornea already described.

314. **Crystalline.** — As the frontal opening of the sclerotica is closed by the cornea, that of the choroid which corresponds with it in position is closed by a transparent double convex lens, called the *crystalline lens*, the axis of which coincides exactly with the optic axis, and which is consequently concentric with the cornea. It is set in the frontal opening of the choroid by means of a series of converging folds of that membrane, which are called the *ciliary processes.* The annular surface formed by these processes, and the crystalline lens which they surround and support, form the posterior side of a compartment in the front of the eyeball, separated completely from the larger compartment behind the crystalline lens.

This arrangement will be more clearly comprehended by the enlarged section of the front of the eye given in *fig.* 180., where 2 is the sclerotica, 3 the cornea, *b* the crystalline lens, and 6 the ciliary processes.

315. **Iris.** — This compartment is partially divided by a thin flat annular diaphragm, called the *iris*, the section of which is shown at 7. This divides the space between the crystalline lens and the cornea unequally into two parts called the *anterior chamber*, *a*, and the *posterior chamber*, *a'*.

The external or anterior surface of the iris is coloured blue, black, or hazel, differently in different eyes, and is the part which, seen through the transparent cornea, gives the characteristic colour to the eye.

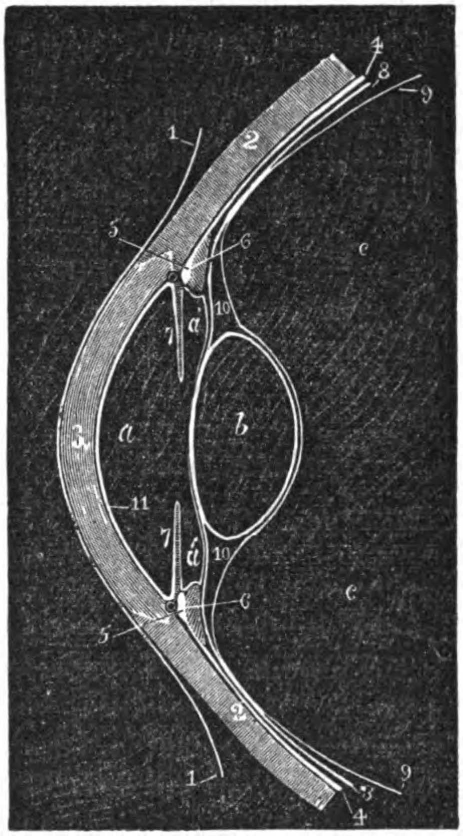

Fig. 180.

316. **Pupil.** — The circular opening surrounded by the iris is called the *pupil*, and is the space through which the light, received through the cornea, is transmitted to the crystalline lens. By this means a pencil of rays is admitted to the crystalline whose external limits are determined by the edges of the iris.

The posterior surface of the iris is covered by a black pigment, contained in a thin transparent membrane, called the *uvea*.

In *fig.* 181. a view of the ciliary processes, 1, which surround and support the crystalline lens is given. That lens, however, being supposed to be removed, the converging folds of which they consist are shown, and the iris, 2, is seen by its dark posterior surface through the space filled by the crystalline, with the pupil, 3, in its centre.

When seen from the front, the pupil appears as a black circular spot P (*fig.* 177.), surrounded by the coloured ring of the iris, because every part of the interior of the eye which could be visible through it is coloured black.

317. **Aqueous humour.** — The compartment of the eye between the cornea and crystalline is filled with a transparent liquid called the *aqueous humour*, which, as its name implies, is a watery fluid, holding in solution very minute quantities of albumen and common salt. The aqueous humour is separated from the cornea by an extremely thin transparent membrane, shown at 11 (*fig.* 180.), called the *membrane of the aqueous humour*, which, however, is represented much too thick in the figure.

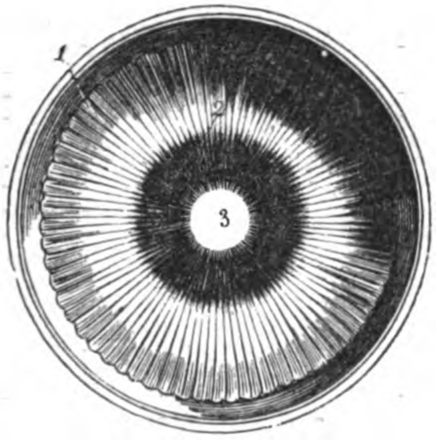

Fig. 181.

The crystalline lens *b* (*fig.* 180.) is enclosed in a transparent capsule, and consists of transparent matter, which increases in density and in its refractive power, proceeding from its external surface inwards and from its edges to its centre.

318. **Vitreous humour.** — The posterior compartment of the eye, *c c* (*fig.* 180.), behind the crystalline which constitutes by far the largest part of the internal cavity, is filled with a transparent liquid called the *vitreous humour*. This is not in immediate contact with the retina, being enclosed in a fine transparent membrane called the *hyaloid*, 9, 9, (*fig.* 180.)

319. **Eyelids. — Conjunctiva.** — The eyelids are not in immediate contact with the sclerotica or the cornea. A fine mucous membrane called the *conjunctiva*, which lines the inner surface of the eyelids is reflected over the fore part of the sclerotica and the anterior surface of the cornea. A part of this membrane is shown in section at 1, 1, in *fig.* 180.

320. **Eyebrows and other accessories.** — Some of the accessories provided for the protection and preservation of the organ of vision have been already noticed. The eyebrows across the edge of the projecting part of the forehead catch the sweat descending from above, and prevent it from falling on the eyes, and aid in shading the eyes from too intense light from above. The eyelids are movable screens, made so as to cover the eye or leave it ex-

posed, as occasion may require. Glands are provided, by which all the parts which move in contact one with another are kept constantly lubricated.

321. **Numerical data of the structure.** — The following are the principal numerical data connected with this organ : —

	100ths of Inch.
Radius of sclerotic coating	39 to 43
Radius of cornea	28 — 32
External diameter of iris	43 — 47
Diameter of pupil	12 — 28
Thickness of cornea	4
Distance of pupil from centre of cornea	8
Distance of pupil from centre of crystalline	4
Radius of anterior surface of crystalline	28 — 39
Radius of posterior surface of crystalline	20 — 24
Diameter of crystalline	39
Thickness of do.	20
Length of optic axis	87 — 95
Index of refraction from air into aqueous humour	1·3366
Index of refraction from air into vitreous humour	1·3394
Index of refraction from air into crystalline humour : —	
At the surface	1·3767
At the centre	1·3990
At the mean	1·3839
Index of refraction from aqueous humour to crystalline humour : —	
At the surface	1·0466
At the mean	1·0353
Index of refraction from vitreous humour to crystalline humour : —	
At the surface	1·0445
At the mean	1·0332

According to Sir D. Brewster, who has supplied the preceding indices of refraction, the focal length of the crystalline is 1·73 inches.

322. **The limits of the play of the eyeball** are as follows : — The optic axis can turn in the horizontal plane through an angle of 60° towards the nose, and 90° outwards, giving an entire horizontal play of 150°. In the vertical direction it is capable of turning through an angle of 50° upwards and 70° downwards, giving a total vertical play of 120°.

323. **Production of the ocular image.**—The structure of the eye being thus understood, it will be easy to explain the effect produced within it by luminous or illuminated objects placed before it.

Let us suppose a pencil of light proceeding from any luminous object, such as the sun, incident upon that part of the eyeball which is left uncovered by the open eyelids.

That part of the pencil which falls upon the white of the eye, w (*fig.* 177.), is irregularly reflected, and renders visible that part of the eyeball. Those rays of the pencil which fall upon the cornea pass through it. The exterior rays fall upon the iris, by which they are irregularly reflected, and render it visible. The internal rays pass through the pupil, and are incident upon the crystalline, which, being transparent, is also penetrated by them, from which they pass through the vitreous humour, and finally reach the posterior surface of the inner part of the eye, where they penetrate the transparent retina, and are received by the black surface of the choroid, upon which they produce an illuminated spot.

The aqueous humour being more dense than the external air, and the surface of the cornea, which includes it, being convex, rays passing from the air into it will be rendered more convergent or less divergent.

In like manner, the anterior surface of the crystalline lens being convex, and that humour being more dense than the aqueous, a further convergent effect will be produced.

Again, the posterior surface of the crystalline being convex towards the vitreous humour, and this latter humour being less dense than the crystalline, another convergent effect will take place. These rays, passing successively through these three humours, are rendered at each surface more and more convergent.

324. **Inverted picture on the retina.** — The eye, therefore, has the optical character and properties of a compound convergent lens, and will consequently form, at some point posterior to it, an optical image of any illuminated object which is presented before it. It is found that the refractive powers of the humours, and the form of their surfaces in eyes of ordinary visual power, are such that the principal focus of the organ is upon the retina at the posterior surface of the cavity, which is filled by the vitreous humour, and consequently an inverted optical picture of any distant object placed before the eye will be projected upon this part of the retina.

325. **Experimental proof of its existence.** — That this phenomenon is actually produced in the interior of the eye may be rendered experimentally manifest by taking the eyeball of an ox recently killed, and dissecting the posterior part, so as to lay bare the choroid. If the eye thus prepared be fixed in an aperture in a screen, and a candle be placed before it at a distance of eighteen or twenty inches, an inverted image of the candle will be seen through the choroid, as if it were produced upon ground glass or oiled paper.

The phenomenon can be still more manifestly shown by making an opening carefully at the upper part of the eyeball, so that the posterior part of the retina may be visible through the vitreous humour. In this case the image of any bright object, such as the window, to which the optic axis may be directed, will be seen depicted on the retina.

The experiment may be more easily performed, according to the method suggested by Magendie, by means of the eye of any albino animal, such as a white rabbit, in which the coats, from the absence of pigment, are transparent. Such an eye being dissected clean, and presented with its axis towards a window, a very distinct image of the window completely inverted will be seen depicted on the posterior semi-transparent wall of the organ.

326. **Eye achromatic.** — That the eye is sensibly achromatic is proved by the fact that the objects we behold are not edged with coloured fringes, as is the case with all lenses which are not achromatic. But if, by any means, an object be seen out of focus, that is, so that its image shall fall either before or behind the retina, the achromatism ceases, and coloured fringes become apparent. The cases in which objects are thus seen out of focus will be presently indicated.

In the analogy observable between the forms and relative densities of the transparent humours which compose this organ, the achromatic combination of lenses is too striking to be casual; and we are irresistibly impressed with the conviction that the combination is made to be nearly achromatic. The two menisci formed by the aqueous and vitreous humours, having the double convex crystalline placed between them of greater density than either, and the two former differing from each other in density, appear to fulfil the conditions of achromatism in a striking manner; and it is doubtless to this combination that is due the apparent freedom from colour in the image depicted on the retina.

327. **Eye aplanatic.** — It is also evident that the eye is aplanatic, or exempt from any sensible spherical aberration, since if it were not, the images on the retina, and consequently the perception of the objects producing them, would be more or less indistinct, which they are not. But if they are seen out of focus, as will presently appear, they become so.

It is probable, as suggested by Sir David Brewster, that the spherical aberration is corrected by the varying density of the crystalline lens, which, having a greater refractive power near its centre, refracts the central rays in each pencil to the same point as its external rays.

It appears, then, that the immediate cause of vision, and the immediate object of perception in the sensorium when we see, is the image thus produced by means of the refractive powers of the humours of the eye.

328. **Other analogies to an optical instrument.** — It may be here observed that the researches of anatomists have shown the existence of many other provisions in the internal structure of the eye, which bring it into still closer analogy with optical instruments. Not only does the iris play the part of the diaphragm provided in telescopes and microscopes to intercept the lateral rays and all stray light (being, however, more perfect than

any ordinary diaphragm, inasmuch as it is capable of enlarging and contracting the opening according as circumstances require), but its posterior surface is coated with a black pigment, so that it cannot reflect the light which it intercepts. The posterior surface of the ciliary processes is covered with the same black pigment which coats the choroid, — a provision which has the same general effect in absorbing any rays of light which may be reflected within the eye, and preventing their being thrown again upon the retina, so as to confuse the image formed upon it. The black colour given to the inner surface of telescopes and microscopes is resorted to for a like purpose.

329. **Conditions of perfect vision.** — In order to have perfect vision, the following conditions must be fulfilled: —

1°. The image on the retina must be perfectly distinct.

2°. It must have sufficient magnitude.

3°. It must be sufficiently illuminated.

4°. It must continue on the retina for a sufficient length of time.

Let us examine the circumstances which affect these conditions.

Distinctness of the image. — The image formed on the retina will be distinct or not, according as the pencils of rays proceeding from each point of the object placed before the eye are brought to an exact focus on the retina or not. If they be not brought to an exact focus on the retina, their focus will be a point beyond the retina or within it.

In either case the rays proceeding from any point of the object, instead of forming a corresponding point on the retina, will form a spot of greater or less magnitude, according to the distance of the focus of the pencil from the retina, and the assemblage of such luminous spots will form a confused picture of the object. This deviation of the foci of the pencils from the retina is caused by the refracting powers of the eye being either too feeble or too strong. If the refracting power be too feeble, the rays are intercepted by the retina before they are brought to a focus; if the refracting power be too strong, they are brought to a focus before they arrive at the retina.

330. The objects of vision may be distributed into two classes, in relation to the refracting powers of the eye: 1st, those which are at so great a distance from the eye, that the pencils proceeding from them may be regarded as consisting of parallel rays; 2ndly, those which are so near that their rays have sensible divergence.

It has been stated that the diameter of the pupil varies from $\frac{1}{8}$ to $\frac{1}{4}$ of an inch in magnitude, the variation depending upon a power of dilatation and contraction with which the iris is endued. Taking the diameter of the pupil at its greatest magnitude of a quarter of an inch, pencils proceeding from an object placed at the distance of three feet from the eye would have an extreme divergence amounting to about a third of a degree; and if the pupil be in its most contracted state when its diameter is only one eighth of an inch, then the divergence of the pencils proceeding from such an object would amount to about a sixth of a degree. It may therefore be concluded that pencils proceeding from all objects more distant from the eye than two or three feet may be regarded as consisting of parallel rays.

The pencils of rays, therefore, proceeding from all such objects, will be made to converge to the principal focus of the eye.

331. **Optical centre of the eye.** — Sir David Brewster concludes that the optical centre of the eye, that is to say, the point at which the axes of secondary pencils intersect the optic axis, is situate in the geometric centre of the eyeball, and consequently must be a little within the crystalline. If, therefore, round this centre we imagine a spherical surface described, whose radius is equal to the focal distance of the combination of the humours of the eye, the image of all objects more distant from the eye than two or three feet will be found on such a surface. Now, since the retina is spread over the surface of the choroid, and since the form of the eye is very nearly spherical, and its diameter but an inch, it follows that the retina is a concave spherical surface, whose centre coincides with the optical centre of the eye, and is at a distance from that centre of about half an inch. If the distance of the retina from this centre be exactly equal to the focal distance of the humours, then the foci of all pencils of parallel rays entering the eye will be formed upon it, and consequently it will receive distinct images of all objects whose distance from the eye exceeds two or three feet. But if the focal distance of the humours be less or greater, then, as already stated, the image on the retina will be indistinct.

332. **Optical remedies for defects in the refracting powers of the eye.** — The remedy for such a defect in vision is supplied by the properties of convergent and divergent lenses, already explained.

If the eye possess too little convergent power, a convergent lens is placed before it, which, receiving the parallel pencils, renders them convergent when they enter the pupil, and this enables the eye to bring them to a focus on the retina, provided the power of the lens be equal to the deficient convergence of the eye.

If, on the other hand, the convergent power of the eye be too great, so that the parallel rays are brought to a focus before arriving at the retina, a divergent lens is placed before the eye, by means of which parallel pencils are rendered divergent before they enter the pupil; and the power of the lens is so adapted to the convergent power of the eye, that the rays shall be brought to a focus on the retina.

The two opposite defects of vision here indicated are generally called, the one *weak-sightedness* or *far-sightedness*, and the other *near-sightedness*.

If the objects of vision be placed so near the eye that the rays composing the pencils which proceed from them have sensible divergence, then the foci of these rays within the eye will be at a distance from the optical centre greater than the principal focus. If, therefore, in this case, the principal focus fall upon the retina, the focus of rays proceeding from such near objects would fall beyond it, and consequently the image on the retina would be indistinct.

333. **Adaptation of the eye to different distances.** — If it be admitted that the formation of a distinct picture at the posterior part of the eye be essential to distinct vision, and that the focus of the eye be regulated by the same principles as that of a convergent lens, it will necessarily follow that, supposing the eye to be so constituted as to have its principal focus on the retina, the foci of all pencils of divergent rays must necessarily be behind the retina. Now, since all objects at less distances from the eye than two feet, transmit pencils sensibly divergent, the foci of all such pencils being behind the retina, the picture on the retina, and consequently the vision of the object, would be necessarily indistinct, and the less the distance of the object from the eye, the greater would be the distance of the foci of the pencils behind the retina, and the more indistinct would be the vision.

Nevertheless, it is found, in fact, that eyes which are capable of distinct vision at distances greater than two feet, are also capable of equally distinct vision at distances considerably within that limit. Thus, most eyes are capable of distinct vision at the distance of eight or ten inches, and many at even less distances. It must therefore be inferred either that there is in the eye such a power of voluntary change, as is sufficient to vary its convergent power on the light transmitted through it, so as to bring forward to the retina the foci of rays diverging from points at eight or ten inches from it, or, that it is so constituted as to bring all pencils, which have a divergence less than those proceeding from objects at eight or ten inches distance, to an exact focus on the retina, without any change in its form or in the state of its humours.

There is, perhaps, no point in physical science upon which more diversity of opinion has prevailed than this. Some eminent physiologists, among whom may be named De la Hire, Haller, Magendie, Simonoff, and Treviranus, have absolutely denied, as a matter of fact, that the eye does undergo any change of form or state in looking at distant and near objects, and the last-mentioned of these philosophers has professed to demonstrate that such a constitution of the humours is possible as would cause all the pencils whose divergence varies within the supposed limits, to come to a focus on the retina. Not only, however, has the validity of the reasoning by which Treviranus supports his hypothesis been called in question, but it has been demonstrated, as a matter of fact, that the state of an eye which sees distinctly objects at eight inches or less distance, is different from the state of the same eye when it sees distinctly objects at distances exceeding two or three feet. This has been established by various experiments.

334. **Experimental proof of voluntary adjustment.** — If we close one eye, and place two needles in the direction of the axis of the other, one at eight inches and the other at twenty-four inches distance, so as not actually to intercept each other, it will be found that the eye cannot see distinctly both needles at the same time, but that by a voluntary act it can render the vision of one or the other distinct. If by this voluntary effort the more distant needle is seen distinctly, the nearer one will be indistinct, and if, on the other hand, the nearer needle be seen distinctly, the more distant one will be indistinct.

It is clear, therefore, that the convergent power of the eye is varied by some action produced upon it; so that in the one case it brings rays which are sensibly parallel to a focus on the retina, while the focus of rays sensibly divergent is behind the retina; and that in the other case, the latter rays are brought to a focus upon the retina, while the focus of the former is in front of it.

335. **Hypotheses which explain this power.** — It appears, therefore, that the power of the eye to refract the pencils of light incident upon it, is to a certain extent under the control of the will; but by what means this change in the refracting power of the organ is made, is not so apparent. Various hypotheses have been advanced to explain it. According to some, the form of the eyeball, by a muscular action, is changed in such a manner as to increase the length of the optic axis, and thus to remove the posterior surface of the retina to a greater distance from the crystalline, when it is necessary to obtain a distinct view of near objects; and, on the contrary, to elongate the transverse diameter of the eye, and shorten the optic axis so as to bring the retina closer to the crystalline, when it is desired to obtain a distinct view of distant objects.

According to others, this change of form is only effected in the cornea, which being rendered more or less convex by a muscular action, gives a greater or less convergent power to the aqueous humour.

According to others, the eye accommodates itself to different distances by the action of the crystalline, which is moved by the ciliary processes either towards or from the cornea, thus transferring the focus of rays proceeding from it within a certain limit of distance to and from the retina; or, by a similar action of the ciliary processes, the crystalline lens may be supposed to be rendered more or less convex, and thus to increase or diminish its convergent power.

336. **Extent of the adjustment.** — To estimate these several hypotheses, it is necessary previously to ascertain what adjusting power the eye must have, to explain the admitted limits of distinct vision. Calculations based on the known refraction of the organ show, that the principal focus is nine tenths of an inch from the centre of the cornea, and that the focus of a pencil diverging from a point four inches distant, would be an inch from the same point. A power of adjustment which would vary the focus at will, through a space not exceeding the tenth of an inch, would therefore be sufficient to explain the adaptation of the eye to distinct vision at different distances, and would show how it is that we see distinctly distant and near objects, within the known limits of vision.

The very minute amount of this adjustment supplies a satisfactory answer to those physiologists who deny, as a matter of fact, any such change in the eye as would explain the phenomena. It is obvious that a change so extremely minute, can easily be imagined to elude all practical means of appreciation.

Admitting, then, the existence of this power of adjustment, it remains to examine the several expedients by which it may be imagined to be exerted.

337. **Dilatation and contraction of the pupil.—Its uses.**— Whatever be the other changes in the internal structure of the eye, it seems to be generally admitted, as a matter of fact, that the pupil is contracted when near objects are viewed, and enlarged when the attention is directed to more distant ones. Such a change of magnitude of the pupil must obviously be produced by the fibrous structure of the iris (*fig.* 181.).

Now this change of magnitude of the pupil is attended with two consequences, both of which are important. It has been shown that the density of the light received from each point of any visible object decreases as the square of the distance of the object increases. Consequently, the number of rays received from each point of a distant object within an opening of given magnitude is less in proportion to the square of the distance than the number received from a near object. To obtain sufficient light, therefore, from the more distant objects, is one of the purposes of the enlargement of the pupil.

But another effect of the change of magnitude of the pupil is a corresponding change of the extent of the surface of the crystalline, which is exposed to the light proceeding from the object. When the pupil is contracted, as it is when the object is near, the light passes through the central portion only of the crystalline; but when the pupil is more dilated, the light is also admitted through its borders.

338. **Its combination with the varying density of the crystalline.** — These circumstances have been proposed by some as the explanation of the variation of the power of vision. It has been already shown that the convergent power of the central part of an ordinary lens is less than that of its borders. But if the material of which the lens is composed vary in its density, so as to give a greater refracting power to the central parts, it may be imagined that the convergent power of these may be greater than that of the borders; and if this be assumed to be the case with the crystalline, it may be conceived that a distinct picture of near objects, might be formed on the retina by the central part of the lens, while a distinct picture of distant ones would be formed by its border.

Admitting this, the contraction of the pupil would explain the distinct vision of near objects, but its expansion would not so satisfactorily explain that of distant ones, since the distinct picture formed by the border of the lens would be rendered more or less confused, by the superposition of the indistinct picture formed by its central part. To this, however, it is answered that when two pictures of the same object are presented, one distinct, and therefore satisfactory, and the other confused, and therefore unsatisfactory, the former engrosses exclusively the attention of the mind, which is altogether unconscious of the latter.

That such a mental phenomenon is in accordance with all the analogies offered by the experience of the senses will be readily admitted. If the ear is affected by sounds, some of which are distinctly articulated while the others are confused, we listen by preference to the former, and soon become unconscious of the latter.

339. **Brewster's experiments.** — Sir David Brewster, to whose researches, more than to those of any other living philosopher, science is indebted for the knowledge we possess of the functions of the eye, made a series of experiments and observations with a view to the solution of this question, from which he inferred that, although the enlargement or contraction of the pupil does actually take place, according as near or distant objects are viewed, it is not itself the cause of distinct vision, but this effect must be ascribed to some action which necessarily accompanies such vari-

ation in the magnitude of the pupil. He concludes that the eye adjusts itself to vision at different distances by two actions; one depending on the will, and the other on the stimulus of light alling on the retina, and that when the voluntary power fails, the adjustment may still be effected by the stimulus of light. As to the peculiar action by which the adjustment is actually produced, he considers that it is nearly certain that the same muscular action which contracts the pupil, brings the crystalline lens closer to it, and consequently brings forward the focus to the retina.

340. **Volkmann's objection.** — To all hypotheses which ascribe the adjusting power of the eye to the mere enlargement or contraction of the pupil, the objection advanced by Volkmann is unanswerable. This objection is based upon the well-known fact that the pupil is enlarged or contracted according as the intensity of the light to which it is exposed is decreased or augmented. If it were admitted that the enlargement of the pupil diminishes the convergent power of the eye, and that its contraction increases that power, it would follow that with every variation of the intensity of the light surrounding us, objects would become distinct or indistinct, which is not in accordance with facts.

These hypotheses may also be set aside by the experiments made with an artificial pupil, consisting of a pin hole in a card. If the two needles placed at different distances, already mentioned, be viewed through such a pin hole, it will be found that the eye still exerts the same power of adjustment by which it can see either distinctly, although, the pin hole being less in magnitude than the pupil, no enlargement or contraction of the latter can have any effect on the phenomenon.

341. **Limits of the power of adaptation to varying distance.** — Whatever be the provisions made in the organisation of the eye, by which it is enabled to adapt itself to the reception of divergent pencils proceeding from near objects, the power with which it is thus endued has a certain limit. Thus, eyes which see distinctly distant objects, and which therefore bring parallel rays to a focus on the retina in their ordinary state, are not capable of seeing distinctly objects brought nearer to them than eight or ten inches. The power of accommodating the vision to different rays is, therefore, limited to a divergence not exceeding that, which is determined by the diameter of the pupil compared with a distance of about ten inches. Now, as the diameter of the pupil is most contracted when the organ is directed to such near objects, we may assume it at its smallest magnitude, or one eighth of an inch, and therefore the divergence of a pencil proceeding from a distance of ten inches would be about 45′.

It may, therefore, be assumed that eyes adapted to the vision of distant objects, are in general incapable of seeing distinctly objects from which pencils have greater divergence than this, or, which is the same, objects applied at less than ten or twelve inches from the eye.

342. **Eyes of feeble convergent power.**—In the case of eyes whose convergent power is too feeble to bring pencils, proceeding from distant objects, to a focus on the retina, they will be, in a still greater degree, inadequate to bring pencils to a focus which diverge from near objects; and consequently such eyes will require to be aided, for near as well as distant objects, by the interposition of convergent lenses. It would, however, be necessary to provide lenses of different convergent powers for distant and near objects, the latter requiring a greater convergent power than the former; and in general the nearer the object viewed, the greater the convergent power required from the lens.

343. **Eyes of strong convergent power.**—In the case of eyes whose convergent power is so great as to bring pencils proceeding from distant objects to a focus short of the retina, and which, therefore, for such distant objects, require the intervention of divergent lenses, distinct vision will be attained without the interposition of any lens, provided the object be placed at such a distance, that the divergence of the pencils proceeding from it shall be such, that the convergent power of the eye bring them to a focus on the retina.

Hence it is that eyes of this sort are called *short-sighted*, because they can see distinctly such objects only, as are placed at the distance which gives the pencils proceeding from them such a divergence, that the convergent power of the eye would bring them to a focus on the retina.

344. **Power of lens required by defective eyes.**—If it be desired to ascertain the focal length of the divergent lens, which such an eye would require to see distant objects distinctly, it is only necessary to ascertain at what distance it is enabled to see distinctly the same class of objects without the aid of a lens. A lens having a focal length equal to this distance, will enable the eye to see distant objects distinctly, because such a lens would give the parallel rays a divergence, equal to the divergence of pencils proceeding from a distance equal to its focal length.

345. **Short-sighted eyes.**—Persons are said to be more or less near-sighted, according to the distance at which they are enabled to see objects with perfect distinctness, and they accordingly require, to enable them to see distant objects distinctly, diverging lenses of greater or less focal length.

As persons who are enabled to see distant objects distinctly have the power of accommodating the eye so as to see objects at ten or twelve inches' distance, so short-sighted persons have a similar power of accommodation, but within proportionally smaller limits. Thus a short-sighted person will be enabled to see distinctly objects placed at distances from the eye varying from three or four inches, according to the degree of short-sightedness with which he is affected.

346. **Causes of short sight and long sight.** — The two opposite defects of vision which have been mentioned, arising from too great or too little convergent power in the eye, may arise, either from a defect in the quality of the humours or in the form of the eye. Thus near-sightedness may arise from too great convexity in the cornea or in the crystalline, or it may arise from too great a difference of density between the aqueous humour and the crystalline, or between the crystalline humour and the vitreous, or both of them; or, in fine, it may arise from defects both of the form and of the relative densities of the humours.

347. **Imperfect transparency of the humours.** — In a certain class of maladies incidental to the sight, the humours of the eye lose in a greater or less degree their transparency, and the crystalline humour is more especially liable to this. In such cases vision is sometimes recovered by means of the removal of the crystalline humour, the organ being thus reduced to two humours, the aqueous and the vitreous; but as the eye owes in a greater degree to the crystalline than to the other humours the convergent power, it is necessary in this case to supply the place of the crystalline by a very strong convergent lens placed before the eye.

348. **Experiment of Scheiner.** — By this well-known experiment a remarkably clear experimental analysis of the convergent power of the eye is obtained.

Let two holes be pierced in a card with a fine needle, at a distance from each other not greater than the eighth of an inch, and therefore less than the diameter of the pupil. Let the card be placed close to the eye, so that the middle of the interval between the holes shall correspond with the optic axis, and, consequently, with the centre of the pupil, and so that the line joining the holes shall be vertical. Let a small object be then placed in the continuation of the optic axis. The rays proceeding from this object and passing through the two holes will then fall within the pupil, and will be brought to a focus at some point upon the optic axis, either before, upon, or behind the retina, according to the distance of the object from the eye and to the converging power of that organ. If the object be moved alternately towards and from the eye, the place of its distinct image will accordingly be moved on the optic axis, and in the same direction, so that it will alternately approach to and recede from the retina, and, in a certain position of the object, its image will be upon the retina. Now, it will be found that there is only one position of the object at which it will be seen singly and distinctly. At all other distances from the eye, two images of it will be visible but indistinct, the line joining them being vertical. As the object is gradually moved from the position which gives a single and distinct image either towards or from the eye, the two images formed will be observed to move gradually from each other in the vertical line, one ascending and the other descending, and to become more indistinct in proportion as they are more separated.

These remarkable phenomena are visible indications of what goes on within the eye, as may be easily demonstrated.

Let P P′, (*fig.* 182.) represent a vertical section of the opening of the pupil, and let A and A′ be the two holes on the screen; let O be the place of the object on the continuation of the optic axis, when it is seen single and distinct. Let R R be the retina, and *o* the distinct image of O produced upon it by the pencils diverging from O, and rendered convergent by the humours of the eye. One of these pencils diverging from O passes through the hole A, and

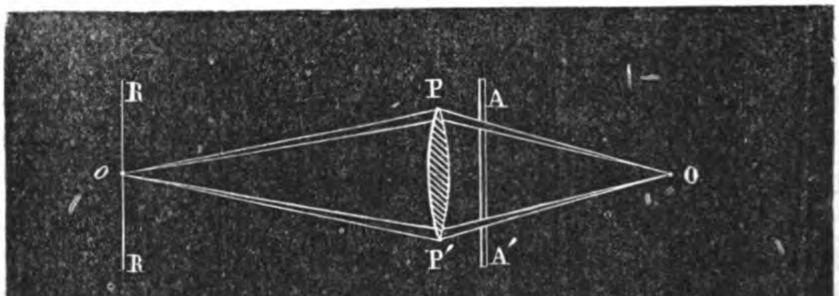

Fig. 182.

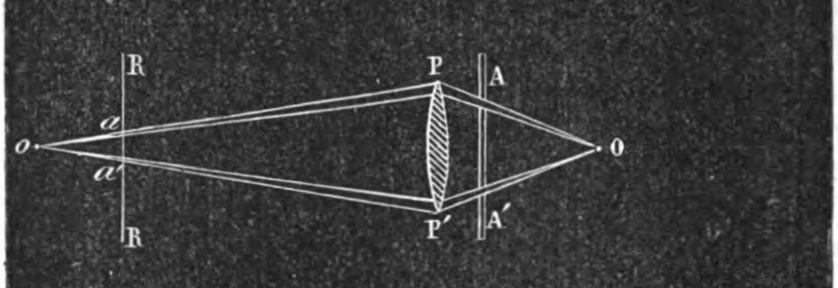

Fig. 183.

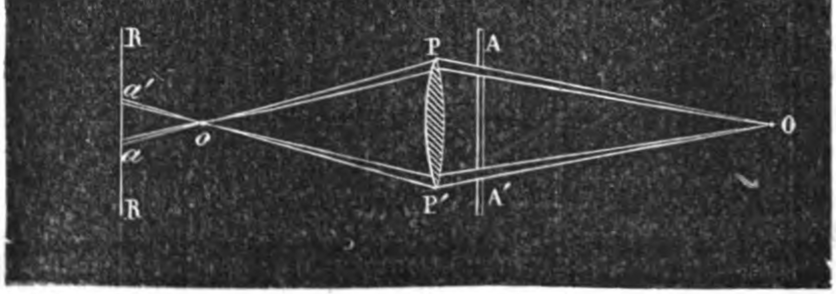

Fig. 184.

through the pupil at P, and is made to converge from P to *o*, the other pencil diverging in like manner from O passes through A′, and through the pupil at P′, and then, like the other, converges to *o*, both pencils uniting to form the distinct image of O upon the retina.

But if the object O be moved nearer to the eye, as shown in *fig.* 183., its image *o* would be formed behind the retina, and the rays of the pencil O P, being intercepted by the retina before arriving at *o*, would form at *a* an indistinct image of O. In like manner the rays of the pencil O P′, being similarly intercepted, would form another indistinct image of O upon the retina at *a′*. The line joining the two indistinct images would be vertical. That the upper image *a* is that produced by the rays passing through the upper hole A, is proved by the fact that if the upper hole A be stopped, the image *a* will instantly disappear. In like manner, if the lower hole A′ be stopped, the lower image *a′* will disappear.

If the object O be gradually moved towards the eye, the place at which its image *o* would be formed will recede farther and farther behind the retina, and the distance *a a′* between the images, as well as their indistinctness, would be proportionally augmented.

If, on the contrary, the object O be removed to a distance from the eye greater than that which is represented in *fig.* 182., its image *o* will be formed, as shown in *fig.* 184., in front of the retina, and the pencils P *o* and P′ *o* which converged from P and P′ to *o* will diverge from that point, and will form two indistinct images, *a a′*, on the retina, the line joining which will be vertical. The lower image *a*, will be that produced by the pencil passing through the upper hole A, as may be proved by the disappearance of *a* upon stopping the hole A. In the same way it may be proved that the upper image *a′* is that produced by the light which passes through the lower hole A′.

349. **Magnitude of the image on the retina.** — In order to obtain a perception of any visible object, it is not enough that the image on the retina be distinct, it must also have a certain magnitude.

Let us suppose that a white circular disc, one foot diameter, is placed before the eye at a distance of 57⅓ feet. The axes of the pencils of rays proceeding from such disc to the eye will be included within a cone, whose base is the disc, and whose vertex is in the centre of the eye. These axes, after intersecting at the centre of the eye, will form another cone, whose base will be the image of the disc formed upon the retina. The common angle of the two cones will, in this case, be 1°.

Let A B, *fig.* 185., be the diameter of the disc. Let C be the centre of the eye, and let *b a* be the diameter of the image on the retina. It is clear, from the perfect similarity of the triangles A C B and *a* C *b*, that the diameter of the image *b a* will have to the diameter of the object B A the same proportion as the distance *a* C of the retina from the centre C has to the distance AC of the object from the same centre. Therefore in this case, since one half the diameter of the eye is but half an inch, and the distance

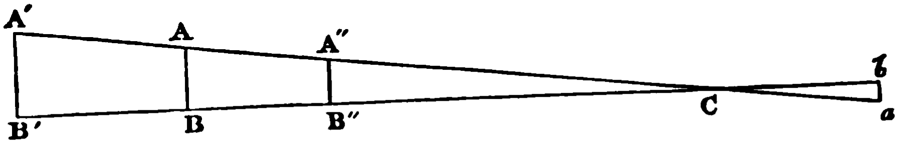

Fig. 185.

AC is in this case supposed to be 57½ feet, the magnitude of the diameter *ba* of the image on the retina will be found by the following proportion: —

$$ab : \mathrm{AB} :: \tfrac{1}{2} : 57\tfrac{1}{2}\times 12 = 690.$$

Therefore we have

$$ab = \frac{\frac{1}{2}\times \mathrm{AB}}{690} = \frac{6}{690} = \frac{1}{115}.$$

The total magnitude, therefore, of the diameter of the image on the retina would in this case be the $\frac{1}{115}$th part of an inch; yet such is the exquisite sensibility of the organ, that the object is in this case distinctly visible.

If the disc were removed to twice the distance here supposed, the angle of the cone C would be reduced to half a degree, and the diameter of the image on the retina would be reduced to one half its former magnitude; that is to say, to the $\frac{1}{230}$th part of an inch. If, on the other hand, the disc were moved towards the eye, and placed at half its original distance, then the angle C of the cone would be 2°, and the diameter of the picture on the retina would be double its first magnitude; that is to say, the $\frac{2}{115}$th of an inch.

In general, it may therefore be inferred that the magnitude of the diameter of the picture on the retina is increased or diminished in exactly the same proportion as the angle of the cone C, formed at the centre of the eye, is increased or diminished.

350. **Visual magnitude.** — This angle is called the visual angle or apparent magnitude of the object; and when it is said that a certain object subtends at the eye a certain angle, it is meant that lines drawn from the extremities of such object to the centre of the eye form such angle.

The *apparent magnitude* of an object must not be confounded with its apparent superficial magnitude, the term being invariably applied to its *linear magnitude.* The apparent superficial magnitude varies in proportion to the square of the apparent magnitude.

Thus, for example, when the disc AB is removed to double its original distance from the eye, the apparent magnitude, or the angle C, is diminished one half, and consequently the diameter *a b* of the picture on the retina is also diminished one half; and since the diameter is diminished in the ratio of 2 to 1, the superficial magnitude of the image, or its area, will be diminished in the proportion of 4 to 1.

351. It is clear from what has been stated also, that when the same object is moved from or towards the eye, its apparent magnitude varies inversely as its distance; that is, its apparent magnitude is increased in the same proportion as its distance is diminished, and *vice versâ.*

It is easy to perceive that the objects which are seen under the same visual angle will have the same apparent magnitude. Thus let A′ B′, *fig.* 185., be an object more distant than A B, and of such a magnitude that its highest point A′ shall be in the continuation of the line C A, and its lowest point B in the continuation of the line C B. The apparent magnitude of A′ B′ will then be measured by the angle at C. This angle will therefore at the same time represent the apparent magnitude of the object A B and of the object A′ B′. It is evident that an eye placed at C will see every point of the object A B upon the corresponding points of the object A′ B′; so that if the object A B were opaque, and of a form similar to the object A′ B′, every point of the one would be seen upon a corresponding point of the other. In like manner, if an object A″ B″ were placed nearer the eye than A B, so that its highest point may lie upon the line C A, and its lowest point upon the line C B, the object, being similar in form to A B, would appear to be of the same magnitude. Now it is evident that the real magnitudes of the three objects A″ B″, A B, and A′ B′, are in proportion to their respective distances from the eye; A′ B′ is just so much greater than A B, and A B than A″ B″, as C B′ is greater than C B, and as C B is greater than C B″.

Thus it appears that if several objects be placed before the eye in the same direction at different distances, and that the real linear magnitudes of these objects are in the proportion of their distances, they will have the same apparent magnitude.

352. **Example of the sun and moon.** — A striking example of this principle is presented by the case of the sun and moon. These objects appear in the heavens equal in size, the full moon being equal in apparent magnitude to the sun. Now it is proved by astronomical observation that the real diameter of the sun is, in round numbers, four hundred times that of the moon; but it is also proved that the distance of the sun from the earth is also, in round numbers, four hundred times greater than that of the moon. The distance, therefore, of these two objects being in the same pro-

portion as their real diameter, their visual or apparent magnitudes are equal.

353. It is evident, from what has been explained, that objects which have equal apparent magnitudes, and are therefore seen under equal visual angles, will have pictures of equal magnitude on the retina, a fact which proves that the visual angle is the measure of the apparent magnitude.

354. If the same object be moved successively to increasing distances, its apparent magnitude will be diminished in the same proportion, exactly as its distance from the eye is increased. Thus, if L M, *fig.* 186., be such an object, its distance E M being expressed by D, and its height L M by H, the visual angle L E M, which measures its apparent magnitude, will be expressed, according to what has been formerly explained, by $\frac{H}{D}$. If the object be now removed to double the distance E M′, the visual angle or apparent magnitude L′ E M′ will be expressed by $\frac{H}{2D}$, which is just one-half $\frac{H}{D}$, the former visual angle; and, in like manner, if the object be removed to three times its first distance, such as E M″, its visual angle or apparent magnitude will be $\frac{H}{3D}$, which is one third of its original apparent magnitude.

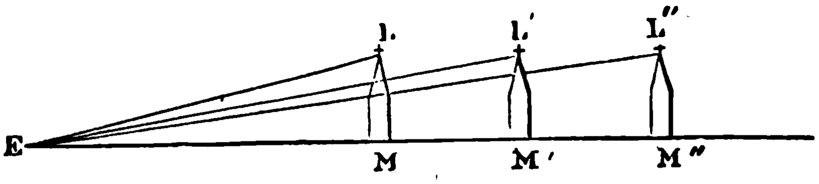

Fig. 186.

355. **Apparent superficial magnitude.** — The apparent superficial magnitude of a body is determined by a section of the body made by a plane at right angles to the lines containing the visual angle. Thus, the apparent superficial magnitude of the sun or moon is determined by a section of those bodies passing through the points where lines drawn from the eye touching them would meet them, which, in consequence of the great distance of these bodies, would be a circular section through their centres, and at right angles to a line drawn from the centre to the eye.

356. **Section of vision.** — This circle, in the case of the sun or moon or other celestial object, is called the *circle of vision;* and a corresponding section of any other object drawn at right angles to the sides of the visual angle would be called the *section of vision.*

For all distant objects, this section is a plane at right angles to the direction in which the object is seen.

357. **The smallest magnitudes which can be distinctly seen.**—If the circular disc A B (*fig.* 185.), which we have supposed to be presented before the eye at a distance of fifty-seven and a half times its own diameter, and which, therefore, subtends at the centre of the eye a visual angle of 1°, be removed to a distance sixty times greater, or to a distance equal to 3450 times its own diameter, it will subtend an angle at C proportionally less, which will therefore be, in this case, an angle of one minute; and if it be removed to double the latter distance, or 6900 times its own diameter, it will subtend a visual angle of thirty seconds. Now it is found that if such an object be directly illuminated by the sun, it will be barely visible. This limit, however, depends as well on the colour of the object as on the degree of its illumination. Plateau affirms that a white disc, such as we have here supposed to be presented to the eye, if the light of the sun shone fully upon it, will be visible when seen under a visual angle of twelve seconds, or the one fifth part of a minute. The disc would subtend this angle at the eye if placed at a distance equal to 17250 times its diameter.

He says also that if the disc, under the same circumstances, were red, it would be distinctly seen until its apparent magnitude were reduced to twenty-three seconds; and that if it were blue, the limit would be twenty-six seconds; but that, if instead of being illuminated by the direct solar light, it were illuminated by the light of day reflected from the clouds, these limiting angles would be half as large again.

358. **Distinctness of vision compared with the magnitude of the pictures on the retina.**—Nothing can be more calculated to excite our wonder and admiration than the distinctness of our perception of visible objects, compared with the magnitude of the picture on the retina, from which immediately we receive such perception.

359. If we look at the full moon on a clear night, we perceive with considerable distinctness, by the naked eye, the lineaments of light and shade which characterise its disc.

Now let us consider only for a moment, what are the dimensions of the picture of the moon formed on the retina, from which alone we derive this distinct perception.

The disc of the moon subtends a visual angle of half a degree, and consequently, according to what has been explained, the diameter of its picture on the retina will be $\frac{1}{230}$th part of an inch, and the entire superficial magnitude of the image from which we derive this distinct perception is less than the $\frac{1}{52000}$th part of a square inch; yet within this minute space we are able to distinguish a multiplicity of still more minute details. We perceive, for example, forms of light and shade, whose linear dimensions do not exceed one tenth part of the apparent diameter of the moon, and which therefore occupy upon the retina a space whose area does not amount to the $\frac{1}{50000000}$th part of a square inch.

360. To take another example, the figure of a man 70 inches high, seen at a distance of 40 feet, produces an image upon the retina the height of which is about one fourteenth part of an inch. The face of such an image is included in a circle whose diameter is about one twelfth of the height, and therefore occupies on the retina a circle whose diameter is about the $\frac{1}{170}$th part of an inch; nevertheless, within this circle, the eyes, nose, and lineaments are distinctly seen. The diameter of the eye is about one twelfth of that of the face, and therefore, though distinctly seen, does not occupy upon the retina a space exceeding the $\frac{1}{40000000}$th of a square inch.

If the retina be the canvas on which this exquisite miniature is delineated, how infinitely delicate must be its structure to receive and transmit details so minute with such marvellous precision; and if, according to the opinion of some, the perception of these details be obtained by the retina *feeling* the image formed upon the choroid, how exquisitely sensitive must be its touch!

361. **Sufficiency of illumination.** — It is not enough for distinct vision that a well-defined picture of the object shall be formed on the retina. This picture must be sufficiently illuminated to affect the senses, and at the same time not so intensely illuminated as to overpower the organ. Thus it is possible to conceive a picture on the retina, so extremely faint as to be insufficient to produce sensation, or, on the other hand, so intensely brilliant as to dazzle the eye, to destroy the distinctness of sense, and to produce pain.

When we direct the eye to the sun, near the meridian, in an unclouded sky, we have no distinct perception of his disc, because the splendour is so great as to overpower the sense of vision just as sounds are sometimes so intense as to be deafening. That it is the intense splendour alone which prevents a distinct perception of the solar disc in this case, is rendered manifest by the fact that if a portion of the solar rays be intercepted by a coloured glass, or by a thin cloud, a distinct image of the sun will be seen.

When we direct the eye to the firmament on a clear night, there are innumerable stars which transmit light to the eye, and which therefore must produce some image on the retina, but of which we are altogether insensible, owing to the faintness of the illumination. That the light, however, does enter the eye and arrive at the retina is proved by the fact, that if a telescope be directed to the stars in question, so as to collect a greater quantity of their light upon the retina, they will become visible.

362. The eye possesses a certain limited power of accommodating itself to various degrees of illumination. Circumstances which are familiar to every one render the exercise of this power evident.

If a person, after remaining a certain time in a dark room, pass suddenly into another room strongly illuminated, the eye suffers instantly a degree of inconvenience, and even pain, which causes the eyelids to close; and it is not until after the lapse of a certain time that they can be opened without inconvenience.

The cause of this is easily explained. While the observer remains in the darkened or less illuminated room, the pupil is dilated so as to admit into the eye as great a quantity of light as the structure of the organ allows of. When he passes suddenly into the strongly illuminated room, the flood of light, arriving through the widely dilated pupil, acts with such violence on the retina as to produce pain, which necessarily calls for the relief and protection of the organ. The iris, then, by an action peculiar to it, contracts the dimensions of the pupil so as to admit proportionally less light, and the eye is opened with impunity.

Effects the reverse of these are observed when a person passes from a strongly illuminated room into one comparatively dark, or into the open air at night. For a certain time he sees nothing, because the contraction of the pupil, which was adapted to the strong light to which it had previously been exposed, admits so little light to the retina that no sensation is produced. The pupil, however, soon dilates, and, admitting more light, objects are perceived which were before invisible.*

There is, however, another cause, which has an important share in producing these and some other phenomena. The sensibility of the retina, as well as that of all the other nerves of sensation, is more or less deadened by intense excitement. Thus, for example, the ear, immediately after being acted upon by a succession of loud sounds, is more or less insensible to the slight impressions made by low sounds; whereas its sensibility is raised to the highest pitch after it has been surrounded for an interval more or less considerable by profound silence. It is the same with the eye: the retina, after exposure to intense light, is more or less insensible to objects feebly illuminated, but after it has continued for some time in obscurity, it recovers its proper sensibility, and such objects make sensible impressions upon it.

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* "Thus, when the lamp that lighted
The traveller at first, goes out,
He feels awhile benighted,
And wanders on in fear and doubt;
But soon the prospect clearing,
In cloudless starlight on he treads,
And finds no lamp so cheering
As that light which Heaven sheds." — MOORE.

363. **Brightness of ocular image.**—If two points from which light radiates be placed at the same distance from the eye, the brightness of their image on the retina will be in proportion to their absolute brilliancy. But if either point be removed to a greater distance, the number of rays passing from it which enter the pupil will be diminished, in the same proportion as the square of its distance is increased, and *vice versâ.* It consequently follows, that the brightness of each point of the image of an object formed upon the retina, will be in the direct proportion of the absolute brilliancy of such point, and in the inverse proportion of the square of its distance from the eye.

Thus, if I express the intensity of the light of the point upon the object, and D its distance from the eye, then the brightness of the image of such point upon the retina will be expressed by $\frac{I}{D^2}$.

It is therefore clear, that the brightness of the image of each point of an object will be diminished, as the square of the distance of the object from the eye is increased.

364. **Apparent brightness the same at all distances.** — It is sometimes inferred from this, though erroneously, that the apparent splendour of the image of the visible object decreases, as the square of the distance increases. This would be the case in the strictest sense, if, while the object were withdrawn from the eye to an increased distance, its image on the retina continued to have the same magnitude; for, in this case, the absolute brightness of each point composing such image would diminish, as the square of the distance increases, and the area of the retina over which such points are diffused would remain the same; but it must be considered that as the object retires from the eye the superficial magnitude of the image on the retina is diminished in the same proportion as the square of the distance of the object from the eye is increased. It therefore follows that while the points composing the image on the retina are diminished in the intensity of their illumination, they are collected into a smaller space, so that what each point of the image on the retina loses in splendour, the entire image gains by concentration.

If the sun were brought as close to the earth as the moon, its apparent diameter would be 400 times greater, and the area of its apparent disc 160000 times greater than at present, but the apparent brightness of its surface would not be in any degree increased. In the same manner, if the sun were removed to ten times its present distance, it would appear under a visual angle ten times less than at present, as in fact it would to an observer on the planet Saturn, and its visible area would be a hundred times less than it is, but the splendour of its diminished area would be exactly the same as the present splendour of the sun's disc.

These consequences, which are of considerable physical importance, obviously follow from the principles explained above.

The sun, seen from the planet Saturn, has an apparent diameter ten times less than it has when seen from the earth.

The appearance from Saturn will then be the same, as would be the appearance of a portion of the disc of the sun, seen from the earth through a circular aperture in an opaque plate, which would exhibit a portion of the disc whose diameter is one tenth of the whole.

365. When the light which radiates from a luminous object has a certain intensity, it will continue to affect the retina in a sensible manner, even when the object is removed to such a distance, that the visual angle shall cease to have any perceivable magnitude. The fixed stars present innumerable examples of this effect. None of these objects, even the most brilliant of them, subtend any sensible angle to the eye. When viewed through the most perfect telescopes they appear merely as brilliant points. In this case, therefore, the eye is affected by the light alone, and not by the magnitude of the object seen.

366. Nevertheless, the distance of such an object may be increased to such an extent, that the light, intense as it is, will cease to produce a sensible effect upon the retina.

Seven classes of the fixed stars, diminishing gradually in brightness*, produce an effect on the retina such as to render them visible to a naked eye. This diminution of splendour is produced by their increased distance. The telescope, however, as has been already stated, brings into view innumerable other stars, whose intrinsic splendour is as great as the brightest among those which we see, but which do not transmit to the retina, without the aid of the telescope, enough of light to produce any sensible effect. Nevertheless, it is demonstrable that, even without the telescope, they do transmit a certain definite quantity of light to the retina; the quantity of light which they thus transmit, and which is insufficient to produce a sensible effect, having to the quantity obtained by the telescope, a ratio depending upon the proportion of the magnitude of the object glass of the telescope to the magnitude of the pupil.

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* The term magnitude is used in astronomy, as applied to the fixed stars, to express their apparent brightness; no fixed star, however splendid, subtends any sensible angle.

367. The quantity and intensity of the light transmitted by an external object to the retina, which is sufficient to produce a perception of such object, depends also upon the light received at the same time by the retina from other objects present before the eye. The proof of this is, that the same objects which are visible at one time are not visible at another, though equally before the eye, and transmitting equal quantities of light of the same intensity to the retina. Thus, the stars are present in the heavens by day as well as by night, and transmit the same quantity of light to the retina, yet they are not visible in the presence of the sun, because the light proceeding from that luminary, directly and indirectly reflected and refracted by the air and innumerable other objects, is so much greater in quantity and intensity as to overpower the inferior and much less intense light of the stars. This case is altogether analogous to that of the ear, which, when under the impression of loud and intense sounds, is incapable of perceiving sounds of less intensity, which nevertheless affect the organ in the same manner as they do when, in the absence of louder sounds, they are distinctly heard.

Even when an object is perceived, the intensity of the perception is relative, and determined by other perceptions produced at the same time. Thus, the moon seen at night is incomparably more splendid than the same moon seen by day or in the twilight, although in each case the moon transmits precisely the same quantity of light, of precisely the same intensity, to the eye; but in the one case the eye is overpowered by the superior splendour of the light of day, which dims the comparatively less intense light proceeding from the moon.

368. **The image must continue a sufficient time upon the retina to enable that membrane to produce a perception of it.** — It is proved in "Astronomy," Chapter XVI. p. 472., that the velocity with which light is propagated through space is at the rate of about 200000 miles per second. Its transmission, therefore, from all objects at ordinary distances to the eye may be considered as instantaneous. The moment, consequently, any object is placed before the eye, an image of it is formed on the retina, and this image continues there until the object is re-

moved. Now it is easy to show experimentally that an object may be placed before the eye for a certain definite interval of time, and that a picture may be painted upon the retina during that interval without producing any perception or any consciousness of the presence of the object.

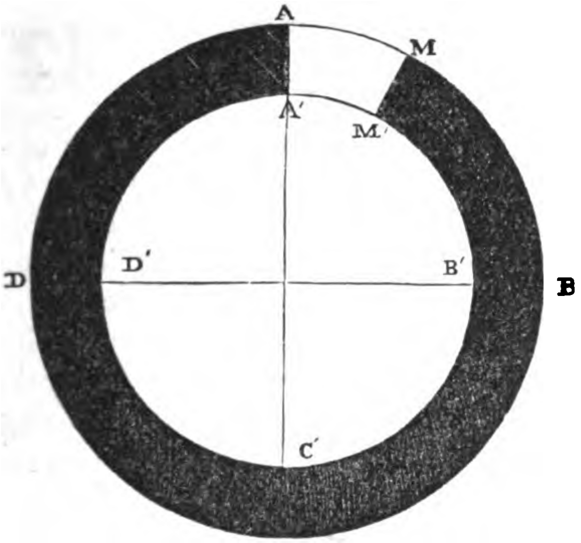

Fig. 187.

To illustrate this, let a circular disc A B C D, *fig.* 187., about 20 inches in diameter, be formed in card or tin, and let a circle A′ B′ C′ D′ be described upon it, about 2 inches less in radius than the disc, so as to leave between the circle and the disc a zone about 2 inches wide. Let the entire zone be blackened, except the space A M M′ A′, forming about the one twentieth of it. Let the disc thus prepared be attached to the back of a blackened screen, so as to be capable of revolving behind it, and let a hole one inch in diameter be made in the screen at any point, behind which the zone A B C D is placed. If the disc be now made to revolve behind the screen, the hole will appear as a circular white spot so long as the white space A M passes behind it, and will disappear, having the same black colour as the screen during the remainder of the revolution of the disc. The hole will therefore be seen as a white circular spot upon the black screen, during one twentieth of each revolution of the disc. If the disc be now put in motion at a slow rate, the white hole will be seen on the screen during one twentieth of each revolution. If the velocity of rotation imparted to the disc be gradually increased, the white spot will ultimately *disappear*, and the screen will appear of a uniform black colour, although it be certain that during the twentieth part of each revolution, whatever be the rate of rotation, a picture of the white spot is formed on the retina.

369. The length of time necessary in this case for the action of light upon the retina to produce sensation may be determined by ascertaining the most rapid motion of the disc which is compatible with a distinct perception of the white spot. This interval will be found to vary with the degree of illumination. If the spot be strongly illuminated, a less interval will be sufficient to produce

a perception of it; if it be more feebly illuminated, a longer interval will be required. The experiment may be made by varying the colour of the space A M of the zone, and it will be found that the interval necessary to produce sensation will vary with the colour as well as with the degree of illumination.

370. **Ocular spectra.** — Since the perception of a visible object is the effect of a certain agitation of the retina, produced by the action of the light proceeding from such object upon it, it follows that the same visible perception will be produced; in other words, the object will be *seen*, if, under any supposable circumstances, the same agitation of the retina should take place in the absence of the object. Whether the act of the memory, in recalling the perception of objects formerly seen, does produce in any degree, however faint, such an agitation of the retina, would be a curious and interesting inquiry; but, meanwhile, it is certain that there are cases in which the agitation of the retina, necessary to produce visual perception, does take place in the absence of the exciting object, and that definite visual perceptions are thus actually produced. Such perceptions are called *ocular spectra*.

The most simple and frequently recurring case of ocular spectra arises from what may be called the nervous inertia of the retina. That membrane does not cease to act at the moment of cessation of the cause which excites it, but continues to vibrate for a certain interval after the removal of that cause :just as the string of a violin continues to vibrate for a certain time after the removal of the bow. It follows that an object must continue to be visible for a certain interval, more or less considerable, after it has been suddenly removed from before the eye. What we see during the interval cannot, therefore, be the object itself, and is consequently an ocular spectrum.

371. **The duration of the impression on the retina after** the removal of the visible object which produced it, varies according to the degree of illumination, the colour of the object, and some other conditions. To illustrate this experimentally, let a circular disc, formed of blackened card or tin, of twelve or fourteen inches in diameter, be pierced with eight holes round its circumference, at equal distances, each hole being about half an inch in diameter, as represented in *fig.* 188.

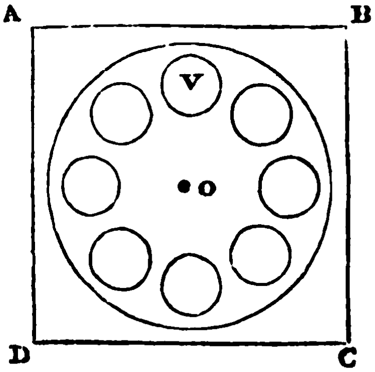

Fig. 188.

Let this disc be attached upon a pivot or pin at its centre O to a board A B C D, which is blackened everywhere, except upon a circular spot at v, corresponding in magnitude to the holes made in the circular plate.

Let this spot be first supposed to be white. Let the circular disc be made to revolve upon the point o, so as to bring the circular holes successively before the white spot at v. The retina will thus be impressed at intervals with the image of this circular white spot. In the intervals between the transits of the holes over it, the entire board will appear black, and the retina will receive no impression. If the disc be made to revolve with a very slow motion, the eye will see the white spot at intervals, but if the velocity of rotation be gradually increased, it will be found that the eye will perceive the white spot permanently represented at v, as if the disc had been placed with one of its holes opposite to it without moving. It is evident, therefore, that in this case the impression produced upon the retina, when any hole is opposite the white spot, remains until the succeeding hole comes opposite to it, and thus a continued perception of the white spot is produced.

If the white spot be illuminated in various degrees, or if it be differently coloured, the velocity of the disc necessary to produce a continuous perception of it will differ. The brighter the colour and the stronger the illumination, the less will be the velocity of rotation of the disc which is necessary to produce a continuous perception of the spot.

These effects show that the stronger the illumination, and the brighter the colour, the longer is the interval during which the impression is retained by the retina.

372. **Why we are not sensible of darkness when we wink.** — This continuance of the impression of external objects on the retina, after the light from the objects ceases to act, is also manifested by the fact that the continual winking of the eyes, for the purpose of lubricating the eyeball by the eyelid, does not intercept our vision. If we look at any external objects, they never cease for a moment to be visible to us, notwithstanding the frequent intermissions which take place in the action of light upon the retina, in consequence of its being thus intercepted by the eyelid.

According to Sir David Brewster, the most instructive experiment on this subject, which requires a great deal of practice to be made with success, is to look for a short time at a window at the end of a long room, and then suddenly to turn the eye to a dark wall. In general, a common observer will in this case see a representation of the window on the wall, in which the dark bars of the sash will appear white, and the panes of glass dark.

A practised observer, however, who makes the observation with great promptness, will see at the moment his eyes are turned to the wall a correct representation of the window. This representation will almost immediately be succeeded by the reversed picture just mentioned, in which the bars are bright and the panes dark.

373. If a lighted stick be turned round in a circle in a dark room, the appearance to the eye will be a continuous circle of light; for in this case the impression produced upon the retina by the light, when the stick is at any point of the circle, is retained until the stick returns to that point.

374. **Flash of lightning.** — In the same manner, a flash of lightning appears to the eye as a continuous line of light, because the light emitted at any point of the line remains upon the retina, until the cause of the light passes over the succeeding points.

Any object moving before the eye with such a velocity that the retina shall retain the impression produced at one point in the line of its motion, until it passes through the other points, will appear as a continuous line of light or colour.

375. **Why an object moving with a great speed becomes invisible.** — But to produce this effect, it is not enough that the body change its position so rapidly, that the impression produced at one point of its path continue until its arrival at another point; it is necessary, also, that its motion should not be so rapid, as to make it pass from any of the positions which it successively assumes, before it has time to impress the eye with a perception of it; for it must be remembered, as has been already explained, that the perception of a visible object presented to the eye, though rapid, is not instantaneous.

The object must remain present before the organ of vision a certain definite time, and its image must continue upon the retina during such time, before any perception of it is obtained. Now, if the body move from its position before the lapse of this time, it necessarily follows that no perception of its presence, therefore, will be obtained. If, then, we suppose a body moving so rapidly before the eye that it remains in no position long enough to produce a perception of it, such object will not be seen.

376. **Example of a cannon ball.** — Hence it is that the ball discharged from a cannon, passing transversely to the line of vision, is not seen; but if the eye be placed in the direction in which the ball moves, so that the angular motion of the ball round the eye as a centre be slow, notwithstanding its great velocity, it will be visible, because, however rapid its real motion through space, its angular motion with respect to the eye (and consequently that of its picture on the retina) will be sufficiently slow to give the necessary time for the production of a perception of it.

377. **Quickness of vision depends on colour, brightness, and magnitude.** — The time thus necessary to obtain the perception of a visible object varies with the degree of illumination, the colour, and the apparent magnitude of the object. The more intense the illumination, the more vivid the colour, and the greater the apparent magnitude, the less will be the time necessary to produce a perception of the object.

If, therefore, the object before the eye be not sufficiently illuminated, or be not of a sufficiently bright colour to impress the retina sensibly, it will then, instead of appearing as a continuous line of colour, cease to be visible altogether; for it does not remain in any one position long enough to produce a sensible effect upon the retina. It is for this reason that a ball projected from a cannon or a musket, though passing before the eye, cannot be seen.

378. If two railway trains pass each other with a certain velocity, a person looking out of the window of one of them will be unable to see the other. If the velocity be very moderate, and the light of the day sufficiently strong, the appearance of the passing train will be that of a flash of colour formed by the mixture of the prevailing colours of the vehicles composing it.

An expedient has already been described to show experimentally that the mixture of the seven prismatic colours, in their proper proportions, produce white light, depending on this principle. The colours are laid upon a circular disc surrounding its edge, which they divide into parts proportional to the spaces they occupy in the spectrum. When the disc is made to revolve, each colour produces, like the lighted stick, the impression of a continuous ring, and consequently the eye is sensible of seven rings of the several colours superposed one upon the other, which thus produce the effect of their combination, and appear as white, or a whitish grey colour, as already explained.

379. **D'Arcy's experiments.** — M. D'Arcy found that the light of a live coal, moving at the distance of 165 feet, maintained its impression on the retina during the *seventh* part of a second, or 0·133°. Dr. Young found that the impression continued *half* a second, or 0·5°; and, more recently, M. Plateau has found it to be very nearly the *third* of a second, or 0·34°, being for

White light - - - - - -	0·35°.
Yellow do. - - - - - -	0·35.
Red do. - - - - - -	0·34.
Blue do. - - - - - -	0·32.

The impression disappears with unequal rapidity: *quickest* in the *white*, less quick in the *yellow*, still less quick in the *red*, and slowest in the *blue*. Hence the impression is least intense in the *blue*, and most intense in the *white*.

380. The duration of the impression also depends on the state of illumination of the surrounding space; thus the impression produced by a luminous object when in a dark room is more durable than that which would be produced by the same object seen in an illuminated room. This may be ascribed to the greater sensitiveness of the retina when in a state of repose than when its entire surface is excited by surrounding lights. Thus it is found that while the varying duration of the impression of the illuminated object in a dark room was one third of a second, its duration in a lighted room was only one sixth of a second.

381. **Continuance of perception depends on intensity of the impression.**—The continuance of the visual impression on the retina, will also depend on the intenseness with which the eye has been directed to the visible object previous to its removal, and on the length of time which it has continued before the eye. According to Müller, the duration of the spectrum will also be augmented by causing the light from the object previously to its removal to act with intermission and not continuously, which might be effected by the apparatus represented in *fig.* 188.

382. **Experiment of Müller.**—"If we gaze," says Professor Müller, "for a considerable time upon a body which presents a continued motion of different parts of its surface in succession, the spectra left on the retina have also an appearance of motion in the same direction, owing to their disappearing from the eye in the same order. This is, in my opinion, the proper mode of explaining certain illusory appearances of motion in objects. If, after looking for a long time at the undulations of a stream of water, we suddenly turn our eyes to the ground at its margin, the ground itself appears to move in the opposite direction to the waves of the water. I have frequently remarked this phenomenon, when, after gazing from my window upon the neighbouring river, I have directed my eyes to the pavement of the street. I observed it also when, being on board a steam-packet, and having looked for some length of time upon the waves which passed, I suddenly turned my eyes towards the deck of the vessel. If we suppose that in these cases spectra were left in the retina by the impressions of the waves, and that they disappeared in the order in which they arose, their successive disappearance, while looking upon the fixed surface of the ground or deck, would necessarily cause an apparent motion of this surface in the opposite direction." *

383. **Optical toys.**—**Thaumatrope.**—Innumerable optical

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* Müller's "Physiology," vol. ii. p. 1180.

toys and pyrotechnic apparatus, owe their effect to this continuance of the impression upon the retina, when the object has changed its position.

Amusing toys, called thaumatropes, phenakisticopes, phantaskopes, &c., are explained upon this principle. A moving object, which assumes a succession of different positions in performing any action, is represented in the successive divisions of the circumference of a circle as in *fig.* 189., in the successive positions it assumes. These pictures, by causing the disc to revolve, are brought in rapid succession before an aperture, through which the eye is directed, so that the pictures representing the successive attitudes are brought one after another before the eye at such intervals that the impression of one shall remain until the impression of the next is produced. In this manner the eye never ceases to see the figure, but sees it in such a succession of attitudes as it would assume if it revolved. The effect is, that the figure actually appears to pirouette before the eye. The effects of catharine-wheels and rockets are explained in the same manner.

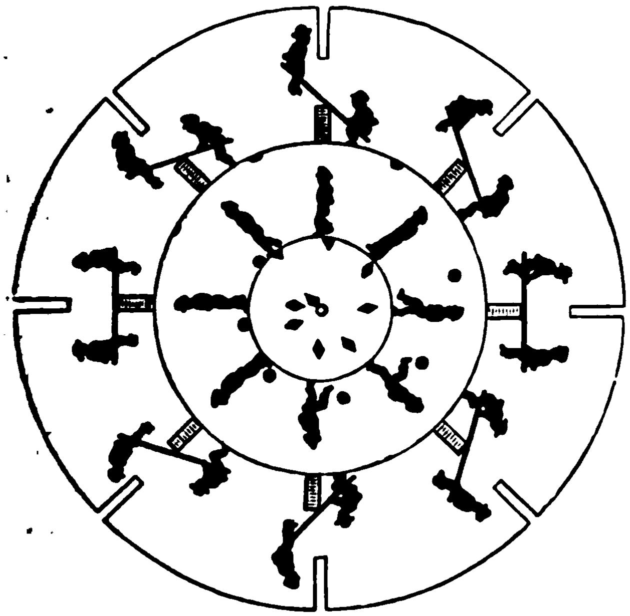
Fig. 189.

384. **The phenakistoscope, or magic disc,** an invention of M. Plateau, is a beautiful instrument, depending on the same principle. It consists of a circular disc of pasteboard, 8 or 9 inches in diameter, having twelve rectilinear slits or chinks in its margin, placed at equal distances, and in the direction of its radii. This disc can be made to revolve rapidly about its axis; and if we look into a mirror through one of the chinks when it is revolving, they will appear to stand still in the mirror, owing to the motions of the object and its image being equal and opposite. Had there been a figure beneath each chink, each figure seen in the mirror would be stationary. If the figures were 11 in number, in place of 12, they would all appear to move in one direction; and if they were 13, they would appear to move in the *opposite* direction. If we now suppose twelve gates to be drawn on a separate disc, smaller

than the main one, and placed upon it so as not to interfere with its slits, these gates will stand still during the revolution of the disc. If we then place thirteen horses with their riders near the gate, *one* horse just before he begins to leap, the second horse with its fore legs raised from the ground, and all the other horses in the different positions of leaping, till the *thirteenth* horse reaches the ground, the effect will be that each horse and its rider will come up to the chink through which we look faster than the gate; and as each gate arrives, the horse will have advanced $\frac{1}{13}$th part of $\frac{1}{12}$th of the circumference of the disc; that is, in one complete revolution it will have moved forward through $\frac{1}{13}$th of the circle. Had there been only eleven slits it would have moved backwards. Now, during this motion the horse has taken thirteen different positions *in succession*, and therefore leaps the gate.

In like manner, there are twelve hedgerows, with several hounds, each of which is represented in thirteen different positions, so that they appear in the act of crossing the hedges, and we have exhibited before us a portion of a fox-hunting scene.

It is obvious that if, instead of a mirror, another person whirls round in an opposite direction, and with the same velocity, a similar disc, the effect will be the same. The similar motion of the two phenakistoscopes could be obtained by machinery.

Another instrument invented by Plateau, he calls the *Anorthoscope*, which, by means of two discs revolving with different velocities, *rectifies*, or makes regular, and multiplies an extremely shapeless and irregular figure.*

385. **Conditions which determine apparent motion.** — In applying this principle to the phenomena of vision, it must be carefully remembered that the question is affected, not by the real, but by the apparent motion of the object; that is to say, not by the velocity with which the object really moves through space, but by the angle which the line drawn from the eye to the object describes per second. Now this angle is affected by two conditions, which it is important to attend to: 1st, the direction of the motion of the object compared with the line of vision; and 2nd, by the velocity of the motion compared with the distance of the object. If the object were to move exactly in the direction of the line of vision, it would appear to the eye to be absolutely stationary, since the line drawn to it would have no angular motion; and if it were to move in a direction forming an oblique angle to the line of vision, its apparent motion might be indefinitely slow, however great its real velocity might be.

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* Brewster's "Optics," p. 421.

For example, let it be supposed that the eye being at E, *fig.* 190., an object O moves in the direction O O′, so as to move from O to O′

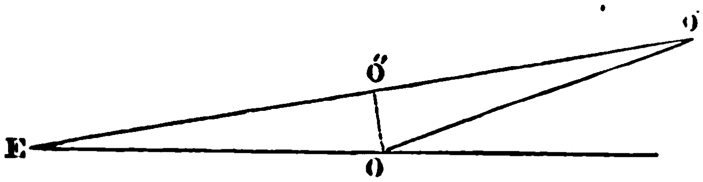

Fig. 190.

in one second. Taking E as a centre, and E O as a radius, let a circular arc O O″ be described. The apparent motion of the object will then be the same as if, instead of moving from O to O′ in one second, it moved from O to O″ in one second.

The more nearly, therefore, at right angles to the line of vision the direction of the motion is, the greater will be the apparent motion produced by any real motion of an object.

386. **How apparent motion is affected by distance.** — A motion which is visible at one distance may be invisible at another, inasmuch as the angular velocity will be increased as the distance is diminished.

Thus if an object at a distance of 57½ feet from the eye move at the rate of a foot per second, it will appear to move at the rate of one degree per second, inasmuch as a line one foot long at 57½ feet distance subtends an angle of one degree. Now if the eye be removed from such an object to a distance of 115 feet, the apparent motion will be half a degree, or thirty minutes per second; and if it be removed to thirty times that distance, the apparent motion will be thirty times slower. Or if, on the other hand, the eye be brought nearer to the object, the apparent motion will be accelerated in exactly the same proportion as the distance of the eye is diminished.

387. A cannon-ball moving at 1000 miles an hour transversely to the line of vision, and at a distance of 50 yards from the eye, will be invisible, since it will not remain a sufficient time in any one position to produce perception. The moon, however, moving with more than double the velocity of the cannon-ball, being at a distance of 240000 miles, has an apparent motion so slow as to be imperceptible.

388. **What motions are imperceptible.**—The angular motion of the line of vision may be so diminished as to become imperceptible; and the body thus moved will in this case appear stationary. It is found by experience that unless a body moves in such a manner that the line of vision shall describe at least one degree in each minute of time, its motion will not be perceptible.

389. Thus it is that we are not conscious of the diurnal motion of the firmament. If we look at the moon and stars on a clear night, they appear to the eye to be quiescent; but if we observe them after the lapse of some hours, we find that their positions are changed; those which were near the horizon being nearer the meridian, and those which were in the meridian having descended towards the horizon. Since we are conscious that this change did not take place suddenly, we infer that the entire firmament must have been in continual motion round us, but that this motion is so slow as to be imperceptible.

390. Since the heavens appear to make a complete revolution in twenty-four hours, each object on the firmament must move at the rate of 15° an hour, or at the rate of one quarter of a degree a minute. But since no motion is perceptible to the eye which has a less apparent velocity than 1° per minute, this motion of the firmament is unperceived. If, however, the earth revolved on its axis in six hours instead of twenty-four hours, then the sun, moon, stars, and other celestial objects, would have a motion at the rate of 60° an hour, or 1° per minute. The sun would appear to move over a space equal to twice its own diameter each minute, and this motion would be distinctly perceived.

The fact that the motion of the hands of a clock is not perceived is explained in the same manner.

391. **Other ocular spectra. — Accidental colour.** — Besides the ocular spectra which produce the various effects above described, and which are precisely similar in form and colour to the actual visible impressions of the objects which produce them when present before the eye, there are various others of a very different character, and which have not been explained in an equally clear and satisfactory manner.

The effect produced by a strongly illuminated image formed on the retina does not appear to be merely the continuance of the same perception after the image is removed, but also a certain diminution or deadening of the sensibility of the membrane to other impressions. If the organ were merely affected by the continuance of the perception of the object for a certain time after its removal, the effect of the immediate perception of another object on the retina would be the perception of the mixture of two colours. Thus, if the eye, after contemplating a bright yellow object, were suddenly directed to a similar object of a light red colour, the effect ought to be the perception of an orange colour; and this perception would continue until the effect of the yellow object on the retina would cease, after which the red object would alone be perceived.

Thus, for example, a disc of white paper being placed upon a black ground, and over it a red wafer which will exactly cover it, if, closing one eye, and gazing intently with the other for a few seconds on the red wafer, the red wafer be suddenly removed so as to expose the white surface under it to the eye, the effect ought to be the combination of the perception of red which continues after the removal of the red wafer, with the perception of white which the uncovered surface produces; and we should consequently expect to see a diluted red disc, similar to that which would be produced by the mixture of red with white. This, however, is not the case. If the experiment be performed as here described, the eye will, on the removal of the red wafer, perceive, not a reddish, but a greenish-blue disc. In like manner, if the wafer, instead of being red, were of a bright greenish-blue, when removed the impression on the eye would be that of a reddish disc.

The following is the explanation which has generally been given of this phenomenon: —

When the eye is directed with an intensity of gaze for some time at the red surface, that part of the retina upon which the image of the red wafer is produced becomes fatigued with the action of the red light, and loses to some extent its sensibility to that light, exactly as the ear is deafened for a moment by an overpowering sound. When the red wafer is removed, the white disc beneath it transmits to the eye the white light, which is composed of all the colours of the spectrum. But the eye, from the previous action of the red light, is comparatively insensible to those tints which form the red end of the spectrum, such as red and orange, but comparatively sensitive to the blues and greens, which occupy the other end. It is therefore that the eye perceives the white disc as if it were a greenish-blue, and continues to perceive it until the retina recovers its sensibility for red light.

The false colour produced by these means has been generally called an accidental colour.

392. **Experiments of Sir D. Brewster.** — The experiment above described may be varied by using wafers of various colours; and it will in each case be found that on the removal of the wafer the accidental colour or ocular spectrum produced will be that which is given in the second column of the following table, supplied by the observations of Sir David Brewster: —

Colour of the Wafer.	Accidental Colour, or Colour of the Ocular Spectrum.
Red - - - -	Bluish-green.
Orange - - -	Blue.
Yellow - - -	Indigo.
Green - - -	Violet, reddish.
Blue - - -	Orange red.
Indigo - - -	Orange yellow.
Violet - - -	Yellow green.
Black - - -	White.
White - - -	Black.

It follows, therefore, from the results in the preceding table, that the primitive and accidental colours are so related to each other, that if the former be reduced to the same degree of intensity as the latter, one will be the complementary colour of the other, or, which is the same, they will be so related that if mingled together they will produce white light,

The experiment may be varied in the following manner: —

If a small particle of red fire be burned in a dark room, so as to illuminate all the surrounding objects with an intense red light, and it be suddenly extinguished, the eye will for a time see a green flame; and this green flame will be visible whether the eye be open or closed.

If, on the other hand, a green fire be burned, it will be succeeded by the perception of a reddish light.

If the eye be directed intently upon the disc of the sun at rising or setting, when it is red, on closing the eyelids a green solar disc will be perceived.

" The phenomena of accidental colours are often finely seen when the eye has not been strongly impressed with any particular coloured object. It was long ago observed by M. Meusnier, that when the sun shone through a hole a quarter of an inch in diameter on a *red* curtain, the image of the luminous spot was *green*. In like manner, every person must have observed, in a brightly painted room, illuminated by the sun, that the parts of any white object on which the coloured light does not fall, exhibit the complementary colours. In order to see this class of phenomena, I have found the following method the simplest and the best. Having lighted two candles, hold before one of them a piece of coloured glass, suppose bright red, and remove the other candle to such a distance that the two shadows of any body formed upon a piece of white paper may be equally dark. In this case one of the shadows will be *red*, and the other *green*. With blue glass, one of them will be *blue*, and the other *orange yellow;* the one having invariably the accidental colour of the other. The very same effect may be produced in daylight by two holes in a window shutter; the one being covered with a coloured glass, and the other transmitting the white light of the sky. Accidental colours may also be seen by looking at the image of a candle, or any white object seen by reflection from a plate or surface of coloured glass sufficiently thin to throw back its colour from the second surface. In this case the reflected image will always have the complementary colour of the glass. The same effect may be seen in looking at the image of a candle reflected from the water in a blue finger glass; the image of the candle is yellowish, but the effect is not so decided in this case, as the retina is not sufficiently impressed with the blue light of the glass.

"These phenomena are obviously different from those which are produced by coloured wafers; because, in the present case, the accidental colour is seen by a portion of the retina which is not affected, or deadened, as it were, by the primitive colour. A new theory of accidental colours is therefore requisite to embrace this class of facts.

"As in acoustics, where every fundamental sound is actually accompanied with its harmonic sound, so in the impressions of light, the sensation of one colour is accompanied by a weaker sensation of its accidental or harmonic colour.* When we look at the *red* wafer, we are at the same time, with the same portion of the retina, seeing *green;* but being much fainter, it seems only to dilute the red, and make it, as it were, whiter, by the combination of the two sensations. When the eye looks from the wafer to the white paper, the permanent sensation of the accidental colour remains, and we see a *green* image. The duration of the primitive impression is only a fraction of a second, as we have already shown; but the duration of the harmonic impression continues for a time proportional to the strength of the impression. In order to apply these views to the second class of facts, we must have recourse to another principle; namely, that when the whole or a great part of the retina has the sensation of any primitive colour, a portion of the retina protected from the impression of the colour is actually thrown into that state which gives the accidental or harmonic colour. By the vibrations probably communicated from the surrounding portions, the influence of the direct or primitive colour is not propagated to parts free from its action, excepting in the particular case of oblique vision. When the eye, therefore, looks at the white spot of solar light seen in the middle of the red light of the curtain, the whole of the retina, except the portion occupied by the image of the white spot, is in the state of seeing everything *green;* and as the vibrations which constitute this state spread over the portions of the retina upon which no red light falls, it will, of course, see the white circular spot green. M. Plateau, to whom we owe so many valuable optical observations and instruments, has published an ingenious theory of accidental colours, in some respects the same, as he himself admits, as that which I had previously explained; in so far, at least, as they both ascribe the accidental colour to an impression of a peculiar nature spontaneously generated in the seat of vision, and not to any relative insensibility to certain rays. To this undoubted truth, M. Plateau has added the following proposition: — That while the combination of *real colours* produces *white*, the combination of *accidental* colours produces the contrary to *white*, or *black;* but I consider this proposition as a mere *verbal illusion*, and the physical fact which it expresses as long known, and as the necessary result of our previous knowledge. The *blackness* which is produced by the union, as it were, of all the accidental colours, is merely the sum of the insensibilities to all the colours, or the inability to see any colour, from the exhaustion of the eye. It cannot, therefore, be called an union of colours. It is the successive deprivation of the power of seeing all the colours of the spectrum.

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* The term *harmonic* has been applied to accidental colours, because the primitive and its accidental colour harmonise with each other in painting.

" The following is M. Plateau's account of his general theory: When the retina is submitted to the action of rays of any colour, it resists this action, and tends to resume its ordinary condition with a force more or less intense. If it is then suddenly withdrawn from the exciting cause, it returns to its ordinary condition by an oscillatory movement, the intensity of which is proportional to the duration of the previous action; — a movement in virtue of which the impression passes at first from the *positive* to the *negative* state, then continues generally to oscillate in a manner more or less regular, while it becomes weaker and weaker. This principle of a regular, or a tendency to a regular, oscillatory movement, is not very consistent with the obliteration of the accidental colour, temporarily or permanently, by involuntary winking, by closing the eyes with different degrees of pressure, by distending the eyes, and by a blow upon the head.*

" A very remarkable phenomenon, in which the eye is not excited by any primitive colour, was observed by Mr. Smith, surgeon, in Fochabers. If we hold a narrow strip of white paper vertically, about a foot from the eye, and fix both eyes upon an object at some distance beyond it, so as to see it double, then if we allow the light of sun, or the light of a candle, to act strongly upon the right eye, without affecting the left, which may be easily protected from its influence, the *left*-hand strip of paper will be seen of a bright *green* colour, and the *right*-hand strip of a *red* colour. If the strip of paper is sufficiently broad to make the two images overlap each other, the overlapping parts will be perfectly white, and free from colour. When equally luminous candles are held near each eye, the two strips of paper will be white. If, when the candle is held near the right eye, and the strips of paper are seen *red* and *green*, we bring the candle suddenly to the left eye, the left-hand image of the paper will gradually change to *green*, and the right-hand image to *red*." *

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* See Plateau's "Essai d'une Théorie generale," &c., Bruxelles, 1834; Edin. Review, April, 1834; and Phil. Mag. December, 1839, vol. xv. p. 435.

393. **Tendency of the eye to complementary impressions.** — From what has been here explained, it will be evident that there is in the organ of vision a natural tendency to the spontaneous production of that tint of colour which is complementary to the one by which the retina has just been strongly excited, and it may, therefore, be expected that an agreeable perception will be produced by dispersing in juxtaposition complementary colours; since the eye in turning from one to the other is always excited by that tint which it is predisposed to receive. And, on the other hand, contrasted colours which differ much from complementary ones produce a disagreeable effect, the eye being as it were disappointed in passing from one to the other. The chromatic relations have so obvious an analogy to musical sounds that they have been called harmonic, disharmonic, or discordant. Complementary colours are harmonic, and colours not complementary, discordant.

394. **Harmonious colours in art.** — The principles resulting from these relations should never be forgotten in the art of decoration, whether of buildings, furniture, or dress; and in fact they are practically applied, though often unconsciously, by all persons of good taste. Thus, in dress, red will accord with green, lilac with yellow, or blue with orange. In drapery, an orange fringe upon a blue stuff is rich and beautiful, while a light yellow upon a blue, or a blue upon a red, is hideous. A lady will throw a scarlet shawl over her shoulders when she wears a green dress, but never with a yellow one.

395. **Why visible objects do not appear inverted.** — A difficulty has been presented in the explanation of the functions of the eye to which, as it appears to me, undue weight has been given. It has been already explained, that the images of external objects which are depicted on the retina are inverted; and it has accordingly been asked why visible objects do not appear upside down. The answer to this appears to be extremely simple. Inversion is a relative term, which it is impossible to explain or even to conceive without reference to something which is not inverted. If we say that any object is inverted, the phrase ceases to have meaning unless some other object or objects are implied which are erect. If all objects whatever hold the same relative position, none can be properly said to be inverted; as the world turns upon its axis once in twenty-four hours, it is certain that the position which all objects hold at any moment is inverted with

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* Brewster's "Optics," p. 435; Lond. and Edin. Phil. Mag. 1832, vol. i. p. 171.

respect to that which they held 12 hours before, and to that which they will hold 12 hours later; but the objects as they are contemplated are always erect. In fine, since all the images produced upon the retina hold with relation to each other the same position, none are inverted with respect to others; and as such images alone can be the objects of vision, no one object of vision can be inverted with respect to any other object of vision; and, consequently, all being seen in the same position, that position is called the erect position.

396. **Direction in which objects are seen.**—The *true direction* of any point in a visible object is that of a straight line drawn from that point to the eye of the observer. The *apparent direction* of such a point is the direction in which it is seen. Now, it by no means follows that these two directions are identical, and in point of fact there are many circumstances under which they are totally different; as, for example, when the rays proceeding from the object are received upon and reflected by an oblique mirror before arriving at the eye. But since the rays which render an object visible in passing through the transparent structure of the eye are refracted and bent out of their course before arriving at the retina, it will be necessary to ascertain whether, by such deflection, they are not turned aside from their proper direction in such a manner as to make the object be seen in the direction of a line different from that which joins the object with the eye, or, in other words, to render the apparent different from the true direction.

To comprehend this question clearly it will be necessary to consider that the rays by which any point, o, *fig.* 191., is rendered visible (so far as they are external to the eye) consist of a diverging conical pencil, whose focus is the point o, and whose base is that part of the crystalline lens to which the light is admitted by the pupil, the diameter of which is represented by P P′. After passing through the crystalline the rays form a convergent pencil, having the same base, whose diameter is P P′, and having its focus o′ on the retina. All the rays composing this convergent pencil

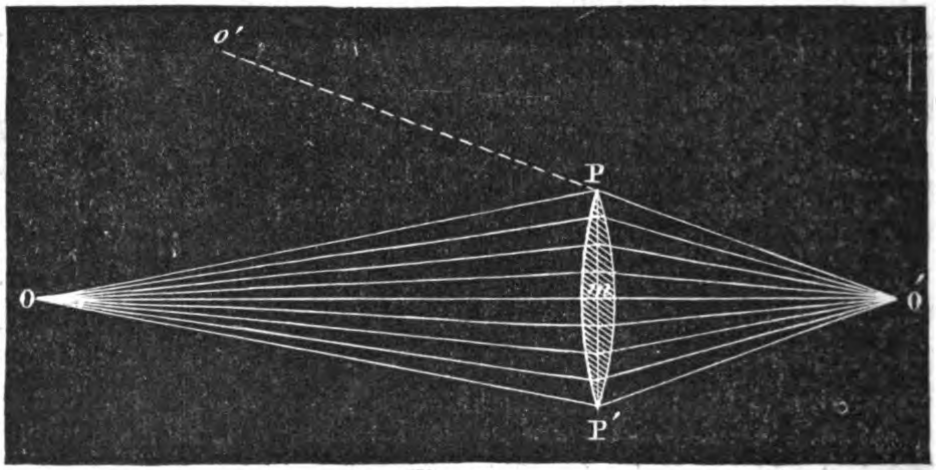

Fig. 191.

P O′ P′ unite in producing the perception of the point O by their combined action. But as these converging rays have severally very different directions, one only, O′ O, having the true direction of the object, it may be asked how it comes to pass that the apparent direction of the object should be that of this particular ray rather than that of any other of the numerous rays composing the pencil?

The answer to this question by *à priori* reasoning, would involve physiological points not compatible with the objects of this volume. But it is easy to show as a matter of fact that each separate ray composing the converging pencil P O′ P′ produces the same impression as to the direction of the visible point O, as does the ray O′ O, which coincides with the true direction of the object.

To establish this, let a card or any similar opaque plate be held before the pupil so as to intercept all the rays of the pencil P′ O P, except those which pass to the highest point P of the pupil. In that case the only rays which will strike the retina will be those which have the direction P O′, and it might therefore be expected that the point O would be seen, not in its true direction O′ O, but in the direction O′ P *o*′, just as though the ray P O were reflected by a mirror placed at P parallel to O′ O. Such, however, will not in fact be the case, but, on the contrary, the point O will be seen in its true direction O′ O, notwithstanding the very different direction of the ray O′ P by which the retina is excited.

Thus, it appears, as Sir David Brewster justly observes, that the line of direction in which the object is seen does not depend on the direction of the ray which produces vision, either before or after it passes through the pupil.

If the refracting apparatus of the eye could be regarded as equivalent to a convex lens of inconsiderable thickness, the apparent direction of each visible point would be that of a line drawn from the point itself to its image on the retina. But since this is not the case, the line joining any point of an object and its image on the retina will not pass through the centre of the lens, but through a point behind the lens coinciding very nearly with the geometrical centre of the eyeball.

When the visible point is situated in the direction of the optic axis, its apparent direction will be rigorously coincident with its true direction. But when it is removed more or less from the direction of the optic axis, so that the pencil P O P′ falls obliquely on the crystalline lens, the converging pencil P O′ P′ will also be oblique, and in that case the apparent direction of O will not be rigorously coincident with its true direction, and there will be a deviation which may be called *ocular parallax;* but this has been shown by Sir David Brewster to be so exceedingly minute in quantity as to produce no appreciable effect in the phenomena of vision; so that for all practical purposes, it may be stated that the apparent and true directions of all visible objects are identical, and that these directions always pass through the centre of the eyeball.

If the optical centre of the eye were not at the centre of the eyeball, the direction of this line of apparent direction would be changed with every movement of the eyeball in its socket; every such movement would cause the optical centre to revolve round the centre of the eyeball, and consequently would cause the line drawn from the optical centre to the object to change its direction. The effect of this would be that every movement of the eyeball would cause an apparent movement of all visible objects. Now, since there is no apparent motion of this kind, and since the apparent position of external objects remains the same, however the eye may be moved in its socket, it follows that its optical centre must be at the centre of the eyeball.

397. **Why the motion of the eyeball does not produce any apparent motion in the object seen.** — Since lines drawn from the various points of a visible object through the centre of the eye remain unchanged, however the eyeball may move in its socket, and since the corresponding points of the image placed upon these lines must also remain unchanged, it follows that the position of the image formed on the eye remains fixed, even though the eyeball revolve in the socket. It appears, therefore, that when the eyeball is moved in the socket, the picture of an external object remains fixed, while the retina moves under it just as the picture thrown by a magic lantern on a screen would remain fixed, however the screen itself might be moved.

Thus, if we direct the axis of the eye to the centre O, *fig.* 192., of any object, such as AB, the image of the point O will be formed at *o* on the retina, where the optical axis D C meets it. The axis of the pencil of rays which proceed from the point O will pass through the centre of the cornea D, through the axis of the crystalline, and through the centre C of the eyeball, and the image of O will be formed at *o*.

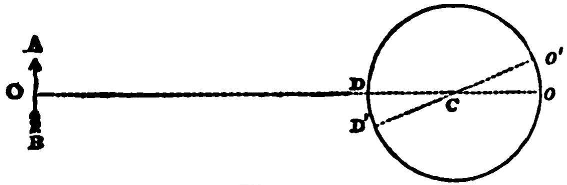

Fig. 192.

Now, if we suppose the eye to be turned a little to the left, so that the optical axis shall be inclined to the line OC at the angle D′CO, the image of the point O will still hold the same absolute position *o* as before; but the point of the retina on which it was previously formed will be removed to *o′*. The direction of the point O will be the same as before; but the point of the retina on which its image will be formed will be, not at the extremity of the optic axis, but at a distance *o o′* from it, which distance subtends at the centre C of the eye, an angle equal to that through which the optic axis has been turned.

It is evident, therefore, that although the eye in this case be moved round its centre, the point O is still seen in the same direction as before.

But if the optical centre of the eye were different from the centre of the eyeball, the direction in which the point O would be seen would be changed by a change of position of the eye.

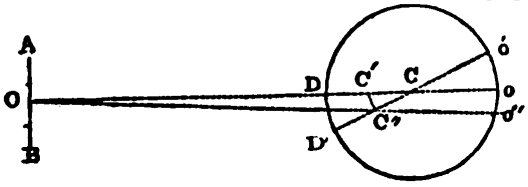

Fig. 193.

To render this more clear, let C, *fig.* 193., be the centre of the eyeball, and C′ the optical centre of the eye. Let the optical axis C D, as before, be first presented to the point O of the object. The image of this point will, as before, be formed at *o*, the point where the optical axis D C meets the retina. Let us now suppose the axis of the eye to be turned aside through the angle D C D′, the optical centre will then be removed from C′ to C″, and the image of *o* will now be formed at the point *o″*, where the line O C″ meets the retina. The direction, therefore, in which O will now be seen, will be that of the line C″ O, whereas the direction in which it was before seen was that of the line C O. The point of the retina at which the image *o* was originally formed is removed to *o′*, while the image is removed to *o″*. Thus there is a displacement not only of the retina behind the image, but also an absolute displacement of the image, and an absolute change in the apparent direction of the object. Since no such change in the apparent direction is consequent upon the movement of the eye in its socket, it follows that the optical centre C′ of the eye must coincide with its geometrical centre C.

398. **Foramen centrale and limbus luteus, or yellow spot.** —That part of the retina which immediately surrounds the point of it to which the optic axis is directed, is attended with several circumstances which, though they are more anatomical than optical, ought not to be passed over here.

The point where the optic axis meets the retina is the centre of a circular yellow spot, called the *limbus luteus*, the radius of which is about the sixteenth of an inch. In its centre, and therefore at the extremity of the optic axis, is what has the appearance of a minute hole, and has accordingly been called from its discoverer, the *foramen centrale* of Soemmering. It is, however, considered by anatomists that this is not a real opening between the vitreous humour and the choroid, inasmuch as a layer of vascular matter covers it, the opening being only in the medullary substance of the retina at that particular point.

The distance between the foramen centrale and the centre of the embouchure of the optic nerve is about the tenth of an inch, and since the radius of the yellow spot is the sixteenth of an inch, it follows that the edge of the yellow spot is about the twenty-seventh of an inch from the centre of the optic nerve.

Taking the radius of the concave spherical surface formed by the retina to be half an inch, 1° upon it will correspond to the 115th part of an inch; and, consequently, the angle subtended by the semidiameter of the yellow spot at the optical centre of the eye will be 7°, and the angle subtended by the distance between the foramen centrale and the centre of the embouchure of the optic nerve will be 11½°.

399. **Local sensibility of the retina.**—The sensitiveness of that membrane is not the same at all points. If we direct the optic axis to any point upon a distant object, a certain extent of that object surrounding the point to which the optic axis is directed will be visible, but not with a uniform vividness and distinctness. The point to which the axis is directed will be seen with greatest distinctness, and the surrounding points will be perceived with less and less distinctness, as they are more distant from this central point, until they altogether disappear.

The extreme mobility of the eye, and the subtle and unconscious action of the will upon it, render it extremely difficult to keep the axis fixed upon a certain point while the visual perception of the surrounding points is attempted to be observed. The moment we desire to ascertain to what visual distance on any side of the central point our perception extends, the optic axis, with the rapidity of thought, directs itself to such points. Nevertheless, by much practice, such self-control can be acquired as will enable an observer to ascertain with some degree of approximation the extent of the *field of vision*, by which term is expressed the circular space surrounding the point to which the optic axis is directed, which includes all the objects which can be perceived by the eye at the same instant.

The circle of the retina surrounding the foramen centrale, which corresponds to this field of vision, includes the entire extent of that membrane which is available for the sense of sight; for, although the range of the eye is really much greater, that extension of its sphere of perception is due to the mobility of the eyeball, by which, as already explained, the optic axis has a play, measured horizontally and vertically, through a considerable angular space.

To determine, by immediate observation, the extent of the field of vision when the optic axis is fixed, let a number of red wafers be attached at short intervals to the circumference of a circle having a whitish ground two feet in diameter, and let a single wafer be attached to its centre. Let the card or pasteboard upon which the wafers are attached be fixed to a vertical wall, so that the central wafer shall be at the level of the eye of the observer standing with his face to the wall. If the observer, closing one eye, the left, for example, stand so that a line drawn from the other eye to the central wafer shall be perpendicular to the plane of the circle, and so that his distance from the wall shall be ten or twelve feet, he will see the entire circle of wafers when the optic axis of his eye is directed to the central wafer. If then he gradually approach the circle, still keeping the optic axis directed upon the central wafer, the circumferential wafers will continue to be visible, but will be gradually less and less distinct. When he approaches to the distance of five feet from the central wafer, a remarkable effect will ensue. Those circumferential wafers which are at and near the right-hand extremity of the horizontal diameter of the circle will suddenly cease to be visible, and a gap will appear in the circle on the right side, extending over a fifth or sixth part of the entire circumference.

If the observer now approach still nearer to the circle, keeping the optic axis still directed to the central wafer, the right-hand wafers will continue to be invisible, until he comes within something less than three feet of the central wafer, when they will suddenly reappear. But upon approaching still nearer, all the circumferential wafers will vanish, the central wafer alone being visible.

400. **Explanation of the phenomena.** — To explain these phenomena, it must be observed that at the distance of ten feet the radius of the circle is seen under a visual angle of 5·7°, which corresponds to the 20th of an inch upon the retina. The retinal image of the circle of wafers will therefore be a circle having a radius of the 20th of an inch described round the foramen centrale; it will therefore fall within the yellow spot; and, as in this position the observer sees with considerable distinctness the circumferential wafers, and with perfect distinctness the central wafer, it follows that the sensibility of the retina corresponding to the yellow spot is within this limit sufficient for distinct vision, the central point, however, being the most sensitive and producing the most distinct perception.

As the observer approaches the circle, the diameter of the image on the retina increases in the same proportion, very nearly, as the distance of the eye from the centre of the circle diminishes. At the distance, therefore, of five feet, the radius of the retinal image is increased to the tenth of an inch, and that part of it which is on the side of the nose consequently passes across the embouchure of the optic nerve; and as this corresponds to that part of the circle which in this position of the observer becomes invisible, it follows that that part of the retina which corresponds with the embouchure of the optic nerve is absolutely insensible.

That this is the true explanation of the phenomenon is proved by the fact that when the observer approaches within less than three feet of the central wafer, the circumferential wafers which were before invisible suddenly reappear. In that case the image of the circle on the retina is so enlarged that its circumference includes within it the entire embouchure of the optic nerve, so that the wafers which at five feet distance projected their images upon the embouchure of the optic nerve, now project them on that part of the retina which lies outside the nerve.

The experiment may be further varied by attaching to the surface of the card several concentric circles of wafers of increasing diameters. When the observer stands at a certain distance from such a card, in the position above described, and with the left eye closed, the wafers on the right of each of the circles, whose radius is between a third and a fifth of his distance from the card, will be invisible, while those which are at a less or greater distance will be perceived.

If the like observations be made with the left eye, the right being closed, similar results will ensue; the wafers which disappear, however, being those on the left side of the circles.

Since it is generally admitted by anatomists and physiologists that the nerves are the only conduits between the organs of sense and the brain, it must appear somewhat inexplicable that the foramen centrale, the only point of the retina where practical anatomists are unable to discover the presence of nervous matter, should not only possess visual sensibility, but should be endowed with that power in a higher degree than any other part of the retina. It cannot, therefore, be matter of surprise that the result of their observations is received with much doubt, more especially as the nervous fibres are highly microscopic, and may be regarded as probably more and more minute, as their sensibility is more exalted. Until, therefore, demonstration more positive and conclusive of the total absence of nervous matter within the vascular covering of the foramen shall be obtained, it will be considered probable that nervous fibres exist there, though their tenuity is such as to escape the observation of microscopic anatomists.

401. **Limits of field of distinct vision.**— Observers have not agreed as to the magnitude of the field of vision, while the optic axis has a fixed direction. I find by my own observa-

tions that objects comprised within a circle of a foot radius, described round the point to which the optic axis is directed, are visible with sufficient distinctness for all the purposes of vision when the eye is placed at the distance of about six feet from the circle. This would correspond to a visual angle of nearly 20° radius, so that if the optic centre of the eye be supposed to be the apex of a cone whose angle is about 40°, all objects within that cone will be visible at the same moment with sufficient distinctness when the optic axis has the direction of the axis of the cone.

When the eye approaches nearer to such a circle, the objects comprised within it, with the exception of those rendered invisible by the insensibility of the embouchure of the optic nerve are still seen, but the perception of them is indistinct and unsatisfactory.

It is probable, however, that these limits of distinct vision, measured from the optic axis as a centre, may be different in different eyes.

Professor Valentin, of Bern, gives the narrow limit of 3° round the optic axis, as the range of distinct vision. This must certainly be an error. The Professor does not state on what authority nor on what kind of experiment or observation his conclusion is based.

A radius of 20° corresponds to about the sixth of an inch upon the retina, and if the conclusion derived from my own observations be correct, it will follow that the portion of the retina available for distinct vision will be a circle described round the foramen centrale as a centre, with a radius of about the sixth of an inch.

402. **Attention necessary to visual perception.** — In enumerating the conditions necessary to insure the distinct perception of a visible object, we have in what precedes included those only which are strictly optical; there is, however, a mental condition not less necessary to perception than the optical conditions already mentioned.

The mind has the power by an act of the will to direct its attention with more or less exclusiveness to certain perceptions or ideas, whether proceeding directly from external objects, or evoked by memory or imagination, in preference to others; and in such cases, although all the conditions of distinct vision above enumerated may be fulfilled, no distinct perception, or no perception at all, may be produced, owing to the attention of the mind being diverted to other objects. This is not peculiar to sight, but common to all sensible impressions. When engrossed in thought upon any subject of deep interest, we often have our eyes open and fixed upon external objects, from which the retina receives impressions fulfilling all the conditions of distinct vision, yet we see nothing. Physiologists explain this by stating that the fibres of the optic nerves, although transmitting to the sensorium the action produced upon the retina, fail to produce a perception there because the sensorium is then preoccupied by other thoughts and perceptions. Although this, instead of explaining the phenomenon, is little more than a statement of it, it is the only solution offered of a question which lies upon the confines of physiology and psychology. "But by this faculty of attention, we also analyse what the field of vision presents. The mind does not perceive all the objects presented by the field of vision at the same time with equal acuteness, but directs itself first to one and then to another. The sensation becomes more intense according as the particular object is at the time the principal subject of mental contemplation. Any compound mathematical figure produces a different impression, according as the attention is directed exclusively to one or the other part of it. Thus, in *fig*. 194., we may in succession have a vivid perception of the whole, or of distinct parts only; of the six triangles near the outer circle, of the hexagon in the middle, or of the three large triangles. The more numerous and varied the parts of which a figure is composed, the more scope does it afford for the play of the attention. Hence it is that architectural ornaments have an enlivening effect on the sense of vision, since they afford constantly fresh subject for the action of the mind." *

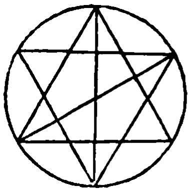

Fig. 194.

403. **Binocular vision.** — The optical phenomena which we have hitherto considered and explained, are such as would be produced in an observer having a single eye, and, as distinguished from certain others now to be explained, may be denominated *monocular*; the peculiar phenomena depending on the simultaneous vision with two eyes being called *binocular*.

404. **Why with two eyes vision is not double.** — The first question which is presented and often asked is, why, having two eyes on which independent impressions are made by the same external object, we do not see that object double?

The first reflection which arises on the proposition of this question, is why the same question has not been similarly proposed with reference to the sense of hearing. Why has it not been asked why we do not hear double? why each individual sound produced by a bell or a string is not heard as two distinct sounds, since it must impress independently and separately the two organs of hearing?

It cannot be denied that, whatever reason there be for demanding a solution of the question, why we do not see double, is equally applicable to the solution of the analógous question, why we do not hear double. Like many disputed questions, this will be stripped of much of its difficulty and obscurity by a strict attention to the meaning of the terms used in the question, and in the discussion consequent upon it. If by seeing double it be meant that the two *eyes* receive separate and independent impressions from each external object, then it is true that we see double. But if it be meant that the *mind* receives two distinct and independent perceptions of the same external object, then a qualified answer only can be given.

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* Müller's "Physiology," vol. ii. p. 1179.

If the two eyes convey to the mind precisely the same impression of the same external object, differing in no respect whatever, then they will produce in the mind precisely the same perception of the object; and as it is impossible to imagine two perceptions to exist in the mind of the same external object which are precisely the same in all respects, it would involve a contradiction in terms to suppose that, in such case, we perceive the object double.

If to perceive the object double mean anything, it means that the mind has two perceptions of the same object, distinct and different from each other. Now, if this distinctness or difference exist in the mind, a corresponding distinctness and difference must exist in the impression produced by the external object on the organs. It will presently appear that cases do occur in which the organs are, in fact, differently impressed by the same external object; and it will also appear that in such cases precisely we *do see double;* meaning by these terms that we have two perceptions of the same object, as distinct from each other as are our perceptions of two different objects.

To render this point more clear, let us consider in what respects it is possible for the impressions made upon the two eyes by the same object to differ from each other.

A visible object impresses the eye with a sense of a certain apparent form, of a certain apparent magnitude, of certain colours, of a certain intensity of illumination, and of a certain visible direction. Now, if the impression produced by the same object upon the two eyes agree in all these respects, it is impossible to imagine that the mind can receive two distinct perceptions of it, for it is not possible that the two visual perceptions could differ from each other in any respect, except in some of those just mentioned. Let us suppose the two eyes to look at the moon, and that it impresses them with an image of precisely the same apparent form and magnitude, of precisely the same colours and lineaments, of precisely the same intensity of illumination, and, in fine, in precisely the same direction. Now, the impressions conveyed to the mind by each of the eyes corresponding in all these respects, the object must be perceived in virtue of both impressions precisely in the same manner; that is to say, it must be seen in precisely the same direction, of precisely the same magnitude, of precisely the same form, with precisely the same lineaments of light and shade, and with precisely the same brightness or intensity of illumination. It is, therefore, in such a case, clearly impossible to have a double perception of it.

But to render intelligible the causes which produce double vision in the exceptional cases which will be presently noticed, as well as single vision in the normal application of the organs of sight, it will be necessary to explain what are the physiological conditions which correspond with the optical ones above explained, and which render absolutely identical the perceptions of the same object produced by the two eyes.

405. **Physiological conditions of single vision.** — The physiological condition which causes identity of perception by two eyes, is simply the perfect identity as to magnitude, colour, brightness, form, and position of the optical pictures of the object formed on the two retinæ. But to understand what constitutes their identity of position, it is necessary that some point or line should be assigned in reference to which the position of the picture is determined. This line must obviously be the optic axis, and the position of the two pictures will be identical if their corresponding points are similarly placed around the foramen centrale of the retina, that being the point, as already explained, through which the optic axis passes. Thus, if any point of one image fall upon the retina at the hundredth of an inch to the left of the foramen centrale, the corresponding point of the other image must also fall at the hundredth of an inch to the left of the foramen centrale of the other eye. In the same manner, if the image of any point fall upon the retina of one eye at any given distance above, or below, or to the right, of the foramen, the image of the same point must fall at the same distance above, or below, or to the right of the foramen of the other eye.

406. **Perfect identity of the two ocular pictures.** — It will be evident from what is here stated, that if the optical pictures of the same object on the two eyes be supposed to be divided by horizontal lines through the foramen, the upper half of one picture will correspond with the upper half of the other, and the lower with the lower; and if they be similarly divided by vertical lines through the foramen, the right-hand half of one will correspond with the right-hand half of the other, and the left with the left.

It is most necessary to observe, however, that when the relative positions of the several parts of the pictures are referred, as they frequently are, to the nose, those parts which are next the nose do not correspond. The right-hand division of the picture in the left eye is *inside* or next the nose, while the right-hand division of the picture in the right eye, which corresponds with it, is *outside* or next the temple. Since these two parts produce one and the same perception, it is necessary to suppose that the nervous fibres which proceed from the external half of one retina must coalesce with those proceeding from the internal half of the other retina before arriving at the sensorium, or, if not that, some other physiological expedient must be provided, in order to combine the impressions produced upon those points of the two retinæ.

In fine, the identical optical pictures upon the two retinæ must be such that if one were imagined to be transferred to the other, so as to place the one foramen upon the other, and any two other corresponding points one upon the other, all the points of the one would fall upon the corresponding points of the other.

407. **Conditions of identity.** — To fulfil these conditions, it is necessary and sufficient that the two optic axes should be directed to the same point of the object, and that the object should be at equal distances from the two eyes. If the optic axes be directed to different points of the object, then the images of different points will be formed at the foramina, and consequently images of different points at all corresponding points of the retina. In that case the two pictures will be different, and the effect will be the same as if two eyes looked at two different objects, and double vision would consequently ensue.

408. **Case in which the pictures are unequal.** — If, while the optic axes are directed to the same point of the object, its distances from the eyes are unequal, the optical pictures will be similarly placed on the two retinæ, but will be unequal in magnitude, their linear dimensions being inversely proportional to the distances of the object from the eyes. But such an inequality of distance as would produce any sensible inequality of the two pictures, can only take place when the distance of the object is so limited that the distance between the eyes will bear a considerable proportion to it; and in that case another effect intervenes, which it is important to notice. The distance of the object will, in short, be so small, that an adjustment of the eye will become necessary for distinct vision; and since the distances of the object from the two eyes are assumed to be unequal, the adjustment for distinct vision of one will be different from the adjustment for distinct vision of the other. If such different adjustments can be simultaneously made, both images will be distinct, but the smaller will be superposed upon the larger, so as to produce indistinctness, all the points superposed except the central one being different.

But if, on the other hand, it be only possible to make the ocular adjustment for distinct vision, at one or the other of the two unequal distances, then one of the pictures will be distinct, and the other more or less confused; but they will still be centrally superposed.

When the distance of a visible object bears so great a proportion to the distance between the eyes that the angle formed by lines drawn from the eyes to any common point in the object may be regarded as evanescent, or so small as to be insensible, the axes of the eyes, which are directed to any point in such an object, will be for all practical purposes parallel. But when the object is at less distances, the angle formed by the same lines will be greater as the distance of the object is less, and, within a certain limit of distance, will acquire sensible magnitude.

409. **Binocular parallax.**—The distance between the centres of the eyes is to some extent different in different individuals, but its average magnitude in adults may be taken at two and a half inches; lines, therefore, drawn to a point twelve feet distant from the nose, would form an angle of 1°, and consequently the axes of the eyes, when directed to such a point, would be inclined to each other at that angle. Now, any angle less than this in magnitude would obviously be insensible, so far as relates to the voluntary effort by which the inclination of the optic axes is varied; but for less distances than twelve feet, the effort which gives them the obliquity corresponding to the binocular parallax, becomes more and more sensible as the distance becomes less. Thus, in looking at a point six feet distant, the parallax, and consequently the inclination of the optic axes, is 2°; at three feet is 4°; at eighteen inches, 8°; and so on.

This angle, formed by the optic axes, when directed to the same object, is called the *binocular parallax* of such object.

410. **Distance estimated by it.**—One of the means by which the distances of visible objects are judged of, is the muscular effort by which the obliquity corresponding to their binocular parallax is given to the optic axes. The greater that effort is, the nearer will be the object looked at.

According to Professor Müller, the effort by which the eyes are adjusted to distinct vision at varying distances, is always simultaneous with that by which the obliquity of the optic axes is made to accord with the binocular parallax. So invariable is this coincidence, that the axes cannot be directed to any near point, even by an effort of the will, without the other internal adjustment of the eye for distinct vision taking place. And if, on the other hand, while the optic axes are directed to any given point, the eye, by an effort of the will, adjusts itself for the distinct vision of any point at a greater or less distance, the axes will involuntarily change their directions, and will converge to a point at the distance of distinct vision.

411. **Cases in which binocular parallax is evanescent.**— These principles have been applied by Professor Müller to explain a multitude of binocular phenomena.

Objects seen at distances at which the binocular parallax is evanescent can never be seen double, for it is easy to prove that their ocular pictures fulfil all the conditions of single vision. The optical axes directed to any point in such an object are necessarily parallel, and images of that point are produced at the foramina, while images of all the surrounding points are produced at corresponding points surrounding the foramen of each retina. The distances of the object from the two eyes being also necessarily equal, the pictures will be of the same magnitude. They are thus absolutely identical in all respects, as well in magnitude as position, and must, consequently, produce a single perception.

The points which in such cases fall within the field of vision are necessarily seen single, however they may differ in their distance from the eyes; for the binocular parallax, being evanescent for all of them, will have no sensible difference, and they will be seen as if they were delineated, as are the various points in a painted landscape, all upon the same surface, and at the same distance from the eyes.

412. **Cases in which binocular parallax is sensible.** — But the case is otherwise for points whose distances from the eyes are within such limits of magnitude, as to produce binocular parallax of sensible amount, and here some very remarkable and interesting phenomena arise.

We have shown above, that when the objects included within the field of vision are placed at distances so considerable, compared with the distance between the eyes, that the binocular parallax shall be evanescent, all the points within the field of vision will have positions in each eye, identical with those which they have in the other, and that, consequently, the vision must always be single.

413. **Horopter defined.**—But if the points be not so distant, and if the binocular parallax have sensible magnitude, it is still possible that the various points, which are viewed, may occupy identical positions in the two eyes, and that the vision of each of them will consequently be single. This will take place, as was first shown by Vieth, and later by Müller, provided such points are placed in the segment of a circle described upon the line joining the centres of the eyes as a base.

This will be easily understood by reference to *fig.* 195., where a a' are the foramina, and and a α and a' α the optic axes converging to the point α. Let b and b' be two corresponding points on the two retinæ, so that the angles marked 1 shall be equal. The visual rays from b b' will then converge to the point β, and since the angles marked 1 are equal, those marked 1′ must also be equal; and since the angles marked 2 are also equal, the angles marked 3 must be equal, and by the known properties of a circle the points α and β must lie in the segment of a circle described upon the line joining the points of intersection of the visual rays as a base.

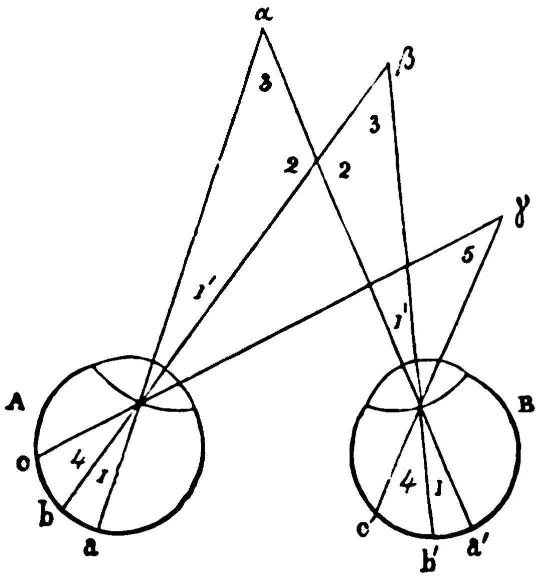

Fig. 195.

In the same way exactly it may be shown that if c and c' have similar positions, the point γ to which they converge will be on the same segment, and so of any other corresponding points.

Thus, it appears that if a segment of a circle be imagined to be described upon the line joining the centres of the eyes as its base, all points in the circumference of such segment will have images in corresponding positions on the two retinæ, and will be seen single.

If such a segment be imagined to revolve round its base it will generate a solid of revolution which will be the *locus* of all points, which will be seen single so long as the binocular parallax is equal to the angle of the segment.

If a line be imagined to be drawn from the middle point of the base of this segment perpendicular to that base, it will meet the segment at α, which point may be regarded as the vertex of this curved surface, which is the locus of the point of single vision. If a tangent plane to that surface be imagined to be drawn through the point α, all points in that plane which are near the point α will coincide nearly with the curved surface, and will be seen single. But the same near coincidence will not take place at other points, such as β and γ; from whence it appears that distinct and single vision will be obtained if a near object be placed directly opposite the nose, so that the lines drawn from the eyes to it shall be equal, and provided the magnitude of such object be not considerable, compared with its distance.*

The surface of single vision corresponding to any given binocular parallax, is called the *horopter*.

Since there is no other example in the nervous system of the corresponding nerves at the two sides of the body referring their respective sensations to the same spot, Professor Müller considers that the cause of the perception of a single image of a point placed in any given horopter, must lie in the organisation of the deeper cerebral part of the apparatus of vision. "The eyes," he says, "may be compared to two branches issuing from a single root, of which every minute portion bifurcates so as to send a twig to each eye."†

414. **Objects out of horopter seen double.**—Whenever an object lies out of the horopter which corresponds to the angle formed by the optic axes, it will be seen double. The most simple experimental illustration of this is the following:—Hold the two forefingers pointed directly upwards before the eyes, opposite the nose, one near the face, and the other much more distant. If we look at either so as to cause the optic axes to converge towards it, the other will be seen double, and the distance between the double images will be greater or less, according as the distance between the fingers is greater or less. The double images will also be indistinct, and will be more indistinct the farther they are apart.

To demonstrate this, let the optic axes of the eyes A and B, *fig.* 196., be directed to a point *a*, so near them as to have con-

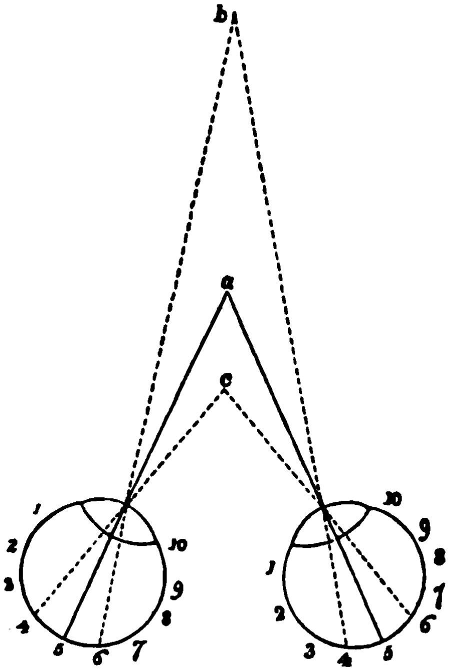

Fig. 196.

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* Professor Müller states that the surface which is the locus of points of single vision is a sphere. This is evidently an error, since the equality of the binocular parallax requires that all sections of such surface, made by a plane passing through the line joining the centres of the eyes, shall be a segment of a circle, the angle of which is equal to the binocular parallax; a condition which is only fulfilled by such a surface of revolution as that above described. I am not aware that this error has been previously pointed out.

† Müller's "Physiology," p. 1197. *trans.*

siderable binocular parallax. Let *b* be a point more distant than *a*. Its image in the eye A will be at 6, and its image in the eye B will be at 4. The point 6 in A, and the point 4 in B, being one to the right, and the other to the left, of the foramen 5, will have positions which do not correspond, and consequently will produce a double image. The eye A will see the object *b* to the left, and the eye B will see it to the right of *a*, and the appearance will thus be that of three objects; *a* seen in the middle, and two similar to *b* seen to the right and to the left of it.

Since, according to what has been explained, the eye is necessarily adjusted for distinct vision at the distance *a*, the images of *b* will be both indistinct, and will be more indistinct the more the distance of *b* exceeds that of *a*.

The apparent distance of the two images of *b* to the right and to the left of *a* will be measured by the angles formed by the lines drawn from the eyes to *b* with those drawn to *a*, and these angles will evidently be augmented, as the distance of *b* from *a* is augmented. Thus, it appears that the distances of the two images of *b* from *a*, an from each other, as well as their indistinctness, will be increased as the distance between *a* and *b* is increased.

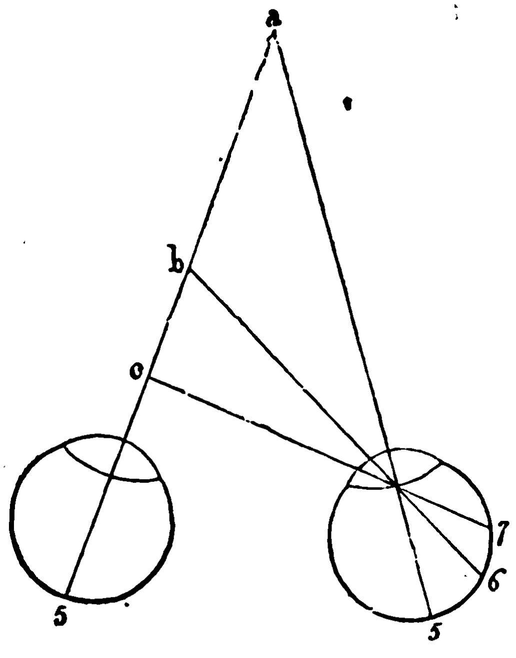

Fig. 197.

That the image of *b* to the left of *a* is that produced in the eye A, may be proved by holding a screen, or the hand, between A and the object *b*. The left-hand image of *b* will then disappear, the right-hand image being still seen. If the screen be held before the eye B, the right hand image of *b* will disappear. If, in fine, the screen be held before both eyes, so as not to interfere with the optic axes, both images of *b* will disappear, and *a* only will be seen.

If an object, in the same manner, be held at *c*, between *a* and the nose, it will also be seen double, and the phenomena may be explained in precisely the same manner. But in this case the image of *c* seen by the eye A will be to the right of *a*, and the image of it seen by the eye B will be to the left of *a*. These statements may be verified, as before, by the interposition of the screen.

The experiment may be varied in several ways. Thus, if, as shown in *fig.* 197., the optic axis of the right eye be directed to an object *a*, while other objects *b* and *c* are placed in the direction of the axis of the left eye, such objects will be seen double, since with the left eye their images will be projected upon *a*, while with the right eye they will be seen to the left of *a*, in the direction 6*b* and 7*c*.

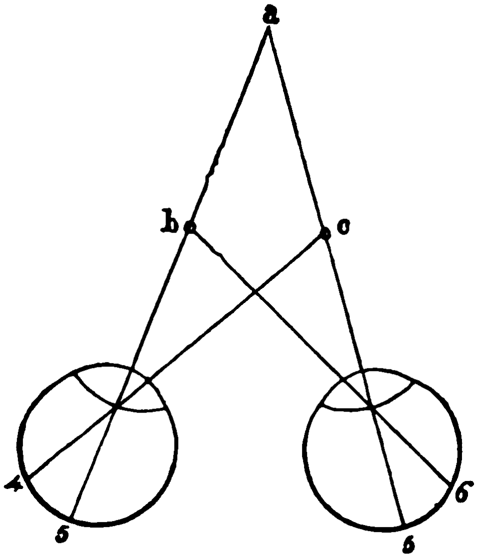

Fig. 198.

In the same manner, the optic axes being supposed to be directed to an object *a*, *fig.* 198., if two other objects *b* and *c* be placed upon them, between *a* and the eyes respectively, an image of *b* will be seen by the right eye to the left of *a*, and an image of *c* by the left eye to the right of *a*, in the direction of the lines 6*b* and 4*c* respectively. The image of *b* seen with the left eye, and that of *c* with the right eye, will be both projected upon *a*.

415. **Double vision why little attended to.** — Since the phenomena of double vision are so evidently connected with the ordinary use of the eyes, it might be expected that instead of attracting the attention of philosophers alone, they would be familiar to every one. Although, however they do constantly present themselves, they in general receive little or no attention, either because the double images are always indistinct, or because our attention is exclusively directed upon the objects which are seen single, and therefore distinct. "In all cases, however, where two objects situated at different distances, and not lying in the same horopter, are seen simultaneously, one or other of them must necessarily appear double. Thus when we look through a window upon a church steeple, either the window-frame or the steeple must appear double, according as the axes of the eyes are directed to the one or the other. Whenever the power of directing the axes of the eyes, so as to meet in the object is, from any cause, lost, double vision must result; hence its occurrence in persons intoxicated, in nervous fevers, in the paroxysms of nervous or hysterical affections, in the state imme-

diately preceding sleep, and in strabismus. This double vision is in no way dependent on any change in the central parts of the nervous system, or in the retina, but is the simple result of the inability to direct both eyes to the object. In the state preceding sleep, and at the moment of falling asleep, our eyes are always rotated strongly inwards; hence at those times all objects, even near objects, appear double. The too great convergence of the eyes can be recognised in the position of the double images; the left-hand image is found to belong to the left eye. In the state of intoxication, also, the eyes are directed inwards."*

416. **Cases in which the two eyes look at different objects.**—In the preceding paragraphs we have considered the cases of visual perception in which the same object is looked at by both eyes, and have shown the conditions under which it will be seen single or double. It remains now to consider a case, which, though not presented in the ordinary use of the eyes, is one which supplies some important illustrations of the physiology of the organ of vision. The case we refer to is that in which the two eyes look at the same time at two different objects.

If two such objects be precisely similar in form, magnitude, colour, and illumination, and if the optic axes of both eyes be directed to them so that their images shall be formed upon the two foramina, they will be seen as one object, and their common position will be the point to which the optic axes converge. If the optic axes in this case be parallel, the two objects will appear as one, placed in their common direction, at such a distance as to render the binocular parallax evanescent.

If, however, the optic axes be not directed to them, but so directed that their images shall be formed at corresponding points of the two retinæ, they will be still seen as a single object, but not so distinctly as when their images are formed at the foramina.

If, in fine, the optic axes be so directed that the two images shall be formed at points of the retina which do not correspond, then the two objects will be seen separately in the directions of lines connecting them with their respective images on the retina.

417. **Experimental illustration.** — This experiment may be performed by mounting two straight tubes like those of a binocular opera glass, but with a provision by which their axes can be placed either parallel to each other, as in the opera glass, or inclined to each other at any desired obliquity. In the ends of these tubes cards may be set, pierced with holes of any desired magnitudes. Opposite these holes may be placed illuminated surfaces of any desired colours, which, when viewed through the

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* Müller's "Physiology," p. 1204. *trans.*

holes, will have the appearance of coloured discs whose apparent magnitudes will be those of the holes.

Now, if we suppose the tubes so adjusted as to be parallel, the holes having equal magnitudes, and that the same coloured surface shall be presented to them, the appearance will be that of two discs of the same magnitude, colour, and illumination, and, the optic axes being parallel, their images will be formed on the foramina of the two retinæ. The appearance, therefore, will be that of a single object at such a distance from the eyes as to render the binocular parallax evanescent.

418. **Case of binocular opera glass.** — In fact, the same ocular phenomenon is actually produced in the common use of the binocular opera glass. The axes of the two tubes in that instrument are set parallel, and the object viewed is supposed to be, and in fact must be, at such a distance as to render the binocular parallax evanescent. The eyes therefore view two distinct images of the same object, which may be regarded as two distinct objects, so placed that when the eyes are directed to them the optic axes are parallel. In this case, as is well known, the vision is single.

419. **Cases in which the optic axes are not parallel.** — If the two tubes above described instead of being parallel are so inclined that their axes shall intersect, a surface having a uniform colour being presented to the two holes, the two discs will be seen as a single disc would be, placed at the intersection of the axes of the tubes. They will, therefore, be seen as a single object.

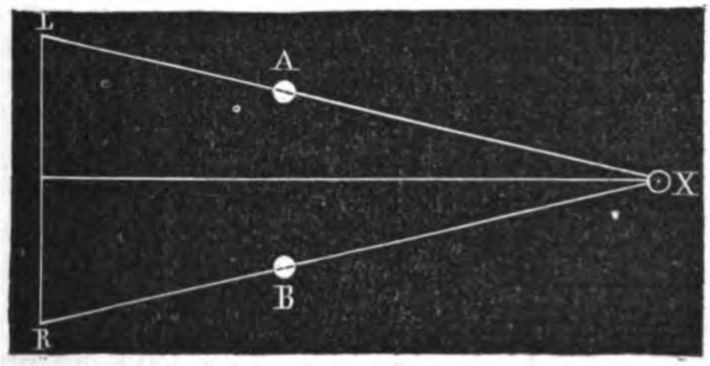

Fig. 199.

To render this more clear, let L A, *fig.* 199., be the tube corresponding to the left, and R B that corresponding to the right eye, X being the point to which the axes of the tubes converge, and A and B the discs visible in the direction of the axes of the tubes.

The two discs thus seen will appear as a single white disc at X, where the visual axes intersect. But in this case the images will be indistinct, since, according to what has been already explained, the convergence of the visual axes to x would be accompanied by an adjustment for distinct vision at the distance x, which would render indistinct both images, the objects being at less distances.

If, in either of these cases the holes, A and B, have different magnitudes, the ocular images having still corresponding positions on the two retinæ, the apparent magnitude of the disc A, seen with the left eye, will be different from that of the disc B, seen with the right eye. It might be expected in such case that the two images would be seen superposed one upon the other, the smaller image being concentrically superposed upon the greater, and therefore rendering the centrical portion of the greater image brighter.

It has appeared, however, from the experiments and the observations of Professor Wheatstone, that such is not the effect, but that the apparent magnitude of the image perceived is intermediate between the two monocular images.

If the horopter be supposed to be divided into two parts, right and left, by a vertical plane passing through the nose, an object placed anywhere in that plane will be at equal distances from the eyes, and images of it having equal magnitudes will be formed at corresponding points on the two retinæ. But if the object be placed in the horopter to the right or to the left of that vertical plane, it will be nearer to one eye than to the other, being always nearer to the eye which is on the same side of the vertical plane. Although, therefore, the two images will be formed on corresponding points of the two retinæ, and, therefore, will be seen in a common direction as a single image, they will nevertheless differ in magnitude; that which is formed in the eye on the same side of the vertical plane with the object being greater than the other. Professor Wheatstone found that in such a case the visual perception produced would be that of an object having an apparent magnitude intermediate between the apparent magnitudes as seen by the two eyes separately.

420. **Cases in which the superposed objects have different colours.** — If the objects, A and B, have different colours, it might be expected that the projection of their images at x would produce the impression of a single image, having a colour due to the mixture of the proper colours of the two objects. Such, however, is not found to be the case: the two images do not coalesce, nor do they appear permanently superposed; but at one time one image, and at another time the other, will be seen; and, at the moment of change, fragments of the two intermingled will be visible. It does not seem to be in the power of the will to

determine the appearance or disappearance of either; but if one picture be more illuminated than the other, that which is less so will appear during shorter intervals.

If the visual lines intersect between the eyes and the discs, as in *fig.* 200., the same results will ensue, but the point X will be between the eye and the objects.

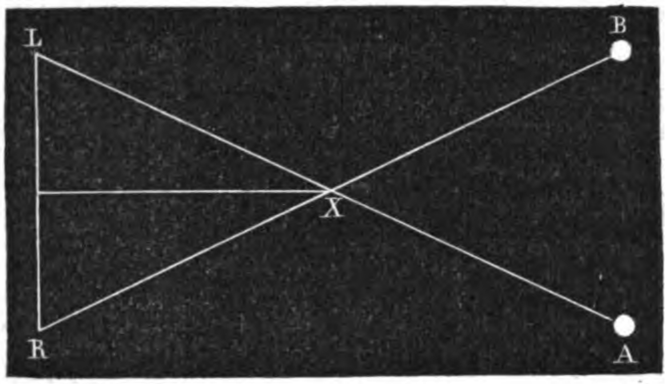

Fig. 200.

421. **Effect of binocular parallax on near objects.** — When an object having relief (such, for example, as a globe) is placed at such a distance from the eyes as to give it sensible binocular parallax, the part which is visible to one eye will be different from that which is visible to the other. Thus, for example, if R, *fig.* 201. be the right, and L the left eye, a ball S being placed before them, the parts seen by the right eye will be *r r'*, while the parts seen by the left eye will be *l l'*. It appears, therefore, that the part *l r'* will be seen at once by both eyes, while *l r* is seen only by the right, and *l' r'* only by the left eye.

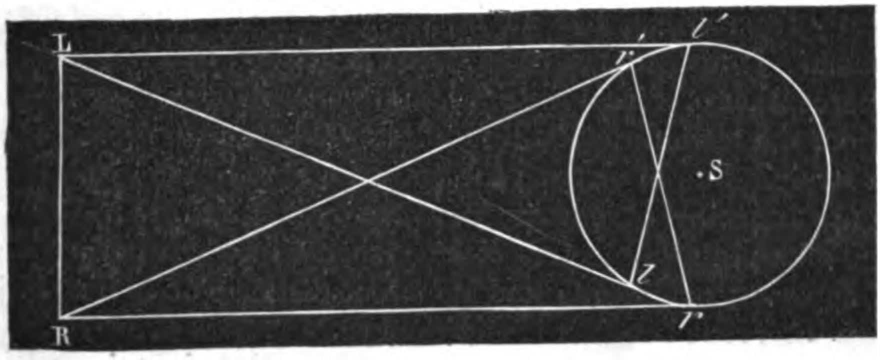

Fig. 201.

422. **Cause of the appearance of relief.**—It is to this circumstance of an object being seen under different aspects by the two eyes, that our perception of relief has been ascribed; and it has

accordingly been adopted as the principle upon which the optical instruments called stereoscopes are constructed.

Although, however, it must be admitted that remarkable appearances of relief are in certain cases produced by this cause, it would be an obvious error to assume that it is either the sole or principal cause of our perception of relief. If it were so, not only would persons deprived of the sight of one eye be incapable of perceiving relief, but even with two eyes we should be incapable of perceiving it in any objects except such as are placed at a distance so small as to have sensible binocular parallax.

423. **The eye supplies no direct perception of magnitude figure, or distance.**— It has been already explained that two similar objects, whose distances from the eye are to each other in the same proportion as their linear dimensions, will have the same apparent magnitude.

In like manner, if an object, such as, for example, a balloon, move from the eye in a direct line, we have no distinct consciousness of its motion, for the line of direction in which it is seen is still the same. It is true that we may infer its motion through the air by the increase or diminution of its apparent magnitude; for, if we have reason to know that its real magnitude remains unchanged, we ascribe almost intuitively the change of its apparent magnitude to the change of its distance; and we consequently *infer* that it is in motion either towards or from us, according as we perceive its apparent magnitude to be increased or diminished. This consciousness of the motion of a body in a direct line to or from the centre of the eye, is not a perception obtained directly from vision, but an inference deduced from certain phenomena. It may therefore be stated generally, that the eye affords no perception of direct distance, and consequently none of direct motion, the term direct being understood here to express a motion in a straight line to or from the optical centre of the eye.

424. **Manner of estimating the real distance.** — The distance of a visible object is often estimated by comparing it with the apparent magnitude and apparent distance of known objects which intervene between it and the eye.

Thus, the steeple of a church whose real height is unknown cannot by mere vision be estimated either as to distance or magnitude, since the apparent height would be the same, provided its magnitude were greater or less in proportion to its supposed distance. But, if between the steeple and the eye there intervene buildings, trees, or other objects, whose average magnitudes may be estimated, a proximate estimate of the magnitude and distance of the steeple may be obtained.

For example, if the height of the most distant building between the eye and the steeple be known, the distance of that building may be estimated by its apparent magnitude, and the distance of the steeple will be inferred to be greater than this.

425. **Appearance of the sun and moon when rising or setting.** — A remarkable illusion, depending on this principle, is deserving of mention here. When the disc of the sun or moon at rising or setting is near the horizon, it appears of enormous magnitude compared with its apparent size when high in the firmament. Now, if the visual angle which it subtends be actually measured in this case, it will be found to have the same magnitude. How then, it may be asked, does it happen that the apparent magnitudes of the sun at setting and at noon are by measure the same, when they are by estimation, and by the irresistible evidence of sense, so extremely different? This is explained, not by an error of the sense, for there is none, but by an erroneous application of those means of judging or estimating distance which in ordinary cases supply true and just conclusions.

When the disc of the sun is near the horizon, a number of intervening objects of known magnitude and known relative distances supply the means of spacing and measuring a part at least of the distance between the eye and the sun; but when the sun is in the meridian, no such objects intervene. The mind, therefore, assigns a greater magnitude to the distance, a part of which it has the means of measuring, than to the distance no part of which it can measure; and accordingly an impression is produced, that the sun at setting is at a much greater real distance than the sun in the meridian; and since its apparent magnitude in both cases is the same, its real magnitude must be just so much greater as its estimated distance is greater. The judgment, therefore, and not the eye, assigns this erroneous magnitude to the disc of the sun.

It is true that we are not conscious of this mental operation; but this unconsciousness is explained by the effect of habit, which causes innumerable other mental operations to pass unobserved.

426. **Method of estimating by sight the magnitude of distant objects.**—As the eye forms no immediate perception of distance, neither does it of magnitude; since, as has been already proved, objects of very different real magnitudes have the same apparent magnitude to the eye, of which a striking example is afforded in the case of the sun and moon. Nevertheless, although the eye supplies no immediate perception of the real magnitude of objects, habit and experience enable us to form estimates more or less exact of these magnitudes by the comparison of different effects produced by sight and touch.

Thus, for example, if two objects be seen at the same distance from the eye, the real magnitude of one of which is known, that of the other can be immediately inferred, since, in this case, the apparent magnitudes will be proportional to the real magnitudes. Thus, for example, if we see the figure of a man standing beside a tree, we form an estimate of the height of the tree, that of the man being known or assumed. Ascribing to the individual seen near the tree the average height of the human figure, and comparing the apparent height of the tree with his apparent height, we form an estimate of the height of the tree.

427. It is by this kind of inference that buildings constructed upon a scale greatly exceeding common dimensions are estimated, and rendered apparent in pictorial representations of them.

On entering, for example, the aisle of St. Peter's at Rome, or St. Paul's at London, we are not immediately conscious of the vastness of the scale of these structures; but if we happen to see at a distant part of the building a human figure, we immediately become conscious of the scale of the structure, for the known dimensions of this figure supply a modulus, which the mind instantly applies to measure the dimensions of the whole. For this reason artists, when they represent these structures, generally introduce human figures in or near them.

428. **Real magnitude may sometimes be inferred from apparent magnitude.** — It has been explained that the apparent magnitude of objects depends conjointly on their real magnitude and their distance. Although, therefore, the eye does not afford any direct perception either of real magnitude or distance, we are by habit enabled to infer one of these from the other.

Thus, if we happen to know the real magnitude of a visible object, we form an estimate of its distance from its apparent magnitude; and, on the other hand, if we happen to know or can ascertain the distance of an object, we immediately form some estimate of its real magnitude.

Thus, for example, the height of a human figure being known, if we observe its apparent visual magnitude to be extremely small, we know that it must be at a distance proportionally great. If we know that at 20 feet the figure of a man will have a certain apparent height, and find that his figure, seen at a certain distance, appears to have only one fifth of this height, we infer that his distance must be about 100 feet.

In like manner, the real magnitude may be inferred from the apparent magnitude, provided the distance be known or can be ascertained. Thus, for example, on entering Switzerland by its northern frontier, when we see in the distance, bounding the horizon, the range of the snowy Alps, the first impression is that of disappointment, their apparent scale being greatly less than we expected; but when we are informed that their distance is so great as sixty or eighty miles we immediately become conscious that, low as they seem to the eye, their real altitude must be enormous.

429. **Eye perceives only angular motion.**—When an object moves in any direction which is not in a straight line drawn to or from the centre of the eye, the direction in which it is seen continually changes, and the eye in this case supplies an immediate perception of its motion; but this perception can be easily shown to be one not corresponding to the actual motion of the object, but merely to the continual change of direction which this motion produces in the line drawn from the object to the eye.

Thus, for example, if the eye be at E (*fig.* 202.), any object which moves from A to B will cause the line of direction in which it is seen to revolve through the angle A E B, just as though the body which moves were to describe a circular arc, of which E is the centre and E A the radius. But if, instead of moving from A to B, the body were to move from A′ to B′, the impression which its motion would produce upon the sight would be exactly the same. It would still appear to be moving from the direction E A′ A to the direction E B B′.

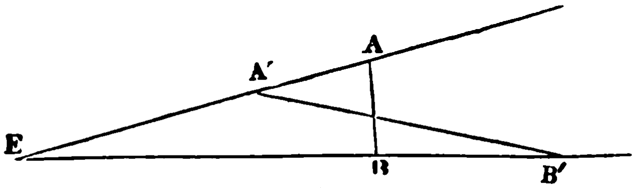

Fig. 202.

In fine, the eye, affording no perception of direct distance, supplies no evidence of the extent to which the body may change its distance during its motion, and the apparent motion will be the same as if the body in motion described a circle of which the eye is the centre.

Hence it is that the only motion of which the eye affords any immediate perception is angular motion; that is, a motion which is measured by the angle which a line describes, one extremity of which is at the centre of the eye, and the other at the moving object.

430. Though the real direction in which a distant object moves cannot be obtained by the direct perception of vision, some estimate of it may be formed by comparing the apparent angular motion of the object with its apparent magnitude.

Thus, for example, if we observe that the apparent magnitude of an object remains constantly the same while it has a certain apparent angular motion, we infer that its distance must necessarily remain the same, and consequently that it revolves in a circle, in the centre of which the observer is placed; or if we find that it has an angular motion, in virtue of which it changes its direction successively around us, so as to make a complete circuit of 360°, and that in making this circuit its apparent magnitude first diminishes to a certain limit, and then augments until it attains a certain major limit, from which it again diminishes, we infer that such a body revolves round us at a varying distance, its distance being greatest when the apparent magnitude is least, and least when its apparent magnitude is greatest. An exact observation of the variation of the apparent magnitude would in such a case supply a corresponding estimate of the variation of the real distance, and would thus form the means of ascertaining the path in which the body moves.

431. **Examples.** — Examples of this are presented in the cases of the sun and moon, whose apparent magnitudes are subject, during their revolution round the earth, to a slight variation, being a minimum at one point and a maximum at the extreme opposite point.

432. **How the apparent motion of an object is affected by the motion of the observer.** — As the eye perceives the motion of an object only by the change in the direction of the line joining the object with the eye, and as this change of direction may be produced as well by the motion of the observer as by that of the object, we find accordingly that apparent motions are produced sometimes in this manner. Thus, if a person be placed in the cabin of a boat which is moved upon a river or canal with a motion of which the observer is not conscious, the banks and all objects upon them appear to him to move in a contrary direction. In this case the line drawn from the object to the eye is not moved at the end connected with the object, as it would be if the object itself were in motion, but at the end connected with the eye. The change of its direction, however, is the same as if the end connected with the object had a motion in a contrary direction, the end connected with the eye being at rest; consequently, the apparent motion of the visible objects which are really at rest, is in a direction contrary to the real motion of the observer.

433. **Example of railway trains.** — In some cases the apparent motion of an object is produced by a combination of a real motion in the object and a real motion in the observer. Thus, if a person transported in a railway carriage meet a train coming in the opposite direction, both extremes of the line joining his eye with the train which passes him are in motion in contrary directions; that extremity which is at his eye is moved by the train which carries him, and the other extremity by the train which passes him. The change of direction of the line is accordingly produced by the sum of these motions; and as this change of direction is imputed by the sense to the train which passes, this train appears

to move with the sum of the velocities of the two trains. Thus, if one train be moved at twenty miles an hour, while the other is moved at twenty-five miles an hour, the apparent motion of the passing train, will be the same as would be the motion of a train moved at forty-five miles an hour, passing a train at rest.

434. **Compounded effects of the motion of the observer and of the object observed.**—If the line, joining a visible object with the eye, be moved at both its extremities in the same direction, which would be the case if the observer and the object were carried in parallel lines, then the change of direction which the line of motion would undergo, would arise from the difference of the velocities of the observer and of the object seen.

If the observer in this case moved slower than the object, the extremity of the line of motion connected with the object would be carried forward faster than the extremity connected with the observer, and the object would appear to move in the direction of the observer's motion, with a velocity equal to the difference; but if, on the contrary, the velocity of the observer were greater than that of the object, the extremity of the line connected with the observer would be carried forward faster than that connected with the object, and the change of direction would be the same as if the object were moved in a contrary direction with the difference of the velocities.

It is easy to perceive that a vast variety of complicated relations, which may exist between the directions and motions of the observer and of the object observed, will give rise to very complicated phenomena of apparent motion. Thus, relations may be imagined between the motion of the observer and that of the object perceived, by which, though both are in motion, the object will appear stationary; the motion of the one affecting the line of direction, in an equal and contrary manner to that with which it is affected by the other; and, in the same manner, either motion may prevail over the other more or less, so as to give the line of direction a motion in accordance with, or contrary to the real motion of the object.

435. **Examples of the planetary motions.**—All these complicated phenomena of vision, are presented in the problems which arise on the deduction of the real motion of the bodies, composing the solar system, from their apparent motions. The observer, placed in the middle of this system, is transported upon the earth, in virtue of its annual motion round the sun, with a prodigious velocity, the direction of his motion changing from day to day, according to the curvature of the orbit. The bodies which he observes are also affected with various motions, at various distances around the sun, the combination of which with the motion of the

earth gives rise to complicated phenomena, the analysis of which is made upon the principles here explained.

436. **Angular or visual distances.** — It is usual to express the relative position in which objects are seen, by the relative direction of lines drawn to them from the eye; and the angle contained by any two such lines, is called the angular or visual distance between the objects. Thus, the angular distance between the objects A and B (*fig.* 202.), is expressed by the magnitude of the angle A E B. If this angle be 30°, the objects are said to be 30° asunder. It is evident from this, that all objects which lie in the direction of the same lines, will be at the same angular distance asunder, however different their real distance from each other may be. Thus, the angular distance between A and B (*fig.* 202.), is the same as the angular distance between A′ and B′.

437. **Visual perception of form and bulk.** — Sight does not afford any immediate perception either of the volume or shape of an object. The information which we derive from it, of the bulk or figure of distant objects, is obtained by the comparison of different impressions made by the same object at different times and in different positions. A body of the spherical form seen at a distance appears to the eye as a flat circular disc, and would never be known to have any other form, unless the impression made upon the eye were combined with other impressions of sight or touch, or of both these senses, which supply the understanding with data, from which the real figure of the object can be inferred. The sun appears to the eye as a flat, circular disc; but, by comparing observations made upon it at different times, it is ascertained that it revolves round one of its diameters in a certain time, presenting itself under aspects infinitely varying to the observer; and this fact, combined with its invariable appearance as a circular disc, proves it to be a sphere; for no body except a sphere, viewed under every aspect, would appear circular.

Although we do not obtain directly from the sense of sight a perception of the shape of a body, we may obtain a perception of the shape of one of its sections. Thus, if a section of the body be made by a plane passing through it, at right angles to the line of vision, the sight supplies a distinct perception of the shape. If an egg, for example, were presented to the eye with its length in the direction of the line of vision, it would appear circular, because a section of it made by a plane at right angles to its length is a circle; but, if it were presented to the eye with its length at right angles to the line of vision, it would appear oval, that being the shape of a section made by a plane passing through its length.

If a body, therefore, presents itself successively to the eye in several different positions, we obtain a knowledge by the sense of sight of so many different sections of it, and the combination of these sections, in many cases, supply data, by which the exact figure of the body may be known.

438. **Visible area.** — As the term "apparent magnitude" is used to express the visual angle under which an object is seen, we shall adopt the term *visible area*, to express the apparent magnitude of the section, made by a plane at right angles to the line of vision; that is to say, to the line drawn from the eye to the centre of the object.

439. **How the shape is inferred from lights and shades.** — Besides receiving through the sight a perception of the figure of the section of the object which forms its visible area, we also obtain a perception of the lights and shades and the various tints of colour which mark and characterise such area. By comparing the perception derived from the sense of touch with those lights and shades, we are enabled by experience and long habit to judge of the figure of the object from these lights and shades and tints of colour. It is true that we are not conscious of this act of the understanding in inferring shape from colour, light, and shade; but the act is nevertheless performed by the mind. It is the character of all mental acts, that their frequent performance produces an unconsciousness of them; and hence it is that when we look at a cube or a sphere of a uniform colour, although the impression upon the sense of sight is that of a flat plane variously shaded, and having a certain outline, the mind instantly substitutes the thing signified for the sign, the cause for the effect; and the conclusion of the judgment, that the object before us is a sphere or a cube of uniform colour, and not, as it appears, a flat plane variously shaded, is so instantaneous, that the act of the mind passes unobserved.

The whole art of the painter consists in an intimate practical knowledge, of the relation between these two effects of perception of sight and touch. The more accurately he is able to delineate upon a flat surface, those varieties of light and shade which visible objects produce upon the sense, the more exact will be his delineation, and the greater the *vraisemblance* of his picture.

What is called relief in painting, is nothing more than the exact representation, on a flat surface, of the varieties of light and shade produced by a body of determinate figure upon the eye; and it is accordingly found, that the flat surface variously shaded, produced by the art of the painter, has upon the eye exactly the same effect as the object itself, which is in reality so different from the coloured canvas which represents it.

440. **Perception of colours.** — The immediate impressions received from the sense of sight are those of light and colour. The

impressions of distance, magnitude, form, and motion, are the mixed results of the sense of sight and the experience of touch. Even the power of distinguishing colours, is not obtained immediately by vision, without some cultivation of this sense. The unpractised eye of the new-born infant obtains only a general perception of light; and it is certain that the power of distinguishing colours, is only acquired after the organ has been more or less exercised, by the varied impressions produced by different lights upon it. It would not be easy to obtain a summary demonstration of this proposition, from the experience of infancy, but sufficient evidence to establish it is supplied by the cases, in which sight has been suddenly restored to adults blind from their birth. In these cases, the first impression produced by vision is that the objects seen are in immediate contact with the eye. It is not until the hand is stretched forth, so as to ascertain the absence of the objects seen from the space before the eye, that this optical illusion is dissipated.

The eye which has recently gained the power of vision, cannot at first distinguish one colour from another, and it is not until time has been given for experience, that either colour or outline is perceived.

441. **Certain defects in vision.** — Besides that imperfection incident to the organs of sight, arising from the excess or deficiency of their refractive powers, there is another class, which appears to depend upon the quality of the humours, through which the light proceeding from visible objects passes, before attaining the retina. It is evident that if these humours be not absolutely transparent and colourless, the image on the retina, though it may correspond in form and outline with the object, will not correspond in colour; for if the humours be not colourless, some constituents of the light proceeding from the object will be intercepted before reaching the retina, and the picture on the retina will accordingly be deprived of the colours thus intercepted. If, for example, the humours of the eye were so constituted as to intercept all the red and orange rays of white light, white paper, or any other white object, such as the sun, for example, would appear of a bluish-green colour; and if, on the other hand, the humours were so constituted as to intercept the blues and violets of white light, all white objects would appear to have a reddish hue. Such defects in the humours of the eye are fortunately rare, but not unprecedented.

Sir David Brewster, who has curiously examined and collected together cases of this kind, gives the following examples of these defects: —

A singular affection of the retina, in reference to colour, is shown in the inability of some eyes to distinguish certain colours of the spectrum. The persons who experience this defect have their eyes generally in a sound state, and are capable of performing all the most delicate functions of vision. Harris, a shoemaker at Allonby, was unable from his infancy to distinguish the cherries of a cherry-tree from its leaves, in so far as colour was concerned. Two of his brothers were equally defective in this respect, and always mistook orange for grass-green, and light green for yellow. Harris himself could only distinguish black and white. Mr. Scott, who describes his own case in the "Philosophical Transactions," mistook pink for a pale blue, and a full red for a full green.

All kinds of yellows and blues, except sky-blue, he could discern with great nicety. His father, his maternal uncle, one of his sisters, and her two sons, had all the same defect.

A tailor at Plymouth, whose case is described by Mr. Harvey, regarded the solar spectrum as consisting only of yellow and light blue; and he could distinguish with certainty only yellow, white, and green. He regarded indigo and Prussian blue as black.

Mr. R. Tucker described the colours of the spectrum as follows: —

Red mistaken for	brown.
Orange ,,	green.
Yellow sometimes	orange.
Green ,,	orange.
Blue ,,	pink.
Indigo ,,	purple.
Violet ,,	purple.

A gentleman in the prime of life, whose case I had occasion to examine, saw only two colours in the spectrum, viz., yellow and blue. When the middle of the red space was absorbed by a blue glass, he saw the black space with what he called the yellow on each side of it. This defect in the perception of colour was experienced by the late Mr. Dugald Stewart, who could not perceive any difference in the colour of the scarlet fruit of the Siberian crab, and that of its leaves. Dr. Dalton was unable to distinguish blue from pink by daylight; and in the solar spectrum the red was scarcely visible, the rest of it appearing to consist of two colours. Mr. Troughton had the same defect, and was capable of fully appreciating only blue and yellow colours; and when he named colours, the names of blue and yellow corresponded to the more and less refrangible rays; all those which belong to the former exciting the sensation of blueness, and those which belong to the latter the sensation of yellowness.

442. **Case of Dr. Dalton.**—In almost all these cases, the different prismatic colours had the power of exciting the sensation of light, and giving a distinct vision of objects, excepting in the case

of Dr. Dalton, who was said to be scarcely able to see the red extremity of the spectrum.

Dr. Dalton endeavoured to explain this peculiarity of vision, by supposing that in his own case the vitreous humour was blue, and therefore absorbed a great portion of the red and other least refrangible rays.

That this opinion was erroneous, however, was proved by the *post mortem* dissection of the eyes of that eminent person, by which it appeared that the vitreous humour was perfectly transparent and colourless.

Sir John Herschel attributes the defect of Dr. Dalton's vision, and other defects of the same class, to a morbid state of the sensorium, by which it is rendered incapable of appreciating exactly, those differences between rays, on which their colour depends.

443. **Memoir of Wortmann.**—M. Wortmann, of Geneva, has recently published an interesting memoir on this subject. The results of his elaborate researches are comprised in the following summary: —

I. Colour blindness has not been studied by the ancients.

II. It has been found only in individuals of the white race.

III. Some of the colour blind see only black and white, and some have the affection so slightly, as only to confound approximating shades of *blue* and *green* in candle light.

IV. There are more of the colour blind than is generally believed.

V. The female sex furnishes a small proportion.

VI. In some cases they may be known by external signs.

VII. There are as many of the colour blind with *blue* as with *black* eyes.

VIII. Colour blindness is not always hereditary.

IX. It does not always affect the males of the same family.

X. It does not always commence at birth.

XI. The colour blind do not judge as we do of complementary colours, or of the contrast of colours.

XII. Several of them are not sensible to the least refrangible rays.

XIII. They see the lines in the spectrum.

XIV. Colour blindness does not arise from any diseased conformation of the eye, or any colouration of the humours of the eye or of the retina.

XV. We may alter the state of colour blindness by very simple means.

XVI. Colour blindness has its origin in the sensorium.

CHAP. XIV.

OPTICAL INSTRUMENTS.

I. SPECTACLES.

444. SPECTACLES are the most universally useful gift, which optical science has conferred on mankind. More wonderful instruments abound. The miracles disclosed to human vision by the telescope and the microscope are known to all. To such marvels, spectacles lay no claim. But to compensate for this, their utility is ubiquitous. In the palace of the monarch and in the cottage of the peasant their beneficent influence is equally diffused. It is remarkable also, that, unlike most other productions of art and science, cost can add nothing to their perfection. Those of the millionaire may be mounted in gold, and those of the humble cottager in iron; but the optical medium, the glass lenses to which they owe their perfection, must be the same.

445. **Visual defects and their remedies.** — The defects of vision capable of being remedied by spectacles are those called weak sight and short sight. The causes which produce these, and the manner in which converging or convex glasses, and diverging or concave glasses, render such vision distinct, have been already explained (342. *et seq.*).

When persons are not very short-sighted, they generally read or work without spectacles, but require their aid when they walk abroad or move in society in large rooms, because the book or the objects of their work can, without inconvenience, be placed at the moderate distance from their eyes which is sufficient to throw the focus back upon the retina; but the more distant objects at which they look when walking abroad or in large rooms are beyond the proper limit of distance, and the focus, being before the retina, must be thrown back by concave spectacles.

Persons whose sight is not very weak, and who can see distinctly distant objects, fail to see nearer ones, but are enabled to see them by the interposition of more or less converging glasses. The nearer the object looked at is, the more convex ought the glasses to be, and hence it comes that very weak-sighted persons require to be provided with more than one pair of spectacles, those adapted to more distant objects being less convex, and those adapted to nearer objects more so.

446. **Form and mounting of spectacles.** — Spectacles consist of two glass lenses mounted in a frame so as to be conveniently

supported before the eyes, and to remedy the defects of vision of naturally imperfect eyes.

Whatever be the defects of sight which they may be used to remove, it is evident that the lenses ought to be so mounted that their axes shall be parallel, and that their centres shall coincide with the centres of the pupils, when the optical axes are directed perpendicularly to the general plane of the face, that is to say, when the eyes look straight forward.

These conditions, though important, are rarely attended to in the choice of spectacles. If spectacles be mounted in extremely light and flexible frames, the lenses almost invariably lose their parallelism, and their axes not only cease to be parallel, but are frequently in different planes. Spectacles ought therefore to be constructed with mounting sufficiently strong to prevent this derangement of the axes of the lenses, and in their original construction care should be taken that the axes of the lenses be truly parallel.

In the adaptation of spectacles, it is necessary that the distance between the centres of the lenses, should be precisely equal to the distance between the centres of the pupils. The clearest vision being obtained by looking through the centres of the lenses, the eyes have a constant tendency to look in that direction. Now if the distance between the centres of the lenses be greater than the distance between the centres of the pupils, the eyes having a tendency to look through these centres, their axes will cease to be parallel, and will diverge as in the case of an outsquint. On the other hand, if the distance between the centres of the lenses, be less than the distance between the centres of the pupils, there will, for a like reason, be a tendency to produce an insquint.

I have myself known persons of defective sight, who had never been able to suit themselves with spectacles, and concluded that they had some defect which spectacles could not remedy. Upon observing the form of their heads, I found, in each case, that the eyes were more distant asunder than eyes generally are, while the spectacles they used, being those made with the lenses at the usual distance, were never, and never could be, so placed as to be concentrical with the eyes, and hence arose the discomfort attending their use. In all such cases I removed the inconvenience, by measuring the distance between the centres of the eyes, and causing proper glasses to be mounted in frames, so that the distance between their centres should correspond with the distance between the centres of the eyes.

I would therefore advise every one who uses spectacles to cause the distance between the centres of their eyes to be exactly measured, and to select for their spectacles mountings corresponding with this distance.

447. **Periscopic spectacles.**—The most perfect vision with spectacles is produced, when the eye looks in the direction of the axis of the lenses, and more or less imperfection always attends oblique vision through them. Persons who use spectacles, therefore, generally turn the head, when those whose sight does not require such aid merely turn the eye.

To diminish this inconvenience, the late Dr. Wollaston suggested the use of menisci (*fig.* 85.), or concavo-convex lenses (*fig.* 88.), instead of double concave or double convex lenses with equal radii, which up to that time had been invariably used.

The effect of these, as compared with double convex and double concave glasses is, that objects seen obliquely through them are less distorted, and, consequently, that there is a greater freedom of vision by turning the eye without turning the head, from which property they were named *periscopic spectacles.*

448. **Eyes having different refracting power.**—In the selection and adaptation of spectacles; it is invariably assumed that the two eyes in the same individual, have exactly the same refracting power. That this is the case is evident from the fact, that the lenses provided in the same spectacles have always the same focal length.

Now although it is generally true, that the two eyes in the same individual have the same refractive power, it is not invariably so; and, if it be not, it is evident that lenses of equal focal length cannot be at once adapted to both eyes.

When the difference of the refractive power of the two eyes is not great (which is generally the case when a difference exists at all), this inequality is not perceived. By an instinctive act of the mind, of which we are unconscious, the perception obtained by the more perfect of the two eyes, in case of inequality, is that to which our attention is directed, the impression on the more defective eye not being perceived.

It might be expected, however, that the inequality would become apparent, by looking alternately at the same object with each of the eyes, closing the other; but it is so difficult to compare the powers of vision of the two eyes when they are not very unequal, by objects contemplated at different times, even though they should be exhibited in immediate succession, that this method fails.

Cases occur not only in which the comparative powers of vision of the two eyes differ, but in which the power of vision even of the same eye, is different when estimated in different directions.

I have known short-sighted persons who were more short-sighted for objects taken in a vertical than in a horizontal direction. Thus, with them, the height of an object would be more perceptible than its breadth, and, in general, vertical dimensions more clearly seen than horizontal. This difference arises from the refractive power of the eye taken in vertical planes, being different from the refractive power taken in horizontal planes; and the defect is accordingly removed by the use of lenses whose curvatures, measured in their vertical direction, is different from their curvature measured in their horizontal direction. The lenses, in fact, instead of having *spherical* surfaces, have *elliptical* surfaces, the eccentricities of which correspond with the variation of the refractive power of the eye.

449. **Spectacles for weak-sighted eyes.**—The convergent power of the lenses necessary for weak-sighted eyes, will necessarily be determined by the degree of the deficiency which exists in the refractive power of the eye. If the eyes be capable of affording distinct vision of objects so distant, that the rays proceeding from them may be regarded as parallel, they will be capable of refracting parallel rays to an exact focus on the retina; but if they are so feeble in their refractive power, as to be incapable of converging rays in the slightest degree divergent to a focus, they will be incapable of seeing distinctly any objects, whose distances from the eye are less than from two to three feet, because the rays composing the pencils from such objects have a divergence which, though slight, the eye is incapable of surmounting, and the pencils accordingly, after entering the eye, converge to a focus not on the retina, but behind it.

Hence we find that persons having feebly refracting eyes, are obliged to remove a printed or written page to a considerable distance from the eye, to be able to read it. The pencils are thus rendered parallel, and therefore such as the eye may bring to a focus on the retina; but this increase of distance from the eye, is attended with the consequence of rendering the light proceeding from the object more feeble, and often too feeble to produce distinct vision. Hence we find that when weak-sighted persons hold a book or newspaper, which they desire to read, at a considerable distance from the eye, they are obliged at the same time to place a candle or lamp near the page, to produce an illumination of the necessary intensity.

Since such eyes are, according to the supposition, adapted to the refraction of parallel rays, the lenses which they require must be such as to render the pencils, proceeding from the objects at which they look, parallel, and consequently they must be lenses whose focal length, is equal to the distance of the objects looked at.

Nothing, therefore, can be more simple than the rule to be followed by such persons in the selection of spectacles. They have only to use for their spectacles, lenses whose focal length is equal to the distance of the objects, which they desire to see distinctly; and if they have occasion to look at objects at different distances, as, for example, at ten and at twenty inches, they ought to be provided with different pairs of spectacles for the purpose, one having a focal length of ten inches, and the other a focal length of twenty inches. When they look at an object, at ten inches from the eye, with spectacles of ten inches focal length, the rays will enter the eye exactly as they would, if the object were at a distance of several feet from them; and those rays, being parallel, will be refracted to a focus on the retina.

It may be asked, in this case, how it happens that if it be necessary for such persons to use spectacles, having a focal length equal to the distance of the object at which they look, they can, nevertheless, see with the same spectacles distinctly objects at distances greater or less, within certain limits, than the focal distance of the spectacles? The answer is, that this arises from the power with which the eye is endued, to adapt itself, within certain limits, to vision at different distances, as has been already explained.

450. **How to determine the refracting power of weak-sighted eyes.**—If the weakness of the sight be such, that the eye is incapable of bringing even parallel rays to a focus on the retina, it will be necessary to use convergent lenses even for the most distant objects. The power of the lenses which are necessary to render the vision of distant objects clear in that case, will supply means of calculating the natural convergent power of the eye; for since the convergent power of the lens, together with the natural convergent power of the eye, bring parallel rays to a focus on the retina, the natural convergent power of the eye, will be equal to the difference between the convergent power of the lens, and the convergent power of an eye capable of bringing parallel rays to a focus on the retina.

To render this more clear, let f be the focal length of a lens, which is equivalent to the refracting power of an eye, which would bring parallel rays to a focus on the retina. Let f' be the focal length of the lens, which is sufficient to enable the defective eye to bring parallel rays to a focus on the retina; and let f'' be the focal length of a lens optically equivalent to the defective eye. We shall then have

$$\frac{1}{f'}+\frac{1}{f''}=\frac{1}{f};$$

consequently we shall have

$$\frac{1}{f''}=\frac{1}{f}-\frac{1}{f'}$$

From this condition the focal length of the eye can be found, since its reciprocal is equal to the difference between the reciprocals of the focal length of an eye adapted to parallel rays, and the focal length of the lens which produces clear vision in the defective eye.

In the same case, spectacles of different convergent power will be necessary when near objects are viewed; for in this case the pencils, having more divergence, will require a more convergent lens to aid the eye in bringing them to a focus on the retina. Such eyes, therefore, will require spectacles of different powers for distant and near objects; and if the power of the eye in adapting itself to different distances be not great, it may even be advisable to provide different spectacles for near objects, which differ in their distance, as already explained in the case of eyes adapted to the refraction of parallel rays.

451. **Spectacles for near-sighted eyes.** — To determine the focal length of the lens, which will enable near-sighted eyes to see distinctly distant objects, it is only necessary to ascertain the distance at which, without an effort, the same eyes can see objects distinctly. This distance determines the degree of divergence of the pencils, which the eyes bring to a focus on the retina. If diverging lenses, whose focal length is equal to this distance, be applied before the eyes, such lenses will give to parallel rays proceeding from distant objects, the same degree of divergence as pencils would naturally have, proceeding from objects whose distance is equal to their focal length; consequently, according to the supposition, the eye will bring such rays to a focus on the retina. The lenses, therefore, which fulfil this condition, will render the vision of distant objects with such eyes, as distinct as would be the vision of objects placed at a distance from the eyes, equal to the focal length of the lenses.

If the excess of the refractive power of short-sighted eyes be so great, and the power of adaptation to varying distances so small, that the same divergent lenses which render distant objects distinct, will not render objects which are near the eyes, but not near enough for distinct vision without spectacles, distinct, then lenses of less divergent power must be used to produce a distinct vision of such objects.

Thus, for example, suppose the case of eyes so near-sighted, as to see distinctly objects only when they are at five inches distance. To enable these eyes to see an object at ten inches distance distinctly, it will be necessary to use divergent lenses; but these lenses must have less diverging power than those which render the vision of distant objects distinct, because the same lenses which would give the necessary divergence to the parallel rays, which proceed from distant objects, would give too great a divergence to the pencils, which proceed from an object at ten inches distance.

II. Magnifying Glasses.

452. Magnifying glasses hold an intermediate place, between the spectacle glasses used by weak-sighted persons and the microscope, and when they possess considerable magnifying power, they are sometimes called simple microscopes ; but the term microscope is more generally applied to that class of optical instruments which consists of a combination of lenses, applicable to the examination of the most minute objects, with amplifying powers much more extensive.

Magnifiers are very variously mounted, according to the uses to which they are applied. The more simple forms, and those which have the least amplifying power, consist of a single converging lens, which may be either double convex, plano-convex, or meniscus.

These glasses are of very extensive use in the arts. In all cases in which the objects operated upon are minute, the interposition of a magnifier is found advantageous, and often indispensable; thus, they are invariably used in different mountings by watchmakers, jewellers, miniature-painters, engravers, and others.

We know no subject respecting which more inexact and erroneous notions prevail, than the amplification or magnifying effect produced by all optical combinations, from the simple convex lens to the most powerful microscope. The chief cause of all this confusion and obscurity, may be traced to a neglect of the proper distinction between visual and real magnitude. The eye, as has been already explained, takes no direct cognisance of real magnitude, which it can only estimate by inference and comparison with the impressions of the sense of touch ; these inferences and comparisons being often attended with complicated calculations and reasoning.

453. **Standard of magnifying power.**—The magnitudes of objects, as they appear with magnifying glasses, are all visual, and not real. When an object, seen by the interposition of such an instrument, is said to be magnified so many times, it is therefore meant that it is so many times greater than it would be, if it were seen with the naked eye; but since it has been shown that the visual magnitude of the same object seen with the naked eye varies, being greater as its distance from the eye is less, it follows that the visual magnitude seen with the naked eye is an indefinite and variable standard, and in order that the visual magnitude of an object taken as the standard of magnifying power

should be definite, it is necessary that the distance at which the object is supposed to be viewed by the naked eye should be stated. When, however, a person without any previous scientific instruction views an object with a magnifier, he becomes instantly conscious of its amplification; that is, that it appears larger than it would appear if he had viewed it without the interposition of the magnifier. The question is, then, at what distance from his eye such a person would suppose the object to be, if looked at without the magnifier; and the reply which has been generally given to this question is, that he would suppose it to be viewed at that distance at which he would see it most distinctly.

This being admitted, microscopists have generally agreed that the visual magnitude viewed with the naked eye, which should be taken as the standard of comparison in expressing the effect of magnifiers, is that which the object would have, when viewed at the distance at which objects are most distinctly seen.

454. **Distance of most distinct vision.** — But here another difficulty arises. In the first place, the distance at which one individual can see an object most distinctly, is not the same as that at which another will see it most distinctly; thus, while a far-sighted person will see most distinctly at the distance of 15 or 16 inches, and cannot see at all at the distance of 5 or 6 inches, a near-sighted person will see most distinctly at the latter distance, and only confusedly and indistinctly at the former. But even the same individual will see the same object most distinctly at one distance, when it is strongly illuminated, and at a much less distance, when it is feebly illuminated.

The distance of most distinct vision is therefore a variable and uncertain standard of comparison.

But there is one thing which is perfectly definite and certain. The visual magnitude of an object, at a given distance, is always the same, and quite independent of the powers and qualities of the eye which views it; it may, or may not, be distinctly seen, or seen at all; but if seen, it can have but one visual magnitude. Thus an object, such as a coin, placed with its surface at right angles to the line of sight, at a distance from the eye equal to 10 times its own diameter, will have a visual diameter of 5¾°, and neither more or less, no matter by what eye it is viewed. Seeing, then, that the distance of most distinct vision varies with different observers, and even with the same observer under different circumstances, and cannot therefore be taken as a standard of reference for visual magnitude, it has been generally agreed that magnifying powers shall be arithmetically expressed, by reference to visual magnitudes seen at 10 inches distance. Thus, if we say that such or such a magnifier magnifies an object three or four times, it is meant that it exhibits that object with a visual magnitude, three or four times as great as that which the same object would have, if viewed with the naked eye at 10 inches distance.

This distance of 10 inches has not been selected arbitrarily. It is considered to be about the distance at which average eyes would see an object most distinctly.* It has the further convenience of lending itself with facility to calculation, by reason of its decimal form. In other countries, the same distance, very nearly, has been adopted as a standard. Thus, French microscopists take 25 centimetres, which is a very small fraction less than 10 inches, as their standard.

455. **Magnifying power of a convex lens.** — This conventional standard being accepted, let us see in what manner an object is made to appear magnified, by the interposition of a single convex lens.

Let E E, *fig.* 203., represent a section of the eye, and *o o′* a small object placed at a much less distance from the eye than is compatible with distinct vision. According to what has been explained, it will appear that the cause of indistinct vision is, in this case, that the image of *o o′*, produced by the humours of the eye, is formed not as it ought to be on the retina at I I′, but behind it at *i i′*. According to what has been explained of optical images, the interposition of a lens, L L, of suitable convexity, will bring forward the image from *i i′* to I I′, and will therefore render the perception of the object distinct.

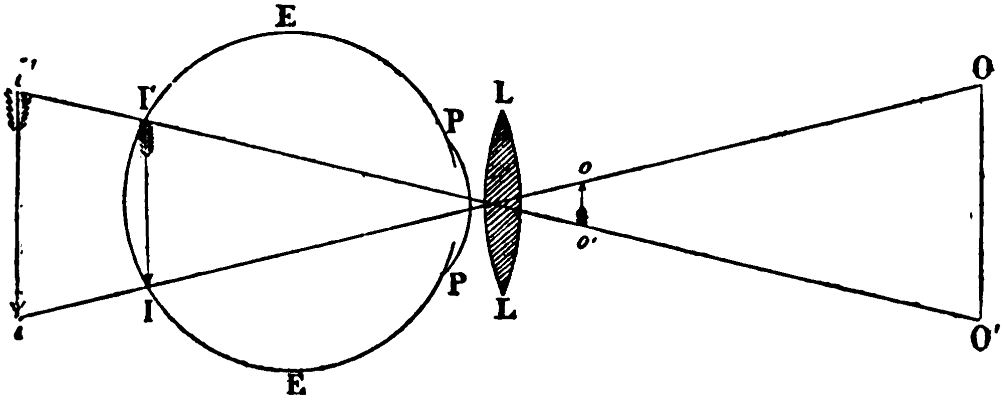

Fig. 203.

Now, it is most important to observe in this case, that the visual magnitude of the object, measured by the angle formed by the lines *o* I and *o′* I′, will be exactly the same as it would be if the eye could have seen the object *o o′* without the interposition of the lens: from which it appears that the lens does not, as is commonly supposed, *directly* augment the visual magnitude of the object, but only enables the eye to see the object with distinctness, at a less distance than it could so see it without the interposition of the lens. We say *directly*, because, although the lens does not augment the visual angle of the object, in the position in which it is actually viewed, yet, by enabling the eye to see it distinctly at a diminished distance, the visual angle of distinct vision, and therefore the apparent magnitude of the object, is increased in exactly the same proportion as the distance at which it is viewed is diminished.

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* Sir David Brewster takes five inches as the distance of distinct vision, and, consequently, his magnifying powers will in all cases be only half those calculated upon the above data.

To understand the magnifying effect of the lens, we must consider that the observer, seeing the object *o o′* with perfect distinctness, obtains exactly the same visual perception of it, as if the object, having the same visual magnitude, were placed at that distance from the eye at which his vision would be most distinct. Let the lines passing through the extremities of the object, therefore, be prolonged to this distance of most distinct vision, and let an object, o o′, be supposed to be placed there, similar in all respects to the object *o o′*, and having the same visual magnitude. It will be evident, from what has been stated, that *o o′*, as seen with the lens, will have precisely the same appearance as the object o o′ would have if seen with the naked eye. The observer, therefore, considers, and rightly considers, that the magnifying power of the lens is expressed by the number of times that o o′ is greater than *o o′*; or, what is the same, by the number of times that the distance of o o′ from the lens, that is, the distance of most distinct vision, is greater than the distance of the object from the lens.

It follows, therefore, generally, that the magnifying power of the lens will be found by dividing the distance of most distinct vision by the distance of the object from the lens.

Adopting this method of estimating the magnifying power, it would follow that the same lens would have different magnifying powers for different eyes, inasmuch as the distance of most distinct vision for short sight is less than that for average sight, and less for average sight than for far sight.

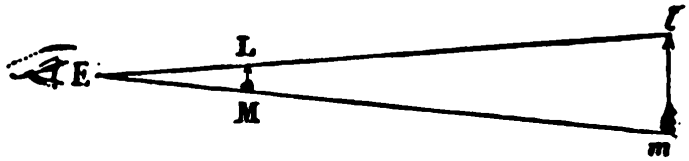

Fig. 204.

To make this more clear, let E, *fig.* 204., represent an average sighted eye; E′, *fig.* 205., a short-sighted eye, and E″, *fig.* 206., a far-sighted eye. Let the same small object, L M, be placed at the same distance from each of them, and let the distance of most distinct vision for the first be E *l*, for the second, E′ *l*′, and for the third E″ *l*″. If, by the interposition of a lens, the object L M be rendered distinctly visible to each of these three eyes, it will appear at *l m* to E, at *l*′ *m*′ to E′, and at *l*″ *m*″ to E″; its apparent magnitude, therefore, to the three eyes, will be in the exact ratio of their respective distances of most distinct vision, and consequently the magnifying power to E′ will be less, and to E″ greater than to E.

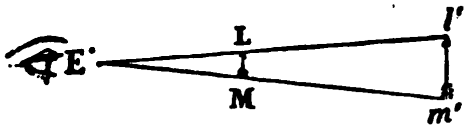

Fig. 205.

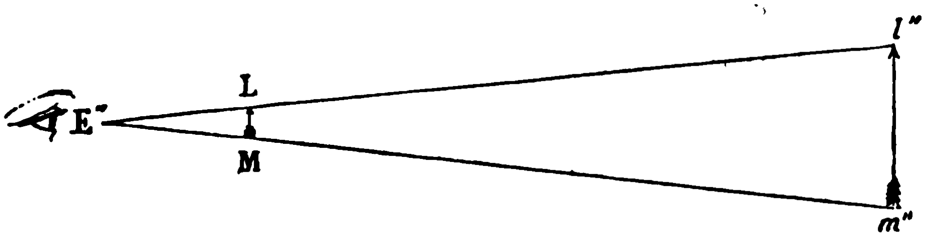

Fig. 206.

It must, however, be observed, that in this, which is the commonly received explanation, a circumstance of some importance is omitted, which will modify the conclusion deduced from it. To produce distinct vision with a given lens, L L, the distance of the object from the lens will not be the same for different eyes; for short sight the object must be nearer, and for long sight more distant than for average sight.

Now, if this variation of the distance from the lens, or of the focus, as it is called, for different eyes, vary in the same proportion as the distance of the most distinct vision (and it certainly does not differ much from that proportion), it will follow, contrary to the received doctrine, that the magnifying power of the same lens will be the same for all eyes, whether they have average sight, long sight, or short sight.

456. **Superficial and cubical magnifying power.**—It is contended by some, that the magnifying power is more properly and adequately expressed, by referring it to the superficial than to the linear dimensions of the objects.

To illustrate this, let us suppose the object magnified to be a square such as *a*, *fig.* 207. Now, if its linear dimensions, that is, its sides, be magnified ten times, the square will be increased to the size represented at A, *fig.* 208.; its height and breadth being each

Fig. 207.

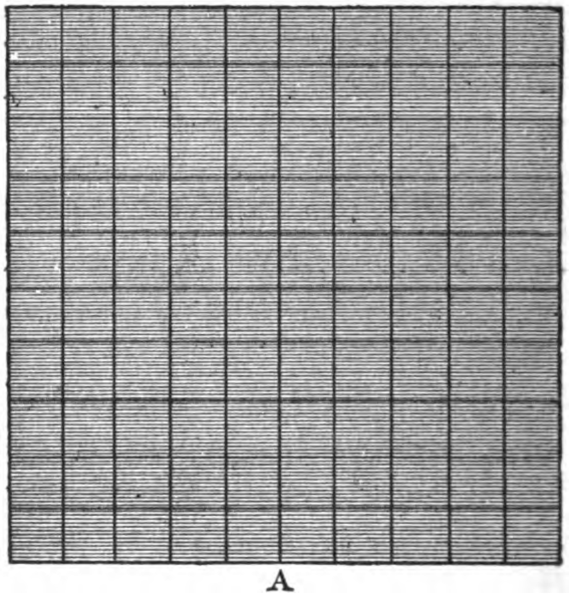

Fig. 208.

increased ten times, and its superficial magnitude being consequently increased 100 times, as is apparent by the diagram.

It is contended, and not without some reason, that when an object, such as *a*, receives the increase of apparent size represented at A, it is much more properly said to be magnified 100 than ten times.

Nevertheless, it is not by the increase of superficial, but of linear dimensions, that magnifying powers are usually expressed. No obscurity or confusion can arise from this, so long as it is well understood that the increase of linear, and not that of superficial dimension, is intended. Those who desire to ascertain the superficial amplification, need only take the square of the linear; thus, if the linear be 3, 4, or 5, the superficial will be 9, 16, or 25, and so on.

It might even be maintained that when an object having length, width, and thickness, a small cube or prism of a crystal, for example, is magnified, the amplification being produced equally on all the three dimensions, ought to be expressed by the cube of the linear increase; thus, if the object, being a cube, be magnified ten times in its linear dimensions, it will acquire ten times greater length, ten times greater breadth, and ten times greater height, and will consequently appear as a cube of 1000 times greater volume.

In this case, however, as in that of the superficial increase, the calculation is easily made by those who desire it, when the linear increase is known.

In all cases in which magnifying lenses are used, except where the lens is large, and the magnifying power low, the eye of the observer should be placed as close as possible to the lens, the pupil being as nearly as possible concentric with the lens; for since the pencils of rays, which proceed from the extreme points of the object, intersect at an angle equal to that formed by lines drawn from the extremities of the object to the centre of the lens, they will diverge after passing through the lens, at the same angle; and the farther the eye is removed from the lens, the more rays it will lose, and beyond a certain limit of distance, a part only of the object will be visible.

457. **Power depends on focal length.** — Since eyes of average sight are adapted to the reception of parallel rays, an object seen through a lens by them will be distinctly visible, only on the condition that its distance from the lens shall be equal to the focal length; for, in that case, the rays which diverge from each point of the object will emerge from the lens parallel, and therefore suitable to the power of the eye.

It is for this reason that the magnifying powers of lenses are estimated, by comparing their focal lengths with the distance of distinct vision. For since the focal length is always the distance of the object from the lens for average eyes, the distance of distinct vision, divided by it, will, according to what has been explained, be the magnifying power of the lens for such eyes.

The focal length of a lens will be less, in proportion as its refracting power upon the light transmitted through it is greater; but the refracting power of the lens depends partly on its convexity, and partly on its material.

With the same material the refracting power will be greater and the focal length less, as the convexity is increased; and, on the other hand, with a given convexity, the refracting power will be greater, and the focal length less, as the refracting power of the material of which the lens is made is greater. Thus, for example, if two lenses be composed of the same sort of glass, that which has the greater convexity will have the less focal length; and if, on the other hand, two lenses, one composed of glass and the other of diamond, have equal convexities, the latter will have a less focal length than the former; because diamond has a greater refracting power than glass.

458. **Lenses of different material.** — It will be evident, from what has been explained, that if two lenses be formed of materials having different refracting powers, such for example as glass and

diamond, so as to have equal focal length, that which has greater refracting power will have the less convexity.

If two lenses therefore be formed, having the same magnifying power, one of glass and the other of diamond, the latter will have less convexity than the former.

From what has been explained on the subject of spherical aberration, it will be understood that the more convex a lens is, the less its diameter must be; for if its diameter exceeds a certain limit relatively to its convexity, the spherical aberration will become so great, as to render all vision with it confused and indistinct. This is the reason why all lenses, of high magnifying power and short focal length, are necessarily small.

But since the spherical aberration depends on, and increases with the convexity of the lens, other things being the same, it follows that if two lenses, composed of different materials, have equal focal lengths, that which has the less convexity will also have less spherical aberration.

459. **Diamond lenses.**—Now since, according to what has been explained above, a diamond lens has less convexity than a glass lens of the same focal length, it will, if it have the same diameter, have less spherical aberration, or, what is the same, it will admit of being formed with a greater diameter, subject to the same aberration.

In lenses of high magnifying powers, and which are consequently of small dimensions, any increase of the diameter which can be made, without being accompanied with an injurious increase of aberration, is attended with the advantage of transmitting more light from each point of the object to the eye, and therefore of rendering the object more distinctly visible. It was on this account that, when single lenses of high magnifying power were thought desirable, great efforts were made to form them of diamond, and other transparent gems having a refracting power greater than that of glass.

Sir David Brewster, who first suggested the advantage of this, succeeded in getting lenses of great magnifying power, made of ruby and garnet; he considered those made from the latter stone to surpass every other solid lens: the focal length of some of those made for him was less than the 1-30th of an inch, the magnifying power being more than 300.

All these and similar efforts made by Messrs. Pritchard and Varley, aided by the genius and science of the late Dr. Goring, have, however, happily for the progress of science, been subsequently rendered unnecessary, by the invention of methods of producing good achromatic object glasses of high power for compound microscopes, so that the range of usefulness of simple microscopes, or magnifying glasses, is now limited to uses and researches in which comparatively low magnifying powers are sufficient.

The most feeble class of magnifying glasses are those occasionally used for reading small type, by persons of very weak sight; they consist of double convex lenses of five or six inches focal length, and having, consequently, a magnifying power no greater than two; they are usually mounted in tortoise-shell or horn, with convenient handles.

460. **Magnifiers for artists.**—Magnifiers of somewhat shorter focal length and less diameter, similarly mounted, are used by miniature-painters and engravers.

Lenses having a focal length of about one inch, set in a horn cell, enlarged at one end like the wide end of a trumpet, the magnitude being made to correspond with the socket of the eye, as represented in *fig.* 209., are used by watchmakers. The wide end, being inserted under the eyebrow, is held in its position by the contraction of the muscles surrounding the eyeball, and the minute work to be examined, is held within an inch of the lens set in the smaller end of the horn case; if the focal length be an inch, the magnifying power of such a glass, for average eyes, will be ten.

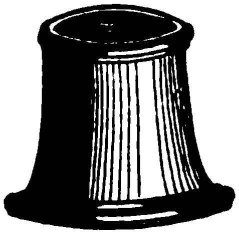
Fig. 209.

Glasses somewhat similarly mounted are used by jewellers, gem-sculptors, and other artists.

To relieve the artist from the fatigue of holding the magnifier in the eye-socket or in the hand, a stand with a movable socket is sometimes resorted to, such as that represented in *fig.* 210. A horizontal arm slides upon a vertical rod, upon which it can be fixed at any desired height by a tightening screw. This arm consists of two joints, connected together by a ball and socket, by which they can be placed at any desired inclination; at the extremity of the lower

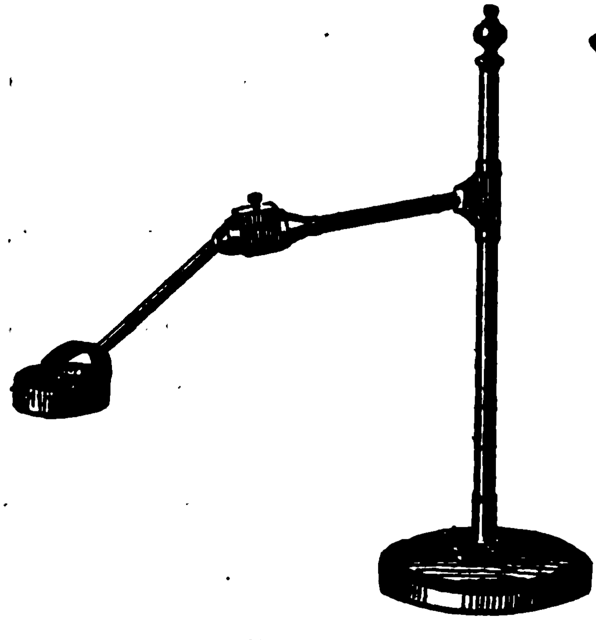
Fig. 210.

arm a fork supports a ring-shaped socket, made to receive the magnifier.

461. **Pocket magnifiers.**—Very convenient pocket magnifiers are mounted in tortoise-shell or horn cases, in the form shown in *fig*. 211. Lenses of different powers are provided, which may be used separately or together. When they are used together, however, the interposition of a diaphragm is necessary to diminish the effects of spherical aberration by cutting off the lateral rays.

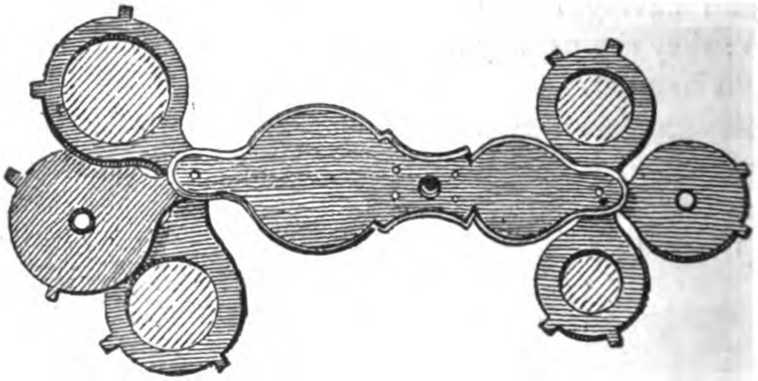

Fig. 211.

Lenses thus mounted are well fitted for medical use, and for certain researches in natural history.

III. The Simple Microscope.

462. **Microscopes, simple and compound.** — When still higher magnifying powers are required, the instrument takes the name of a *microscope*

Microscopes are of two kinds, *simple* and *compound*.

In the simple microscope, the object under examination is viewed directly, either by a simple or compound converging lens.

In the compound microscope, an optical image of the object, produced upon an enlarged scale, is thus viewed.

The use of single lenses, as simple microscopes, is rendered difficult by the prevalence of aberration, which necessarily attends great converging power.

463. **Coddington lens.** — One of the most convenient forms of simple microscope, consisting of a single lens is that which has received the name of the Coddington lens, from its supposed invention by the eminent mathematician of that name. The lens,

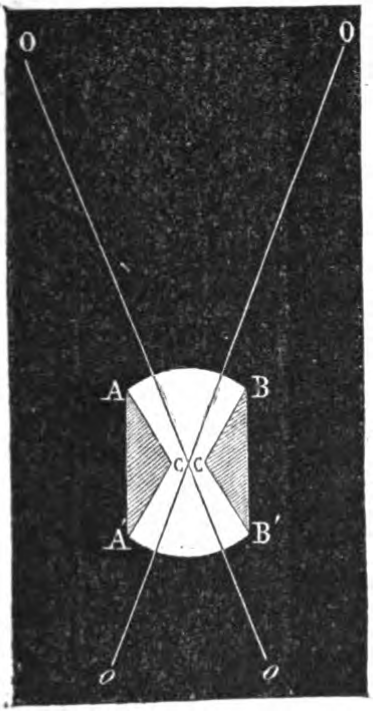

Fig. 212.

however, appears to have been one, of the numerous contributions of Sir David Brewster to optical science.* To form this lens let a solid ball or sphere of glass, about ½ inch in diameter, be cut round its equator, so as to form round it an angular groove, leaving two spherical surfaces on opposite sides uncut. The angular groove is then filled up with opaque matter, the circular edge of the groove serving as a diaphragm between the two spherical surfaces. A section of such a lens is shown in *fig.* 212., where A B and A′ B′ are the two spherical surfaces left uncut, and A C A′ and B C B′ the section of the angular groove filled with opaque matter. The course of the rays passing through it from any point such as o, is shown by the lines o *o*, and it will be evident from the mere inspection of the figure, that the effect of the lens upon the rays will be precisely the same, wherever the point o may be placed; this lens therefore, gives a large field equally well defined in all directions, and since it is no matter in what position it is held, it is very convenient as a hand and pocket glass; it is usually mounted in a small case, such as is shown in *fig.* 213., which can be carried in the waistcoat pocket.

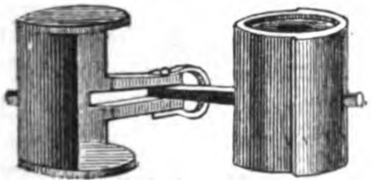

Fig. 213.

464. **Doublets and triplets.** — Magnifying glasses of low powers, such, for example, as those which range from 5 to 40, may be constructed with much advantage in one or the other of the above forms. When, however, higher powers are necessary, the use of such lenses, with very short focal length, is attended with much practical inconvenience, which has been removed by the use of magnifiers, consisting of two or more lenses combined. The combinations of this kind which are found most efficient, consist of two or three plano-convex lenses, with their convex sides towards the eye; these are called *doublets* and *triplets*.

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* See Brewster's "Optics," p. 470.

Let E E and D D, *fig.* 214., represent the two lenses of a doublet, and let *o o* be a small object placed before D D, at a distance from it less than its focal length. According to what has been explained, D D will produce an imaginary image of *o o* at *i i*, more distant from D D than *o o*, so that an eye placed behind D D would receive the rays from *o o*, as if they had diverged from the corresponding points of *i i*.

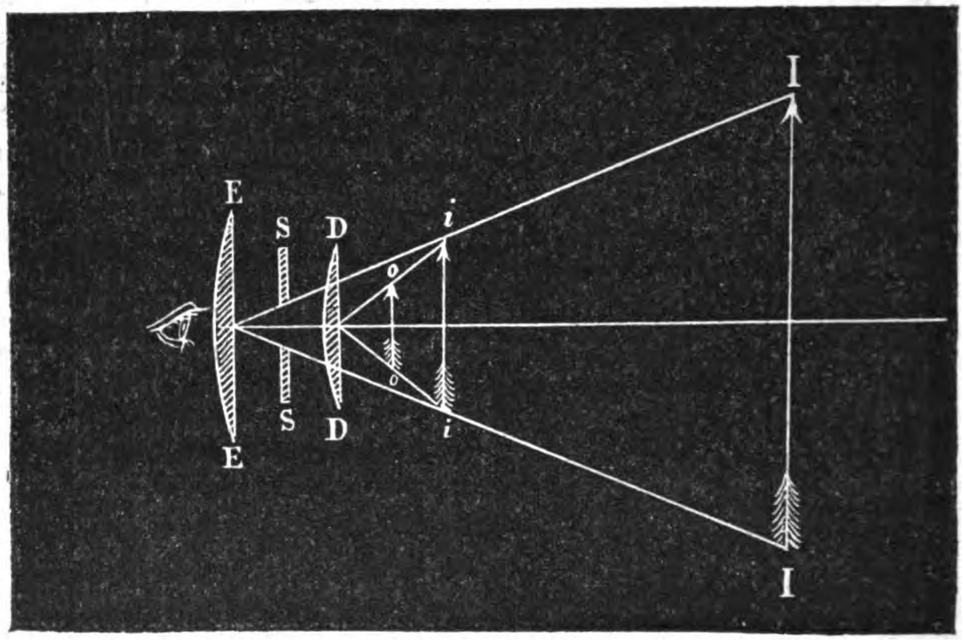


Fig. 214.

But instead of being received by an eye placed behind D D, these rays are received by the other lens E E ; the image *i i* therefore plays the part of an object before the lens E E, and being at a distance from E E less than the focal length of the latter, an imaginary image of *i i* will be produced at I I ; the rays, after passing through E E, entering the eye as if they had come from the corresponding points of I I.

To cut off all scattered rays not necessary for the formation of the image, a stop or diaphragm, S S, consisting of a circular disc of metal, with a hole in its centre, is interposed between the two lenses.

Such a combination, when high powers are necessary, has several advantages over an equivalent single lens. In the first place, the effect of spherical aberration is much less ; and secondly, the object can be placed at a much greater distance from the anterior lens D D, and can consequently be more conveniently manipulated, if it be desired to dissect it, or to submit it to any other process ; it can also be illuminated by a light thrown upon that side of it which is presented to the glass. This could not be done if it were nearly in contact with the glass, which must necessarily be the case by reason of its very short focal length, if a single lens were used.

465. **Wollaston's doublets.**—It was recommended by Dr. Wollaston, the inventor of these doublets, to give the two lenses composing them unequal focal lengths, that of E E being three times that of D D.

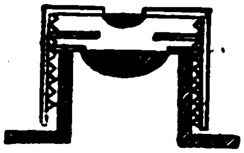


Fig. 215.

The lenses are usually set in two thimbles, one of which screws into the other, as shown in *fig.* 215., so that they can be adjusted as to their mutual distance, so as to produce the best effect. When still higher powers are sought, the lens D D is replaced by two plano-convex lenses, in contact, which taken together play the part of the single lens D D in the doublet; this combination is called the triplet.

When a very low magnifying power is required, the lenses E E and D D may be separated, by unscrewing.

466. **Mounting doublets.** — The lenses, whether of a doublet or a triplet, being thus properly mounted, expedients must be adopted to enable the observer to apply them conveniently to the object under examination. The most simple method of effecting this would be to hold the lens to the eye with one hand, and to present the object before it at the proper distance with the other. But even in this case it would be necessary that the lens should be attached to a convenient handle, and unless the magnifying power were very low, the steadiness necessary to retain the object in the focus could not be imparted to it, and while the observation would be unsatisfactory, the fatigue of the observer would be considerable. When high powers are used, every motion of the object is as much magnified as the object itself,

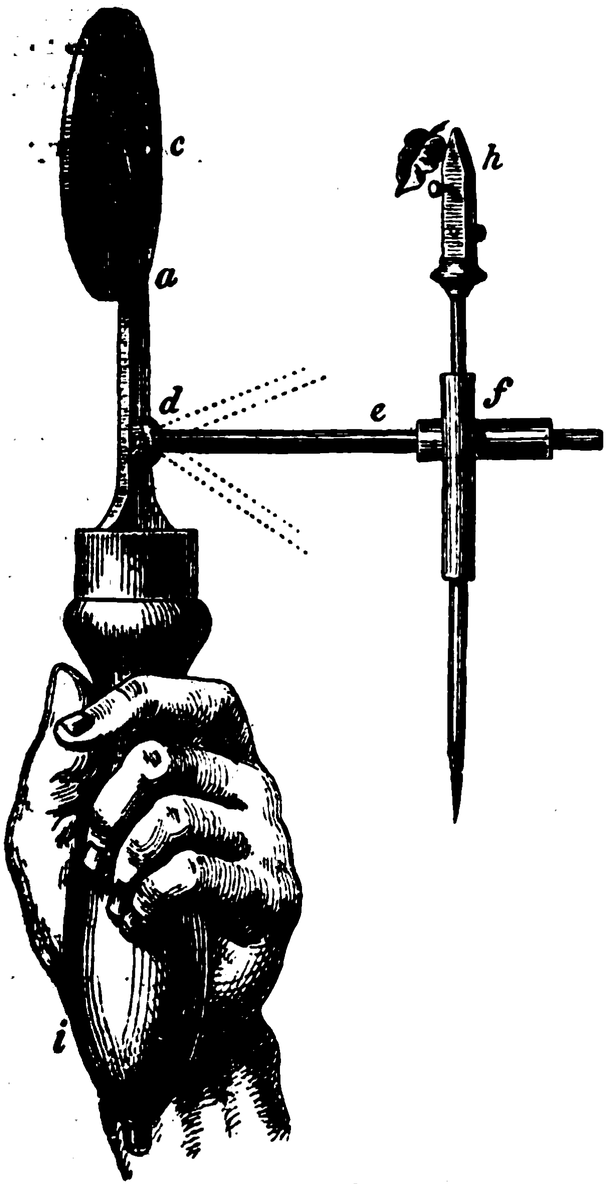


Fig. 216.

and consequently in such cases the most extreme steadiness is indispensable.

Whatever be the form of the mounting, therefore, it is necessary that the object should be supported by some piece attached to that by which the doublet itself is supported, so that it may be steadily held in the axis of the lenses, and that its distance from them may be varied at pleasure, by some smooth and easy motion, by which the observer can bring the object to the proper focus.

One of the most convenient forms of mounting, for a common hand microscope is shown in *fig.* 216.

The doublet is inserted in a socket *c* made to fit it; the screen *b* protects the eye from the light by which the object is illuminated; an arm *e* is jointed at *d*, so that it can be turned flat against *a*, when the instrument is not in use, and can be inclined to *a*, at any desired angle. This arm being round, a sliding tube *f* is placed upon it, fixed to another tube at right angles to it, in which a vertical rod slides, to the upper end of which is attached a forceps *h* or any other convenient support of the object under examination.

Several doublets or triplets of various powers may be provided, any of which may be inserted at pleasure in the socket *c*.

When still greater steadiness is required, and greater bulk and higher price do not form an objection, the arm and socket bearing the doublet are fixed upon a vertical pillar, screwed to a table with proper accessories for adjusting the focus and illuminating the object.

467. **Chevalier's mounting.** — The arrangement adopted in the simple microscopes of Charles Chevalier, shown in *fig.* 217., may be taken as a general example of this class of mounting.

The case in which the instrument is packed serves for its support when in use. A square brass pillar T T, screwed into the top of this case X, has a square groove cut along one of its sides, in which the square rod G is moved upwards and downwards by a rack and pinion R; at the top of this rod, a horizontal arm *a* is attached, at the end of which a socket is provided to receive the doublets; several of which having different powers are supplied with the instrument.

The object under observation is supported on the stage P, firmly attached to the upper end of the square pillar T T; in this stage is a central hole, through which light is projected on its lower surface when the object is transparent, and the quantity of this light is modified by means of an opaque disc D, pierced with holes of different magnitudes.

By turning this disc on its centre, any one of these holes may be brought under the object; when the object is not transparent, the opening in the stage is stopped, and it is viewed by light thrown upon its upper surface.

A square box B, sliding upon the pillar T T, with sufficient friction to maintain it at any height at which it is placed, carries a reflector M, by which light is projected upwards to the opening of the stage P, this light being more or less limited in quantity by the orifice of the diaphragm D, which is presented in its path.

In this instrument the object is brought into focus, by moving the arm which carries the doublet up and down, by means of the rack and pinion R,

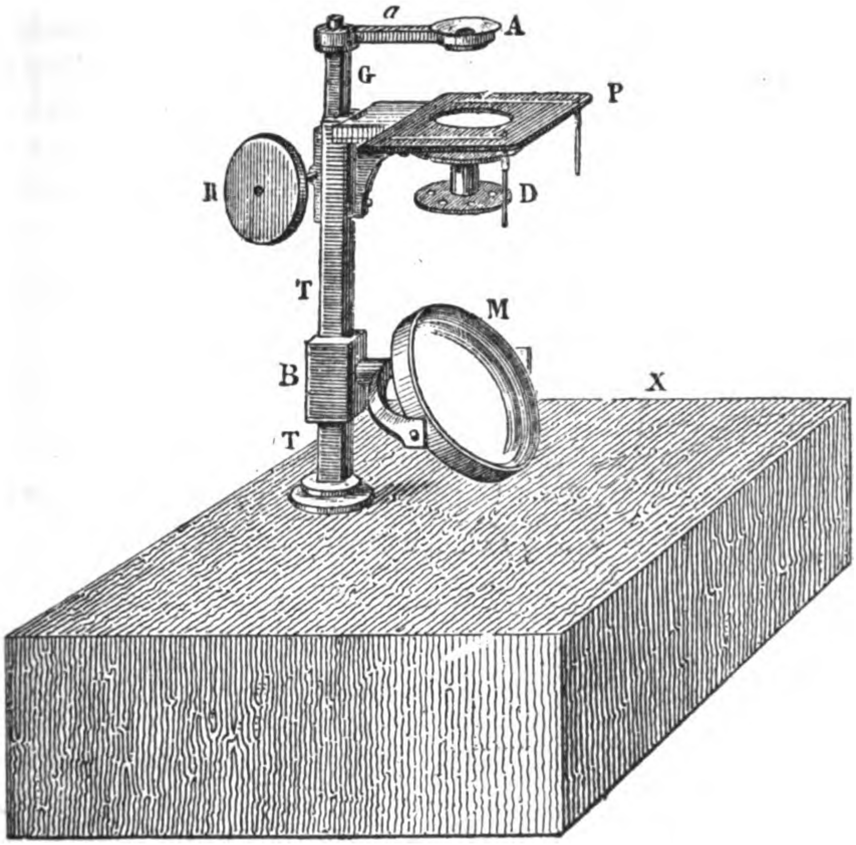


Fig. 217.

the stage, supporting the object, being fixed. The same effect might be, and is in some microscopes, produced by moving the stage, supporting the object, to and from the lens: but when the instrument is applied to dissection, it is necessary to keep the subject dissected immovable, and, therefore, not only to maintain the stage stationary, but to render it so solid and stable that it will bear the pressure of both the hands of the operator while he manipulates the dissecting instruments. On this account the stage is often made larger than is represented in the figure, and supported by a separate pillar.

The arm *a* carrying the doublet is also sometimes fixed in a square socket on the top of the rod G, so that it can be moved to and fro in the socket, while the socket itself can be turned upon the rod G; by this combination of motions, the observer can with great convenience move the lens over every part of the object under examination.

Simple magnifiers, with provisions similar to these, are made by the principal opticians, Messrs. Ross, Leland, and Powell, Smith and Beck, Pritchard, Varley, and others.

When the object has not sufficient transparency to be seen by light transmitted through it from below, it may be illuminated by a light thrown upon it from above by a lamp or candle, and condensed, if necessary, to obtain greater intensity, by means of a concave reflector or convex lens.

IV. The Compound Microscope.

468. This instrument, in its most simple form, consists of a magnifying lens or combination of lenses, by means of which an enlarged optical image of a minute object is produced, and another magnifying lens, or combination of lenses, by which such image is viewed, as an object would be by a simple microscope.

The former is called the *object glass*, or *objective*, since it is always directed immediately to the object, which is placed very near to it; and the latter the *eye glass*, or *eye piece*, inasmuch as the eye of the observer is applied to it, to view the magnified image of the object.

469. **Refracting microscope.**— Such a combination will be more clearly understood by reference to *fig.* 218., where *o* is the object, L L the object glass, and E E the eye glass.

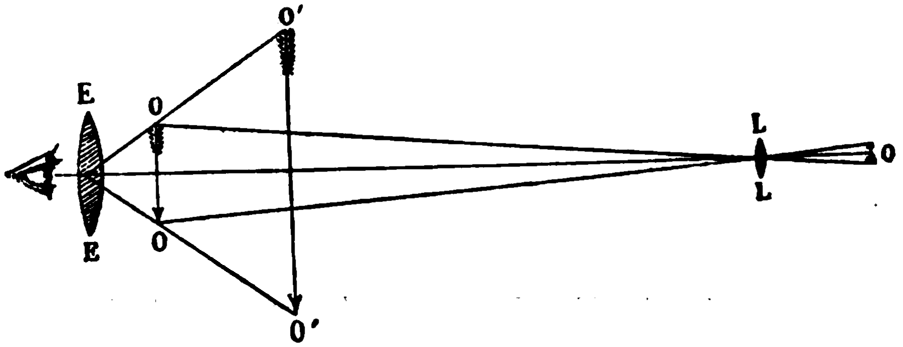


Fig. 218.

The object glass, L L, is a lens of very short focal length, and the object *o* is placed in its axis, a very little beyond its focus. According to what has been explained (158. *et seq.*), an image O O, of *o*, will be produced at a distance from the object glass L L, much greater than the distance of *o* from it: this image will be inverted, with relation to the object; its linear magnitude will be greater than that of the object, in the proportion of O L to *o* L; and consequently its superficial magnitude will be greater, in the proportion of the squares of these lines.

The image O O, thus formed, may be considered as an object viewed through the magnifying glass E E, and all that has been explained, relating to the effect of such a lens, will be applicable in this case. The observer will adjust the eye glass E E, at such a distance from O O as will enable him to see the image most distinctly, and the impression produced will be, that the image he looks at, is at that distance from his eye at which he would see such an object most distinctly, without the interposition of any magnifying lens; let this distance be that of a similar image o′ o′, and the impression will be that the object he beholds has the magnitude o′ o′.

The distance of most distinct vision with the naked eye, and the distance from the image at which the eye glass must be placed to produce distinct vision, both vary for different eyes, but they vary almost exactly in the same proportion, so that the magnifying effect of the eye glass upon the image o o will be the same, whether the observer be long-sighted or short-sighted; in estimating the magnifying power, therefore, of such a combination, we may consider, in all cases, the distance of the eye glass E E from the image o o to be equal to its focal length, and the distance of o′ o′ from the eye glass to be 10 inches.

To estimate the entire amplifying effect of such a microscope, we have only to multiply the magnifying power of the object glass by that of the eye glass; thus, for example, if the distance of the image o o from the object glass be 10 times as great as the distance of the object from it, the linear dimensions of the image o o will be ten times greater than those of the object; and if the focal length of the eye glass be ½ an inch, the distance of most distinct vision being 10 inches, the linear dimensions of o′ o′ will be 20 times those of o o, and therefore 200 times those of the object; the linear magnifying power would in that case be 200, and, consequently, the superficial magnifying power 40000.

The eye and object glasses are usually mounted at the distance of 10 or 12 inches asunder, adjustments nevertheless being provided, by which their mutual distance can be varied within certain limits.

470. **Field glass.** — A convex lens is generally interposed between the object glass and eye glass, which, receiving the rays diverging from the former, before they form an image, has the effect of contracting the dimensions of the image, and at the same time increasing its brightness. The effect of such an intermediate lens will be understood by reference to *fig.* 219., where F F is the intermediate lens. If this lens F F were not interposed, the object glass L L would form an image of the object *o* at o o; but this image being too large to be seen at once with any eye glass, a certain portion of its central parts would only be visible. The lens F F, however, receiving the rays before they arrive at the image o o, gives them increased convergence, and causes them to produce a smaller image o′ o′, at a less distance from the object glass L L. The dimensions of this image are so small, that every part of it can be seen at once with the eye glass.

The portion of the image which can be seen at once with the eye glass, is called the *field of view* of the microscope.

It is evident, from what has been stated, that the effect of the lens F F is to increase the field of view, since by its means the entire image of the object can be seen, while without its interposition the central parts only would be visible.

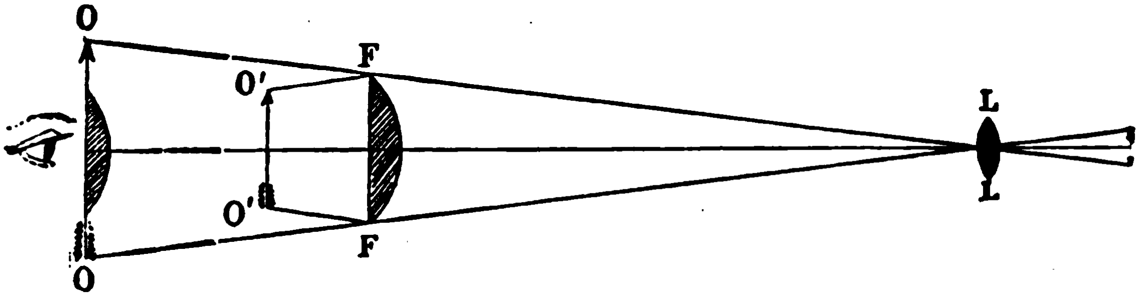


Fig. 219.

The lens F F has, from this circumstance, been called the *field lens.*

But the increase of the field is not the only effect of this arrangement. The light which would have been diffused over the surface of the larger image O O, is now collected upon that of the smaller image O′ O′; and the brightness, therefore, will be increased in the same proportion as the surface of O O is greater than the surface of O′ O′, that is, in the proportion of the square of O O to the square of O′ O′.

Another effect of the field lens is to diminish the length of the microscope, for the eye glass, instead of being placed at its focal distance from O O, is now placed at the same distance from O′ O′.

471. **Reflecting microscope.** — In this brief exposition of the general principle of the microscope, the image, which is the immediate subject of observation, is supposed to be produced by a convex lens; such an image, however, may also be produced by a concave reflector, and being so produced, may be viewed with an eye glass, exactly in the same manner as when produced by a convex lens.

Microscopes have accordingly been constructed upon this principle, and are distinguished as *reflecting microscopes*; those in which the image is produced by a lens being called *refracting microscopes.*

The principle of a reflecting microscope will be understood by reference to *fig.* 220., where L L is the concave reflector, of which C is the centre; the object *o* is placed towards the reflector, at a distance from C greater than half the radius, and an inverted image of it is formed at O O, which, as in the case of the refracting microscope, is looked at with an eye glass E E.

The great improvements which have taken place within the last twenty years in the formation of the object glasses of refracting microscopes, have rendered these so very superior to reflecting microscopes, that the latter class of instruments has fallen so completely into disuse, that it will not be necessary here to notice them further.

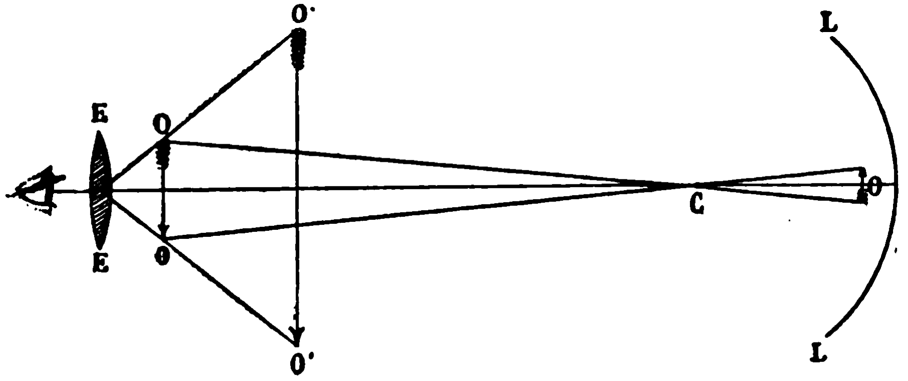


Fig. 220.

472. **Conditions of efficiency.**—In what has been explained, the general principle only of the microscope has been developed: many important circumstances of detail upon which its efficiency mainly depends must now be noticed.

These conditions are essentially identical with those necessary for the perfection of natural vision, and are consequently,—1°, sufficient visual magnitude; 2°, sufficient distinctness of delineation; and 3°, sufficient illumination.

The visual angle under which the image is seen will depend on the magnitude of the image and the shortness of the focal length of the eye glass. The optical conditions which set practical limits to these will presently appear.

The greater the visual angle is, the more perfect must be the distinctness of the image, both as respects delineation and colour, since all errors will necessarily be magnified in the exact proportion of the amplitude of the visual angle.

The distinctness of the image as to form and colour will depend on the extent to which the aberrations, spherical and chromatic, are corrected by the material and form of the lenses.

The expedients by which these aberrations are effaced have been already explained (164. *et seq.*, and 207. *et seq.*).

The illumination of the image will depend on conditions connected with the angular aperture of the object glass, which have also been already fully explained.

473. **Angular aperture.**—The practical method of determining the angular aperture is as follows:—

Let *m m*, *fig.* 221., be the microscope, the object end being fixed upon a pivot, so that the eye end can be moved over a graduated semicircle. Let a small luminous object, such as the flame of a candle, be placed in the direction *r r*, at the distance of 6 or 8 feet, so that the rays proceeding from it to the object glass may be considered as parallel. If the microscope be directed towards the candle, all the rays will fall perpendicularly on the object glass, and will evidently pass through it to the eye glass. If the microscope be then turned on the pivot to the left, the rays will fall more and more obliquely on the object glass, and a less and less number of them will pass to the eye glass.

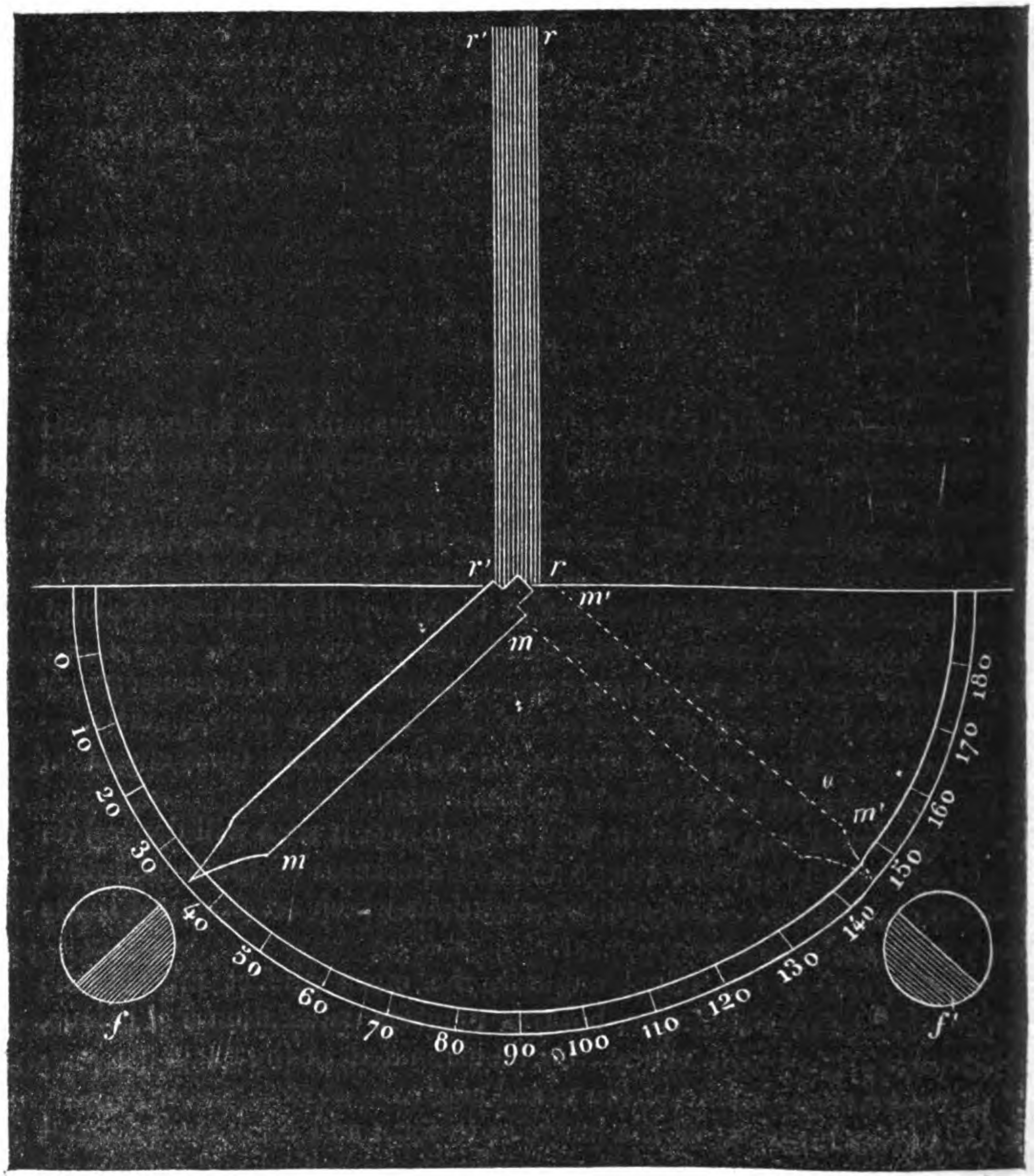


Fig. 221.

When such a position as *m m* is given to the microscope, those rays only which fall upon the border of the object glass upon the right of the observer will arrive at the eye glass, and the field of view will then appear, as shown at *f*, half illuminated and half dark. If the microscope be moved beyond this position, the field will be entirely dark, no rays being transmitted to the eye glass.

If the microscope, on the contrary, be moved to the other side of the graduated semicircle, the same appearances will be produced, and when it assumes the position *m′ m′*, the field will be again half illuminated, and beyond that point it will be dark.

The arc of the graduated semicircle, included between the two positions *m m* and *m′ m′*, will then be the measure of the angular aperture of the object glass, since that arc will correspond with the greatest obliquity, at which rays diverging from the object to the object glass, can pass through the latter, so as to arrive at the eye glass.

474. **To produce perfect achromatism.**—From what has been explained (207. *et seq.*), it is evident that compound object lenses may be produced by which an achromatic image will be formed; but unless the lenses which form the eye piece, that is, the eye glass and the field glass, also form an achromatic combination, the chromatic aberration which has been effaced by the perfection of the object glass will be more or less reproduced by the imperfection of the eye piece.

This defect might, it is true, be remedied by making both the field glass and eye glass achromatic; but independently of other objections to such an expedient, it would be needlessly expensive, and the same purpose is attained in a much more simple manner; since, by a suitable combination of lenses, differing in form and material, the dispersion produced by a simple converging lens can be neutralised, so that the extreme coloured images, red and violet, may be made to coalesce. It is easy to conceive that the forms of the lenses may be so modified as to produce, not the coincidence of these extreme images, but their interchange of position, so that the violet image which was nearest to the lens shall be most distant from it, and the red, which was most distant from it, shall be nearest to it.

The chromatic aberration produced by a simple converging lens, in which the violet image is nearest to the lens, and the red most distant from it, being designated *positive chromatic aberration*, that of a compound converging lens, such as has been just described, in which the positive chromatic aberration is *over corrected*, and in which, consequently, the red image is nearest to, and the violet most distant from, the lens, is called *negative chromatic aberration*.

It is evident that the forms of the component parts of a compound lens, may always be such as to give it any required degree of negative chromatic aberration.

Now the lenses composing the eye piece, being simple converging lenses, will necessarily have positive chromatic aberration. If the lenses composing the object piece be so formed and combined that they shall have a degree of negative chromatic aberration precisely equal to the positive chromatic aberration of the eye piece, it is plain that the two contrary aberrations will neutralise each other, and the result will be that the image seen through the eye piece will be, for all practical purposes, achromatic.

To make this more evident, let L L, *fig.* 222., be the compound object glass, consisting of a double convex lens of crown glass, and a plane or convex lens of flint glass, formed so as to produce negative chromatic aberration; let F F be the field glass, E E the eye glass, and O the object.

Let V V R R be the coloured images of the objects, which would be produced by L L, if F F were not interposed; these images will be slightly concave towards L L, and since L L is supposed to have negative aberration, the red images R R will be nearest to it, and the violet ones, V V, most remote from it.

But the rays which would converge upon the various points of these images being intercepted by the field glass F F, are rendered more convergent by it, and the images are accordingly formed nearer to it. This lens, F F, also increases the convergence of the violet rays, which are most refrangible, more than it increases that of the red rays, which are least refrangible. The consequence of this is, that the violet and red images are brought closer together than they were, as shown in the figure; but still the violet images are more distant from F F than the red, so that the chromatic aberration of L L and F F conjointly is still negative, though less than the aberration of L L alone.

There is another effect produced by the lens F F, which it is important to notice. The images produced by L L, which were slightly concave towards F F, are changed in their form, so as to be slightly concave towards E E.

In fine, then, the rays diverging from the images R′ R′ V′ V′, after passing through the eye glass E E, have their divergence diminished, so as to diverge from more distant points, I I. The divergence of the violet rays, V V, being most refrangible, is most diminished, and that of the red rays, R R, being least refrangible, is least diminished. If their divergence were equally diminished, a series of coloured images would be formed at I I, the violet being nearer, and the red farther from E E; but the divergence of the violet, which is already greater than the red, is just so much greater than the latter, that the difference of the effects of E E upon it is such as to bring the images ogether at I I.

Thus it appears that the positive aberration of the eye glass E E is exactly equal to the negative aberration of L L and F F taken conjointly, so that the one exactly neutralises the other, all the coloured images coalescing at I I, and producing an image altogether exempt from chromatic aberration.

There is another important effect produced by the eye glass; the images R′ R′ V′ V′, which are slightly concave towards E E, are rendered straight and flat at I I.

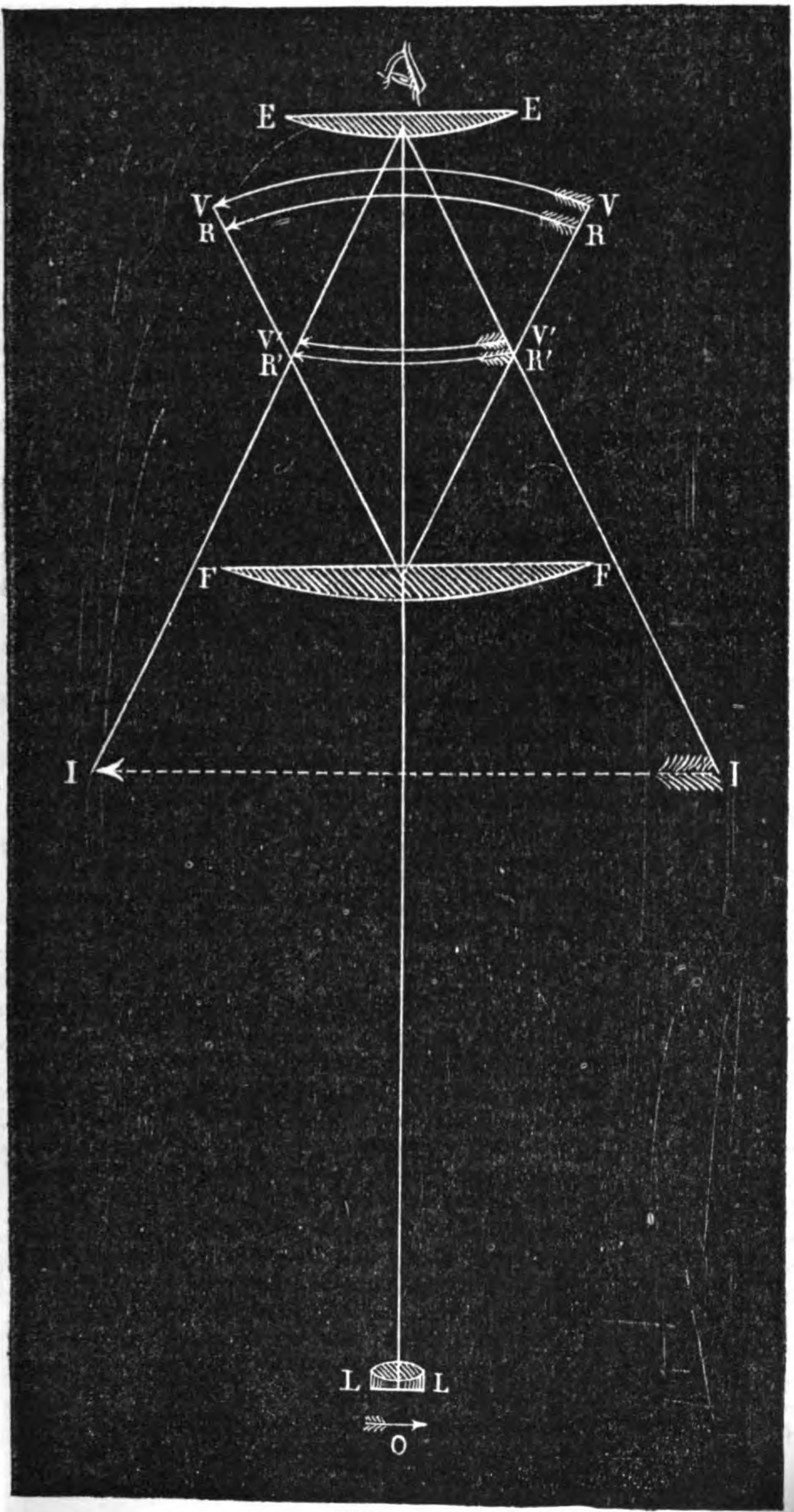


Fig.

Thus, it appears that, by this masterly combination, a multiplicity of defects, chromatic, spherical, and distortive, are made, so to speak, to efface each other, and to give a result, practically speaking, exempt from all optical imperfection.

There is still another source of inaccuracy which, though it is more mechanical than optical, demands a passing notice. All the lenses composing the microscope require to be set in their respective tubes, so that their several axes shall be directed upon the same straight line with the greatest mathematical precision. This is what is called *centring* the lenses, and it is a process, in the case of microscopes, which demands the most masterly skill on the part of the workman. The slightest deviation from true centring would cause the images produced by the different lenses to be laterally displaced, one being thrown more or less to the right and the other to the left, or one upwards and the other downwards; and even though the aberrations should be perfectly effaced, the superposition of such displaced images would effectually destroy the efficiency of the instrument.

475. **Compound object pieces.** — In what precedes, we have, to simplify the explanation, supposed the object glass to consist of a single achromatic lens, a circumstance which never takes place except when very low powers are sufficient. A single lens, having a very high magnifying power, would have so short a focus and such great curvature, that it would be attended with great spherical aberration, independently of other objections; great powers therefore, have been obtained by combining several achromatic lenses in the same object piece, so that the rays proceeding from the object are successively refracted by each of them, and the image submitted to the eye glass is the result of the whole.

The optical effect of such a combination will be more clearly understood by reference to *fig.* 223., where L L, L′ L′, and L″ L″, represent a combination of three achromatic object glasses.

Let O O be the object, placed a little within the focus f of the lens L L. The image of O O, produced by L L, would then be an imaginary one in the position I I; (158. *et seq.*). After passing through L L, the rays, therefore, fall upon L′ L′, as if they diverged from the several points of the image I I, which may, therefore, be considered as an object placed before the lens L′ L′. Let f' be the focus of L′ L′; the image of I I produced by L′ L′ will therefore be imaginary, and will be at I′ I′; the rays, after passing through L′ L′ will fall upon L″ L″, as if they diverged from the several points of I′ I′. This image I′ I′ therefore may be considered as an object placed before the lens L″ L″. Let f'' be the focus of the lens; the image of I′ I′ produced by L″ L″ will then be I″ I″, and will be real; this will then, in fact, be the image transmitted to the eye piece.

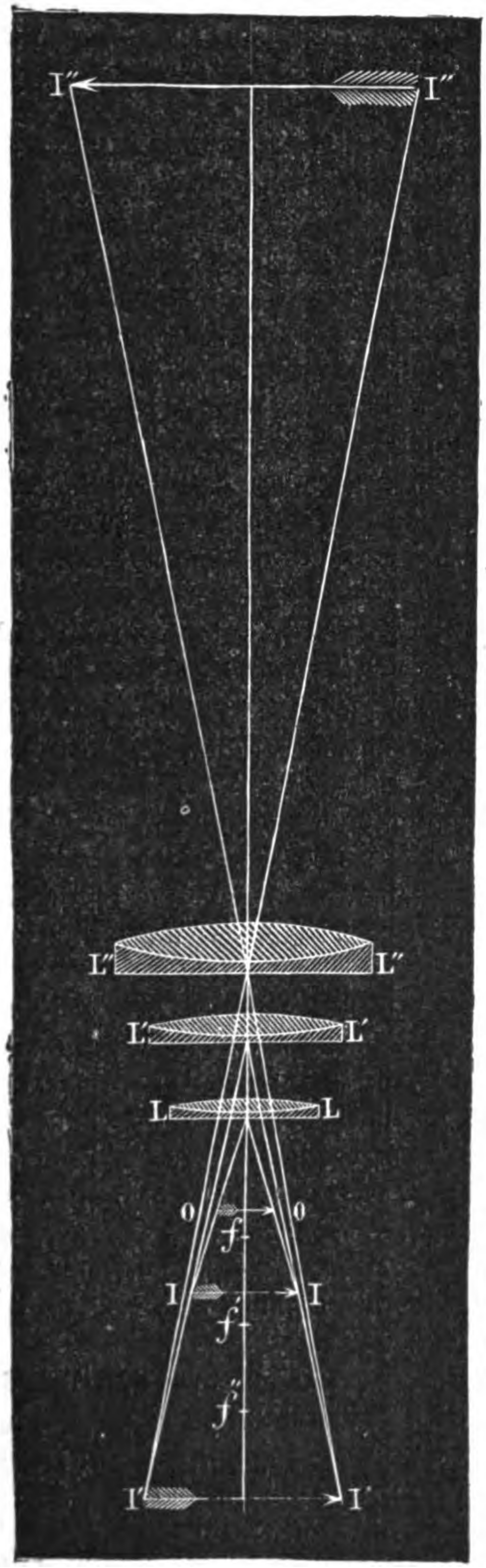


Fig. 223.

To render the diagram more easy of comprehension, we have not here attempted to represent the several distances in their proper proportions.

The compound lenses, of which object pieces consist, are generally, as represented in the figure, plane on the sides presented towards the object. This is attended, among other advantages, with that of allowing a larger angle of aperture than could be obtained, if the surface presented to the rays diverging from the object were convex.

The extreme rays diverging from each point of the object, fall upon the surface of the object glass with a greater and greater obliquity, as they approach its borders, and since there is an obliquity so extreme, that the chief part of the rays would not enter the glass at all, but would be reflected from it, the angle of aperture must necessarily be confined within such limits, that the rays passing through the borders of the lens will not be so oblique as to fall under this condition. If the surface of the object glass presented to the object were convex, it is evident that the rays diverging from an object at a given distance from it, would fall upon its borders with greater obliquity, than if it were plane, and, consequently, such an object glass would allow of a less angle of aperture, than a plane or convex one with its plane side towards the object.

476. **Adjusting object pieces.** — Improvements have recently been made in object glasses, by which angles of aperture have been obtained so great, as not to admit even of a plane surface being presented to the diverging pencil, and it has accordingly been found necessary, in such cases, to give the object glasses the meniscus form, the concave side being presented to the object. By this expedient angles of aperture have been obtained so great as 170° with a plane or convex glass; the extreme of pencils having such an angle would fall upon the surface of the lens at angles of five degrees, and those of the lateral pencils at even less. With such obliquities the chief part of the rays would be reflected, and, consequently, the borders of the lens would be inefficient.

The three achromatic lenses here described being mounted, so that their axes shall be precisely in the same straight line, constitute what is generally called an *object glass*, but which, perhaps, might with more convenience and propriety be denominated an *object piece.*

In the superior class of instruments, where magnifying power is pushed to so extreme a limit as 1500 or 2000, the utmost precision in the balance of the aberrations must necessarily be realised, since the slightest imperfection so prodigiously magnified would become injuriously apparent.

477. **Glass cover of slider achromatised.** — The extreme degree of perfection, which has been attained in the best class of microscopes may be imagined when it is stated that an object which is distinctly visible under a power of 1500 or 2000, when it is exposed to the object glass uncovered, will be sensibly affected by aberration if a piece of glass, no more than the 100th of an inch in thickness, be laid upon it. Infinitesimally small as is the aberration produced by such a glass film, it is sufficient, when magnified by such a power, to be perceptible, and to impair in a very sensible manner the distinctness of the image.

As it has been found necessary, for the preservation of microscopic objects, to cover them with such thin films of glass, through which, consequently, they are viewed, adjustments are provided in microscopes with which the highest class of powers are supplied, by which even the small aberration due to these thin plates of glass, thus covering the objects, can be corrected. This is effected by mounting the lenses, which compose the triple object piece, in such a manner that their mutual distances, one from another, can be varied within certain small limits, by motions imparted to them by fine screws. This change of mutual distance produces a small effect upon the aberrations, rendering their total results negative, to an extent equal to the small amount of positive aberration, produced by the thin glass which covers the object.

478. **Eye pieces.** — The eye glass and the field glass are both plane or convex lenses, having their plane sides turned towards the eye; they are set in opposite ends of a brass tube, varying in length from two inches downwards, according to their focal lengths, the distance between them and, consequently, the length of the tube, being always equal to half the sum of their focal lengths.

479. **Use of various powers.** — In the prosecution of microscopic researches, the use of very various magnifying powers is indispensable; the higher powers would be as useless for the larger class of objects, as the lower ones for the smaller. But even for the same object, a complete analysis cannot be accomplished without the successive application of low and high powers: by low powers the observer is presented with a comprehensive view of the entire form and outline of the object under examination, just as an aeronaut who ascends to a certain altitude in the atmosphere obtains a general view of the country which would be altogether unattainable upon the level of the ground; by applying successively higher powers, as has been already explained, the smaller parts and minuter features of the object are gradually disclosed to view, just as the aeronaut, in gradually descending from his greatest altitude, obtains a view of objects which were first lost in the distance, but at the same time loses, by too great proximity, the general outline.

The microscope makers, therefore, supply in all cases an assortment of powers, varying from 30 or 40 upwards; observations requiring powers under 40, being more conveniently made with magnifying glasses or simple microscopes. For this purpose it is usual, with the best instruments, to furnish six or eight object pieces and three or four eye pieces, each eye piece being capable of being combined with each object piece. The number of powers thus supplied will be equal to the product of the number of object pieces, multiplied by the number of eye pieces.

The powers, however, may still be further varied, by provisions for changing the distance between the object and eye pieces, within certain limits. For this purpose, the tube of the instrument is sometimes divided into two, one of which moves within the other, like the tube of a telescope, the motion being produced by a fine rack and pinion: in this case the eye piece is inserted in one of the tubes, and the object piece in the other. By combining this provision with a proper assortment of object pieces and eye pieces, all possible gradations of power between the highest attainable, and the lowest which is applicable, can be obtained.

480. **Magnitude of field.** — The actual magnitude of the space which can be presented at once to the view of the observer, will vary with the magnifying power; but in all cases it is extremely minute. Thus, with the lowest class of powers, where it is largest, it is a circular space, the diameter of which does not exceed the 8th or 10th of an inch; it follows, therefore, that no object, the extreme limits of whose linear magnitude exceed this, can be presented at once to the view of the observer. Such objects can only be seen in their *ensemble*, by means of less powerful magnifying glasses, or with the naked eye.

481. The field of view, with powers from 100 to 300, varies in diameter from the 15th to the 40th of an inch; from 300 to 500 it varies from the 40th to the 70th of an inch; and from 500 to 700 from the 70th to the 100th of an inch.

It will thus be understood, that even with the moderate power of 700, an object to be included wholly within the field of view, must have a magnitude such as may be included within a circle, whose diameter does not exceed the 100th of an inch. These observations will be more clearly appreciated by reference to the annexed diagram (*fig.* 224.), where A is a circle whose diameter is the 6th of an inch; B one whose diameter is the 12th of an inch; C the 25th; D the 50th; and E the 100th.

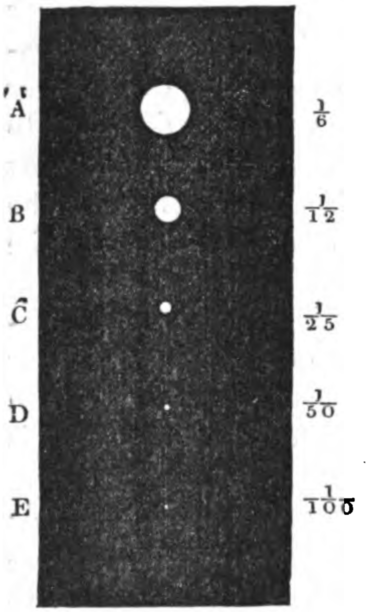


Fig. 224.

But when still higher powers are used, the actual dimensions of the entire space comprised within the field of view will be so very minute, that an object which would fill it, and still more, smaller objects included within it, would not only be altogether invisible to the naked eye, but would require considerable microscopic power to enable the observer to see them at all.

The actual dimensions of the field of view, which correspond to each magnifying power, vary more or less in different instruments. Those which I have given above are taken from a microscope made by Charles Chevalier, which is in my possession. The difference however, in this respect, between one instrument and another, i not considerable, and the above will serve as a fair illustration of the limits of the field of instruments in general.

The entire dimensions of the field of view, therefore, being so exceedingly minute, it will be easily understood that some difficulty will attend the process by which a small object, or any particular part of an object, can be brought within it: thus, with the moderate power of 500, the entire diameter of the field being no more than the 70th of an inch, a displacement of the object to that extent, or more, would throw it altogether out of view. If, therefore, the object, or whatever supports it, be moved by the fingers, the sensibility of the touch must be such as to be capable of producing a displacement thus minute.

If the object be greater in its entire dimensions than the field of view,—a circumstance which most frequently happens,—a part only of it can be exhibited at once to the observer; and to enable him to take a survey of it, it would be necessary to impart to it, or to whatever supports it, such a motion as would make it pass across the field of view, as a diorama passes before the spectators, disclosing in slow succession all its parts, and leaving it in the power of the observer to arrest its progress at any desired moment, so as to retain any particular part under observation.

The impracticability of imparting a motion so slow and regular, by the immediate application of the hand to the object, or its support, will be very apparent, when it is considered that while the entire object may not exceed a small fraction, say, for example, the 20th of an inch in diameter, the entire diameter of the field of view may be as much as 20 times less, so that only a 20th part of the diameter of the object would be in any given position comprised within it.

482. **Mechanism to move and illuminate the object.** — These and similar circumstances have rendered it necessary, that the want of sufficient sensibility and delicacy of the touch in imparting motion to the object, shall be supplied by a special mechanism, by means of which the fingers are enabled to impart to the object, an infinitely slower and more regular motion, than they could give it without such an expedient. The means by which this is accomplished will be presently explained.

We have seen that the intensity with which the microscopic image is illuminated, depends on the angle of aperture, other things being the same; but however large that angle may be, when considerable magnifying power is used, it is necessary that the object itself should be much more intensely illuminated, than it would be by merely exposing it to the light of day, or that of the most brilliant lamp. It is therefore necessary to provide expedients, by which a far more intense light can be thrown upon it.

483. **To focus the instrument.**—The instrument is said to be in *focus* when the observer is enabled to see with the eye glass the magnified image of the object with perfect distinctness; this will take place provided the mutual distances between the eye piece, the object piece, and the object are suitably adjusted; and this adjustment may be accomplished by moving any one of these three towards or from the other two, while these last remain fixed: thus, for example, if the object and the object piece remain unmoved, the instrument may be brought into focus by moving the eye piece to or from the object piece. The rack and pinion, already described, which moves the tube in which the eye piece is inserted, can accomplish this. This provision, however, is not made in all microscopes.

If the eye piece and the object be fixed, the instrument may be brought into focus, by moving the object piece to or from the object. To effect this, it would be necessary that the object piece should be inserted in a tube, moved by a rack and pinion, like that of the eye piece.

In fine, if the object piece and eye piece be both fixed, the instrument may be brought into focus by moving the object, or whatever supports it, to or from the object glass.

All these methods are resorted to in the different forms in which microscopes are mounted by different makers.

484. **To render objects translucent.** — Since nearly all material substances, when reduced to an extreme degree of tenuity, are more or less translucent, and since almost all microscopic objects have that degree of tenuity, by reason of their minuteness, it happens that nearly all of them are more or less translucent; and where, in exceptional cases, a certain degree of opacity is found, it is removed without interfering with the structure, by saturating the object with certain liquids, which increase its translucency, just as oil renders paper semi-transparent. The liquid which has been found most useful for this purpose, is one called *Canada balsam.* When the object is saturated with this liquid, it is laid upon a slip of glass, about two inches long and half an inch wide, and is covered with a small piece of very thin glass, made expressly for this purpose, the thickness in some cases not exceeding the 100th of an inch. It is usual to envelop the oblong slip of glass, in the middle of which the object is thus mounted, with paper gummed round it, a small circular hole being left uncovered on both sides of the glass, in the centre of which the object lies.

The slips of glass thus prepared, with the objects mounted upon them, are called *sliders;* and the objects so mounted are so placed that the axis of the object piece shall be directed upon that part of them which is submitted to observation, provisions being made to shift the position of the slider, so as to bring all parts of the object successively under observation. Further provisions are also made to throw a light upon the object, by which it will be seen as an object is on painted glass.

Since, however, there are some few objects which cannot be rendered translucent, expedients must be provided, by which they can be illuminated upon that side of them which is presented to the microscope. It is often necessary, also, even in the case of translucent objects, that they should be viewed by means of light thrown upon that side of them which is turned to the object glass.

485. **Mounting and accessories.**—These general observations being premised, we shall proceed to explain the method by which the optical part of the instrument is mounted, and the several accessories by which the object is supported, moved, and illuminated.

Let us suppose, for the present, that the eye piece E E, *fig.* 225., and the object piece *o*, are mounted in a vertical tube, with whose axis A A A, the several axes of the lenses, accurately coincide. Let *d d* be a diaphragm, or blackened circular plate, with a hole in its centre, placed in the focus of the eye glass, by which all rays of light not necessary to form the image shall be intercepted. Let D be a milled head, by turning which the tube which carries the eye piece can be moved within certain limits to and from the object piece, and let D′ be another milled head, by which the tube which carries the object piece can be moved within certain limits to and from the object, or by which the entire body B B of the microscope, carrying the object piece and eye piece, can be moved to and from the object.

Let S S be a flat stage of blackened metal or wood, having a circular hole in its centre, as shown in plan at S′ S′, and let it be fixed by proper arrangements, so that the axis A A A of the microscope shall pass through the centre of the circular aperture, and so that its plane shall be at right angle to that axis. Let a slider, such as we have described above, upon which an object is mounte , be laid upon this stage, so that the object shall be in the centre of the hole, and therefore in the axis A A A of the microscope, as shown at S′ S′.

Let M M be a concave reflector, receiving light either from a lamp or a window, and reflecting it upwards towards the opening in the slider, in converging rays, so as to condense the light with more or less intensity upon the under side of the object; if the convergence produced by M M be insufficient, it may be augmented by the interposition of a convex lens C C. This may or may not be interposed, according as the object is smaller or greater, and requires a more or less intense illumination.

The light thus thrown upon the lower side of the object, the latter, being sufficiently translucent, is rendered visible by it.

If the object be opaque, it may be illuminated from above by several expedients; being placed upon a blackened plate resting on the stage S S, light proceeding from a window or a lamp may be condensed upon it by a concave reflector M′ M′, or by a convex lens L L. These arrangements are only applicable when the object is at such a distance from the object piece, that the light reflected by M′ M′ or L L shall not be wholly or partially inter-

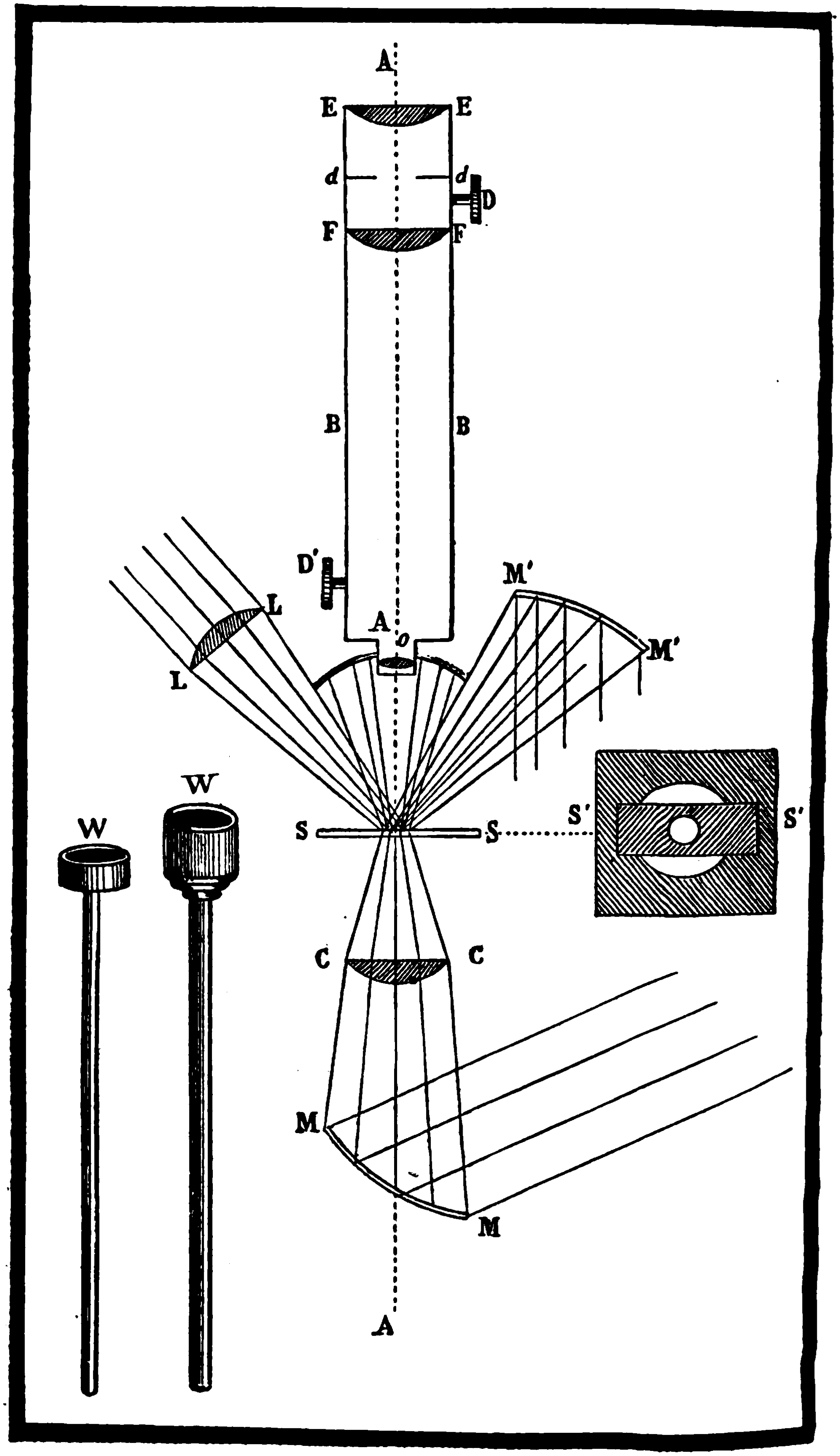


Fig. 225.

cepted by the object piece. This would always be the case, however, when very high powers are used, and when, consequently, the object must be brought very close to the object piece. In that case the object is supported upon a small piece of blackened cork, or in a dark cell of the form represented at W W; this support is placed in the centre of the opening of the stage, so as not to intercept any but the central rays reflected from M M; upon the end of the object piece a concave reflector, having a hole in its centre, through which the object piece passes, is fixed; the light proceeding from M M, and falling upon this reflector, is reflected by it, so as to converge upon the object, and thus to illuminate it.

486. **Lieberkuhn.—Disc of diaphragms.**—A concave illuminator thus mounted is called, from its inventor, a *Lieberkuhn.*

In the illumination of objects it is frequently necessary to limit, to a greater or less extent, the diameter of the pencil of light thrown from the reflector, M M, upon the object. Although this may partly be accomplished by varying the distance of the reflector from the object, or by the interposition of a convex lens, such expedients are not always the most convenient, and a much more ready and effectual method of attaining this end is supplied by providing below the stage, S S, a circular blackened disc, capable of being turned upon its centre in its own plane. This disc is pierced with a number of holes of different diameters, as shown in *fig.* 226., and it is so mounted, that the openings in it, by turning it round its centre, may be brought successively under the object. This is easily done by fixing the centre of this disc at a distance from the centre of the stage, equal to the distance between the centre of the disc and the centres of the holes made in it.

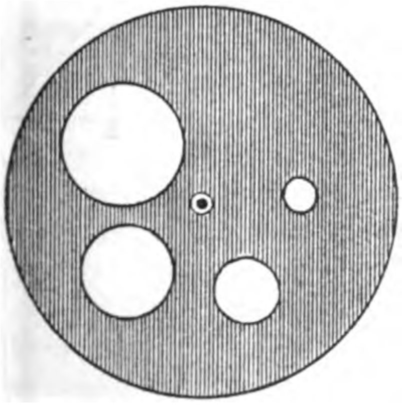


Fig. 226.

This appendage is called the *disc of diaphragms*, and is of great use in the illumination of objects, as will appear hereafter.

As the effect of the illuminators varies not only with their distance from the object, but also with the direction in which the light directed from them falls upon the object, provisions are made in mounting the microscope, by which various positions may be given to them, so that the light may fall upon the object in any desired manner.

487. **Illuminating apparatus.**—In the frame in which the illuminator, M M, is mounted, it is customary to place two reflectors, one at each side, one concave and the other plane. By the former a converging, and by the latter a parallel, pencil of light is reflected towards the object.

In this general illustration we have supposed the axis of the instrument to be vertical; it may, however, have any direction whatever; but whatever be its direction, the stage, S S, must always be at right angles and concentric with it. The eye piece and object piece are also supposed to be set in the same straight tube, with their axes set in the same straight line. This arrangement, though most commonly adopted, is neither necessarily nor always so. The tube which carries the eye piece may, on the contrary, be inclined at any desired angle with that which carries the object piece; for this purpose it is only necessary to place in the angle formed by the two tubes a reflector, so inclined that the rays coming from the object piece shall be reflected along the axis of the tube which carries the eye piece.

488. **Method of rendering axes of eye piece and object piece at right angles.**—Thus, for example, if the tube which carries the object piece be vertical, a plane reflector, M M, *fig.* 227., receiving the rays coming in a vertical direction from the object piece, will reflect them horizontally to the eye piece E E.

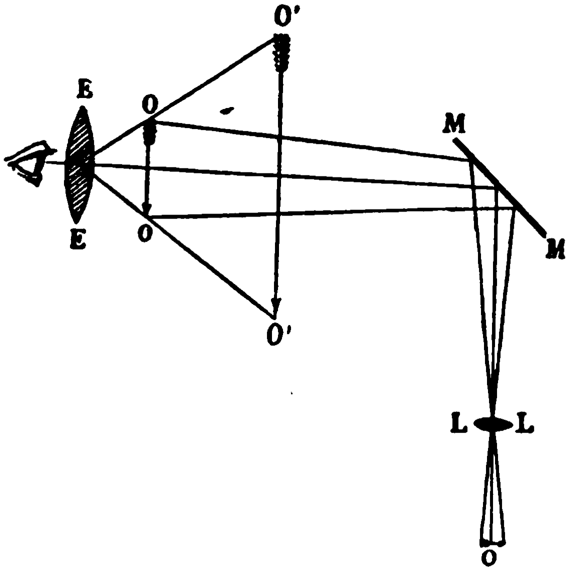


Fig. 227.

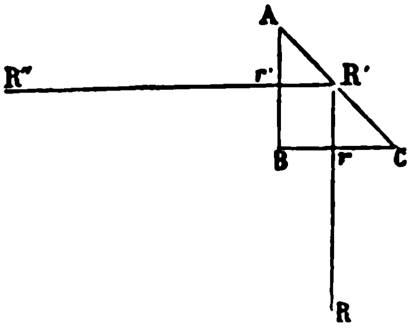


Fig. 228.

The same object would be attained with more advantage, and less loss of light, by means of a rectangular prism, A B C, *fig.* 228., the vertical ray, R R being reflected by the back, A C, of the prism in the horizontal direction R' R''.

Since a single reflection thus made produces an inverted image, it is sometimes preferable to accomplish the object by two successive reflections, as shown in *fig.* 229., where the ray, A B, is successively reflected at B and C to the eye at D. And the same object may be attained more advantageously by means of a quadrangular prism, as shown in *fig.* 230.

Much practical convenience often arises from the adoption of this expedient; thus, while the object tube is directed vertically downwards, to an object supported on a horizontal stage, or floating on or swimming in a liquid, the eye tube may be horizontal, so that the observer may look in the level direction. In this case the two tubes are fixed at right angles, the reflecting surface being placed at an angle of 45° with their axes. We shall see hereafter a case in which, by the adoption of an oblique tube, several observers may at the same time, looking through different eye pieces, see the same object through one and the same object glass.

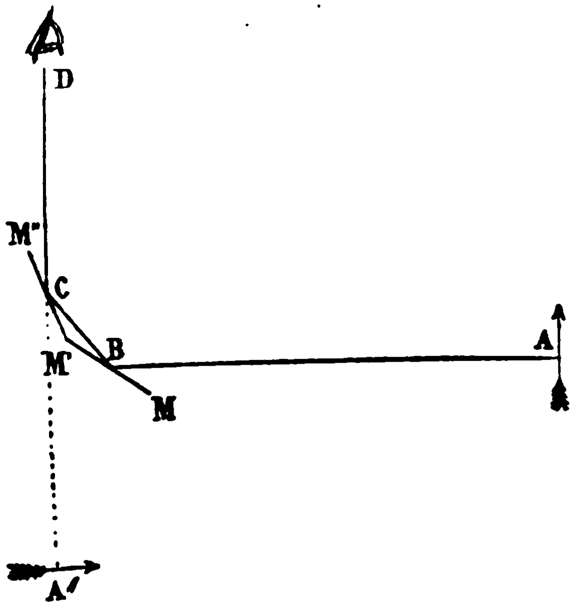


Fig. 229.

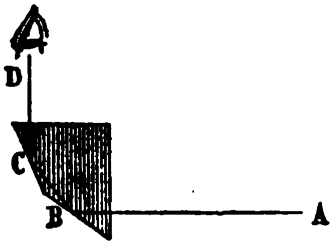


Fig. 230.

489. **The support and movement of the object.**—The appendage of the microscope, adapted for the support of the object, called *the stage*, has been already described in its most simple form.

Since every motion or disturbance by which the stage may be affected will necessarily be increased, when seen through the microscope, in the exact proportion of the magnifying power, it is of the utmost importance that it should be exempt from all tremor, and that it should have strength sufficient to bear, without flexure, the pressure of the hands in the manipulation of the object. When a high power is used, the focal adjustment of the instrument requires to be so exact, that a displacement of the object, which would be produced by the slightest pressure of the fingers upon a stage not very firmly supported, would throw it out of focus.

Fine screws are applied, in various ways, to focus the instrument by varying, at pleasure, the distance between the stage and the object glass. Generally two classes of adjustment are provided for this purpose. The first, called the coarse adjustment, by which the stage is moved towards the body of the instrument, or the latter towards the stage with a quick motion, so as to bring the object approximately to the focus. Another much finer screw is provided, called the fine adjustment, which produces a much slower motion of the same kind, by means of which the instrument is accurately focussed.

490. **Mechanism of the stage.** — Slow motions, in different directions, are imparted to the stage on which the object is placed by similar means. These stages are variously constructed, but generally consist of two or more flat plates movable, one upon the other, by means of fine screws, one of which imparts a motion right and left, and the other a motion backward and forward. These screws are moved by milled heads placed at the edges of the stage, as shown in *fig.* 231.

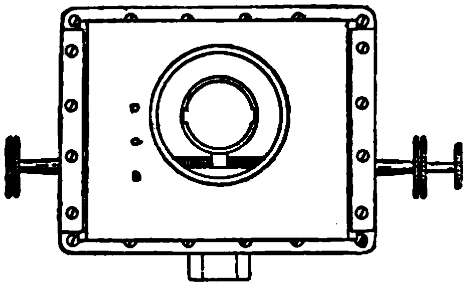


Fig. 231.

In *fig.* 232. a circular stage is shown, which is capable of being turned in its own plane round its centre, while, by means of the screws v and v′, it can be moved transversely in two directions at right angles to each other.

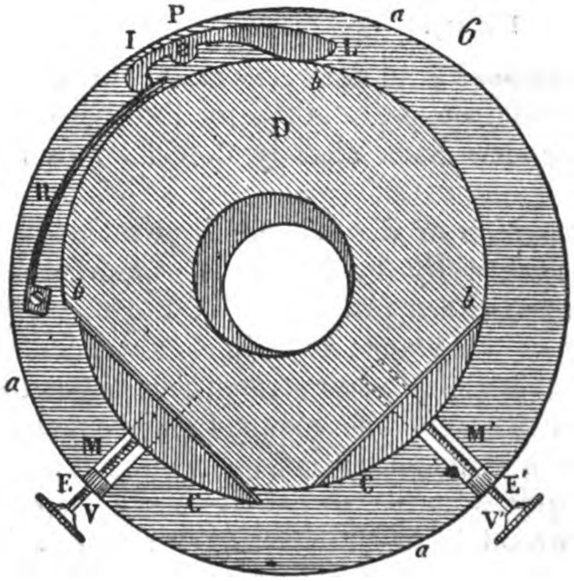


Fig. 232.

491. **Various forms of the instrument.** — The methods of mounting microscopes, so as to adapt them to the convenience and the ease of observers, are very various, depending on the purposes to which they are applied, their price, the exigencies of the purchaser, and the skill, taste, and address of the maker.

The qualities which it is desirable to confer upon the stand and mounting of the instrument are, simplicity of construction, easy portability, smoothness and precision in the action of all the moving parts, and such combinations as may cause any tremor imparted to the stand to be distributed equally over every part of the mounting. These capital objects are attained very completely in all the mountings of the best makers, British and foreign.

492. **Frauenhofer's mounting.** — One of the most simple models for the mounting of a compound microscope was contrived by Frauenhofer so early as 1816, long before achromatic lenses were produced. This model, owing to its great simplicity, convenience, and cheapness, is still extensively used for the lower priced instruments, especially by the continental makers.

The body of the instrument is attached to a vertical pillar, *fig.* 233., and its axis is permanently vertical. It is focussed by a rack and pinion, worked by a milled head on the right of the observer. The stage is fixed in its position and placed on the top of a short tube, in the lower part of which the reflector is suspended on a horizontal axis, so that it can be placed at any desired obliquity to the axis of the instrument, and thus can always throw a beam of light upwards to the object. One side of this mirror is concave, and the other plane.

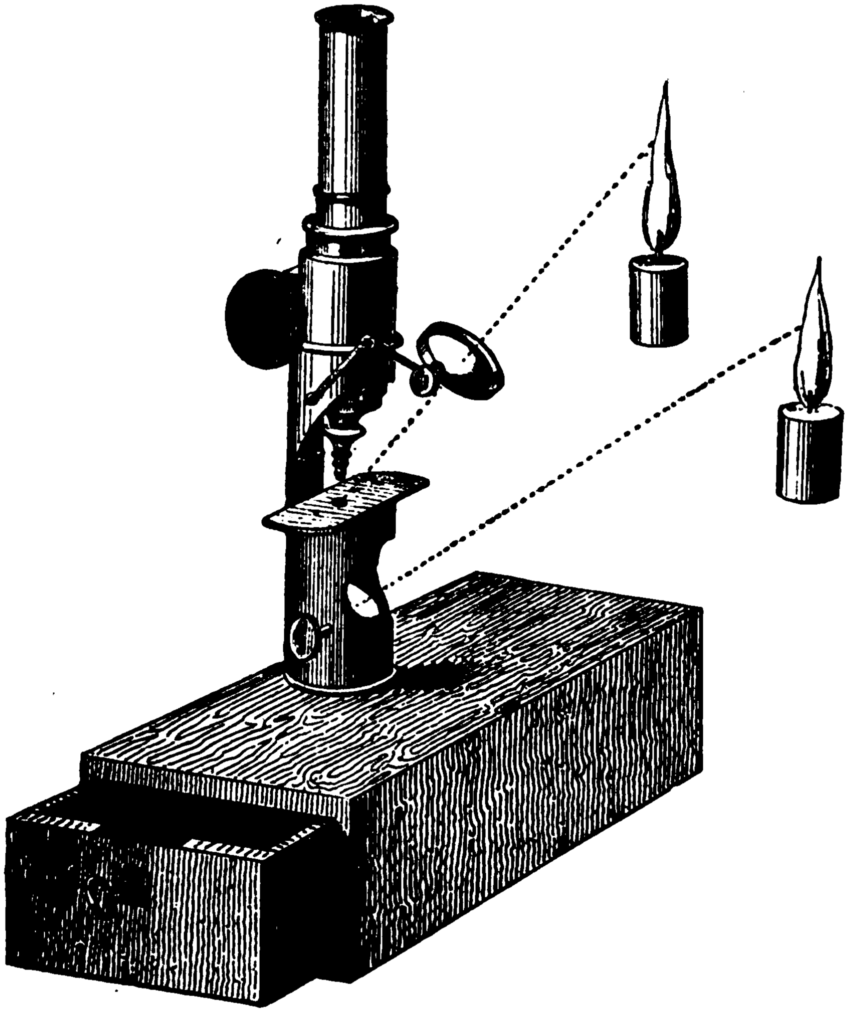


Fig. 233.

For the illumination of opaque objects, a lens is attached by a jointed arm to the upper part of the pillar, on which the instrument is supported.

M. Lerebours, of Paris, makes excellent microscopes on this model, with a triple achromatic object piece and other accessories, which he sells at the very moderate price of 90 francs (*3l. 12s.*). Several thousands of these have been sold.

Most of the better class of instruments are so mounted, that any direction whatever can be given to the axis of the body. Various mechanical expedients are used for accomplishing this, most of which are analogous to the methods of mounting telescopes. In some, the instrument with its appendages is supported upon two uprights of equal height by means of trunnions, which pass through its centre of gravity, so that it turns upon its supports like a transit instrument, the axis of the body being capable of assuming any inclination to the vertical. The observer, therefore, may at pleasure look obliquely or vertically downwards, or obliquely upwards, as may suit his purpose.

Similar motions are also produced by mounting the instrument upon a single pillar by means either of a cradle joint, such as is generally used for telescope stands, or a ball and socket. Stands of this form are attended with the advantages of offering great facility for moving the instrument horizontally round its axis.

In the attainment of all these objects, as well as in the production of eye pieces and object pieces of capital excellence, the leading makers of London, Paris, Berlin, and Vienna, have honourably rivalled each other, and it may be most truly said, to their credit, that if some have excelled others in particular parts of the instrument, there is not one who has not in some way or other contributed, by invention or contrivance, to the perfection either of the optical or mechanical parts.

Much, however, is also due to the eminent philosophers and professors, who have more especially devoted their attention to those parts of science, in which the microscope is a necessary means of observation, and foremost among these is the patriarch of optical science, Sir David Brewster. It would be difficult to name the part of the instrument, or of its accessories or appendages, for the improvement of which we are not deeply indebted to this eminent man. Among the more recent philosophers who have contributed to the advancement of micrography, and by whose researches and suggestions the makers have been guided, may be mentioned Messrs. Goring, Lister, Coddington, Quecket, Mandl, Dujardin, Le Baillif, Seguier, De la Rue, and numerous others.

The eminent makers of the British and continental capitals are well known. Good instruments of the low priced sort are made by nearly all the opticians; but those who have more especially devoted their labours to the microscope, are Messrs. Ross, Smith, and Beck; Powell and Lealand; Pritchard, Varley, and Pillischer, in London; Messrs. Nachet, Charles Chevalier, and George Oberhauser, of Paris; MM. Ploessel and Schieck, of Vienna; and M. Pistor, of Berlin.

Without the intention of assigning any relative precedence to these artists, we shall now present a brief description of some of the instruments, according as they are severally mounted by them.

493. **Chevalier's universal microscope.** — The mounting of this instrument offers many conveniences and advantages to the observer.

A mahogany case A, *fig.* 234. (p. 356.), containing a drawer B, in which the instrument and its appendages are packed when out of use. serves as its support. A strong brass pillar, C C, is firmly screwed into the top of the case, and upon this pillar the entire instrument is supported.

The pillar C C sometimes is made in two lengths, which are screwed one upon the other, by which means the height of the instrument may be varied at pleasure, either one or both lengths being used.

An arm E *c* is attached by a joint at E to the summit of the pillar C C, so that it can be moved on the joint E with a hinge motion, and may thus be placed at any angle with the pillar C C. In the figure it is represented at right angles with C C.

To the middle D of the arm E *c*, a square brass bar D F G is attached at right angles to E *c*, so that when E *c* is at right angles to C C, the bar D F G is parallel to C C. In the face of the bar D F G, which is presented to C C, a rack is cut.

Two square pieces P and M are fitted to the bar D F G, and are moved at pleasure upwards and downwards upon it by means of pinions, having milled heads O and N.

To the square piece P is attached the stage Z, upon which the object is placed, and maintained in its position by two springs, one of which is shown in the figure. This stage is provided with several adjustments, which have been already explained; it will be sufficient for the present to observe that it is capable of being moved upwards and downwards with the square piece P, to which it is attached, by turning the milled head O, and that a slower motion, to give more exact adjustment, is imparted to it by a fine screw having a milled head at Q.

To the square piece M is attached the illuminator H, on one side, K, of which is a concave reflector, and on the other, I, a smaller plane reflector. This illuminator has two motions, a horizontal or lateral one upon a joint at M, by which it can be placed at pleasure either vertically under the centre of the stage Z, or at a limited distance on one side or other of the vertical through the centre of the stage. The circular illuminator is suspended at two points diametrically opposite in a semicircular piece, and may be placed at any desired inclination to the vertical, and with either reflector upwards by means of the milled head L.

From the lowest part of the pillar C C a piece projects, having a cavity corresponding with the size and form of the bar D F G, into which that bar enters when it is vertical as represented in the figure, and in which it is held by the pin at G.

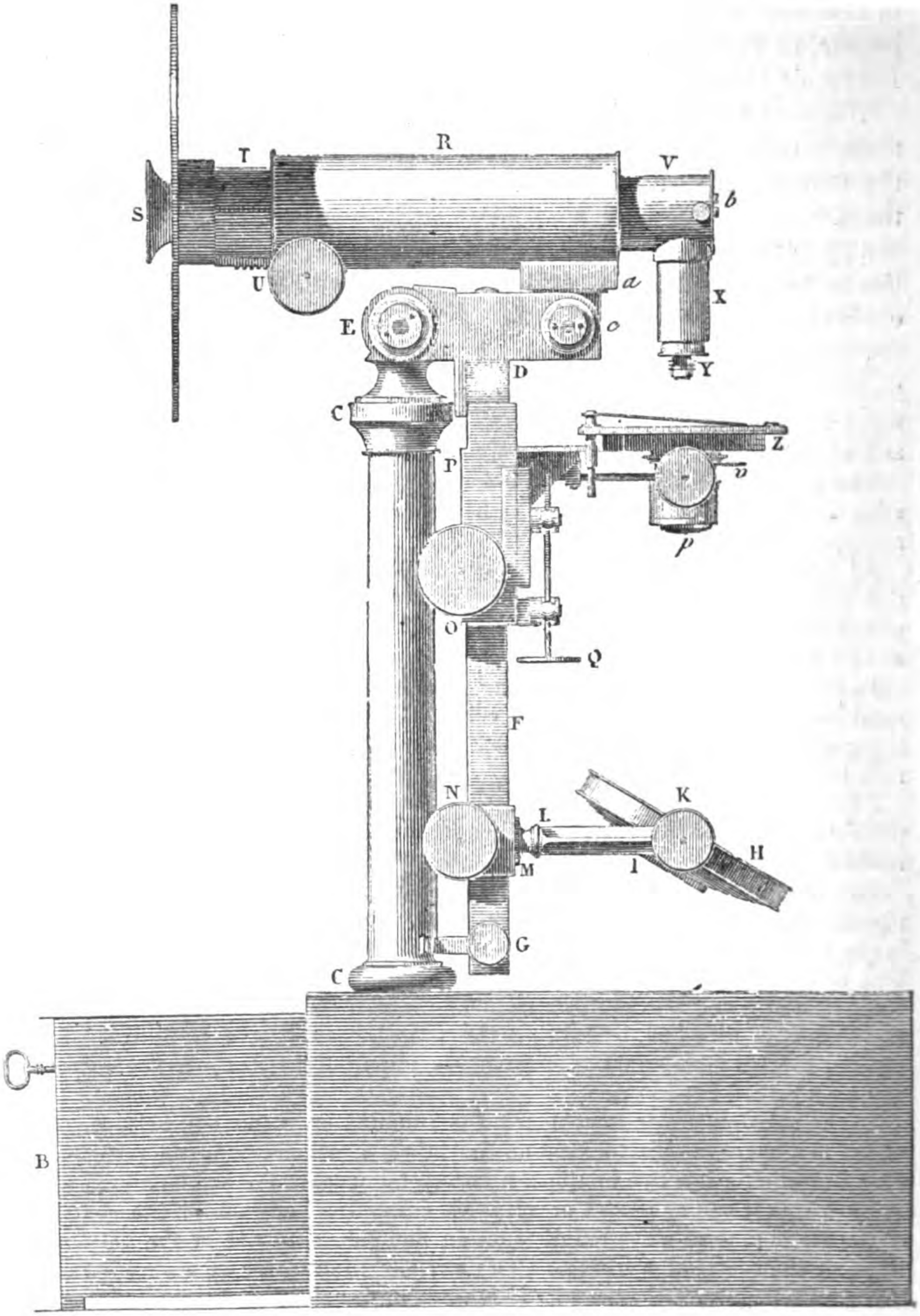


Fig. 234.— Chevalier's Universal Microscope.

The body, R, of the microscope, as shown in the figure, is horizontal. The eye tube T is moved backwards and forwards in the body R by a pinion U working in a rack. The eye piece S is inserted in this tube, and the eye is protected from the light by a circular blackened screen, seen edgeways in the figure. The rectangular tube V X is inserted by a bayonet-joint in the remote end of the body R, in which it is capable of being turned, so that the object tube X shall be horizontal, to enable the observer with greater facility to screw on or to change the object pieces at Y.

The body is attached to the bar E *c* by a joint at *c*, upon which it can be turned, by which means other positions can be given to the instrument.

The position in which the instrument is generally used is that represented in the figure. Various other positions, however, may be given to it. Thus, the object piece may be directed upwards by turning the rectangular piece in the tube of the body, and the stage with its appendages in that case is placed above the object glass. This is convenient when chemical substances are observed, which by evaporation might tarnish the instrument.

By the removal of the rectangular piece V X the object piece may be inserted directly in the tube of the body, so that its axis shall coincide with that of the body. By this arrangement the instrument may be placed with its axis vertical, or inclined to the vertical at any desired angle, by means of the joint E.

494. **Ross's improved microscope.** — Mr. Ross holds a place in the foremost rank of philosophical artists, and deservedly enjoys an European celebrity.

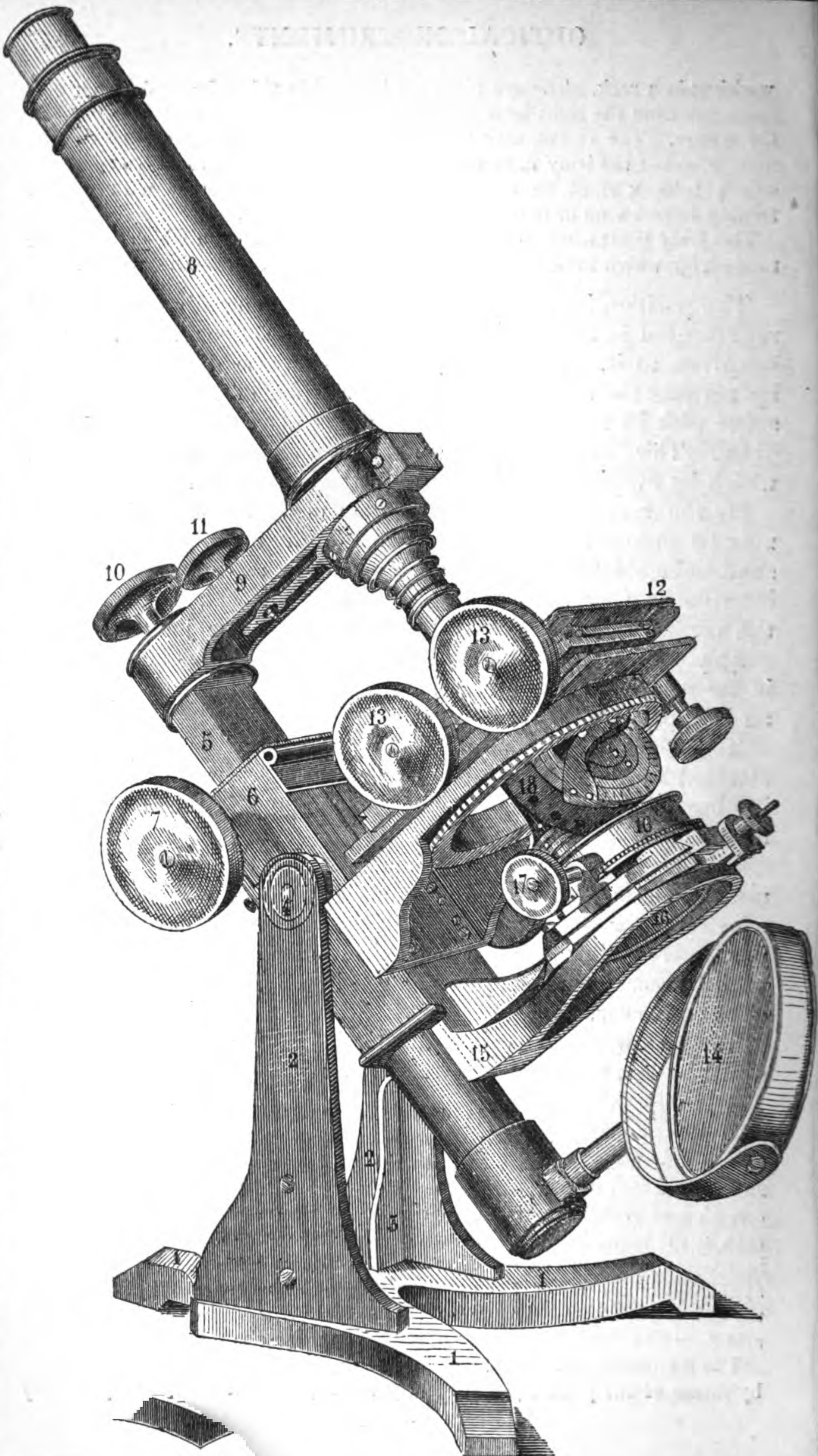
To his labours, perseverance, and genius, much of the perfection attained in the construction of object lenses is due. The adjusting object piece, already described (477.), is one of his recent inventions.

In the progressive improvement which the microscope has undergone in his hands, the stand and the mounting, with the provisions for the arrangement of the accessories, have of course been more or less modified from time to time, and are at present varied according to the price of the instrument and the purposes of the observer.

We shall here give a short description of the most recent form given by him to his best instruments.

Upon a tripod, 1, 1, (*fig.* 235., p. 358.), are erected two upright pieces, 2, 2, strengthened by inside buttresses, 3. These uprights support a horizontal axis, 4, which passes nearly through the centre of gravity of the instrument, and upon which it turns, so that the axis of the body may be placed in any direction, — vertical, horizontal, or oblique. The square bar, 5, having a rack at the back, is moved in the box, 6, by the pinion, 7. The body, 8, is inserted in a ring at the end of the arm, 9, which latter is fixed upon a pin at the end of the rod, 5, upon which it turns, so as to remove at pleasure the object piece from over the stage, to change or clean the lenses. The arm 9, can be fixed in its position by the pin, whose milled head is 10.

The instrument is focussed first by moving the body to and from the stage by means of the pinion, 7, and rack, 5, the adjustment being completed by



a much slower motion imparted to the body by the milled head, 11, which is connected with a screw and lever, by one revolution of which the body is moved through the 300th part of an inch. An elastic play is allowed to the body, so as to guard against injury by the accidental contact of the object piece with the slider.

The usual rectangular motions are imparted to the stage, 12, through the extent of an inch, by the milled heads, 13, which act on pinions by which the racks are driven, which carry the stage right and left, and backward and forward. The illuminating mirror, 14, is supported in the usual way, so as to be placed at any desired angle with the axis of the instrument. Below the stage is fixed an arm, 15, capable of being moved up and down by rack and pinion. This arm supports a tube, 16, intended to receive apparatus to modify the light transmitted by 14 to the object. Various apparatus for condensing and otherwise modifying the illumination are provided, which fit into this tube, 16. A motion of revolution round its axis is given to this tube by the milled head, 17. By these means, the effect of oblique light can be shown on all parts of the object. A condenser, 18, invented by Mr. Gillet, of a peculiar construction, provided with a series of diaphragms formed in a conical ring, is inserted beneath the stage.

Polarising apparatus, and other appendages, can also be attached to the secondary stage.

With his largest and best instruments, Mr. Ross supplies four eye glasses and eight object glasses, by which thirty-two varieties of power and illumination may be obtained. The object glasses vary from 2 inches to a 12th of an inch in focal length, and from 12° to 170° in angular aperture.

495. **Messrs. Smith and Beck's microscopes.** — The largest and most efficient class of instruments constructed by these artists do not differ much in their mounting from those of Mr. Ross, above described. Like the latter, they are supported by a horizontal axis, between two strong vertical pillars, screwed into a tripod base. The instrument with its appendages, turning on the horizontal axis, can thus be placed at any obliquity whatever with the vertical. The coarse adjustment of this microscope is made by a rack and pinion, by which the entire body is moved to and from the stage. The object piece is set in a tube, which moves within the principal tube of the body, the motion being imparted to it by a fine screw with a milled head, which constitutes the fine adjustment. Two different kinds of stage are supplied, one called the lever stage, consisting of three plates of brass, the lowest of which is fixed, and the other two provided with guides and slides, and a lever by which they may be moved, together or separately, in directions at right angles to each other; the other form of stage also has two motions at right angles to each other, one produced by rack and pinion, and the other by a screw whose axis is carried across the stage, and is turned by the left hand, while the rack and pinion is turned by the right hand.

Messrs. Smith and Beck also construct other forms of micro-

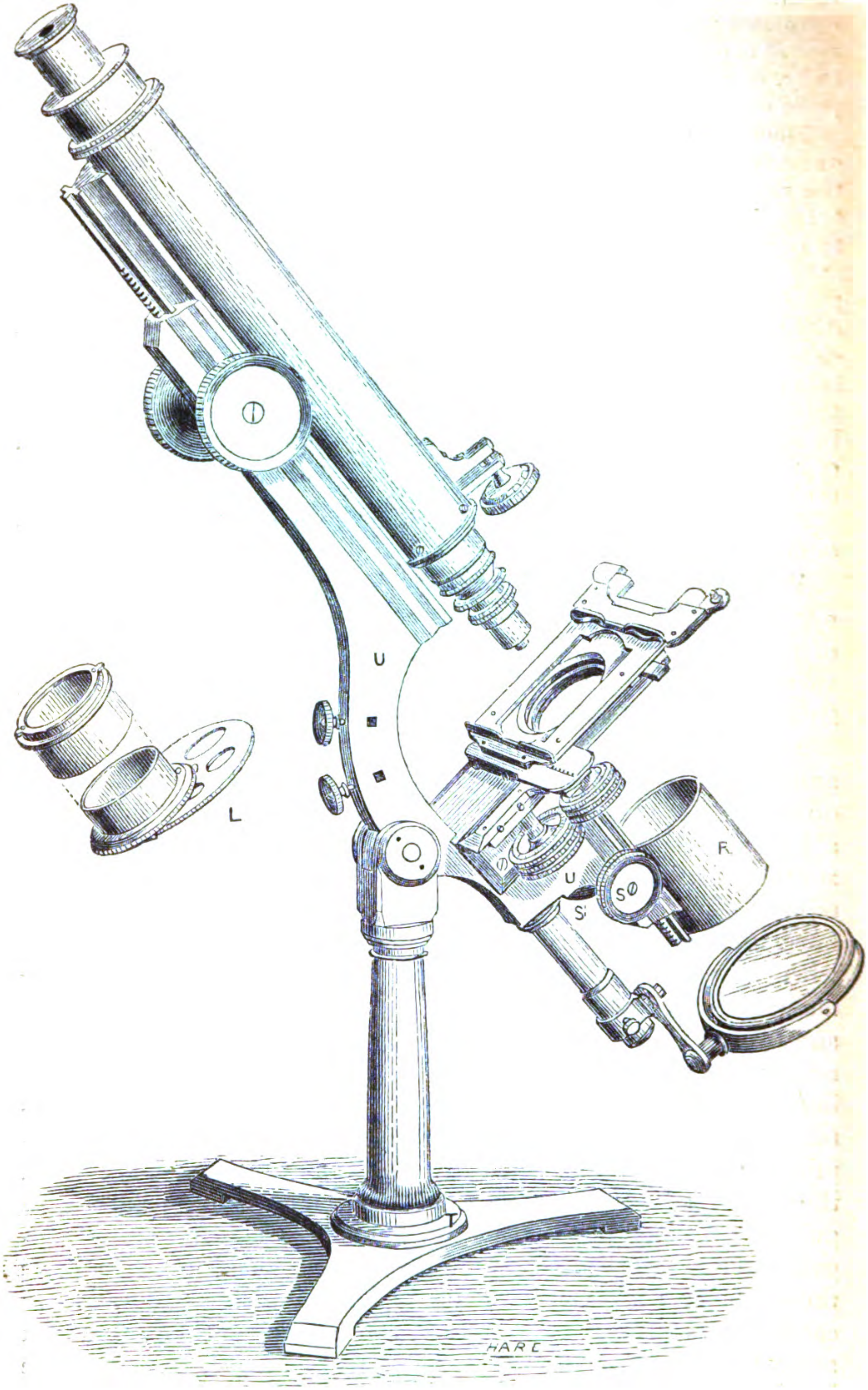


Fig. 236.—**Messrs Smith and Beck's smaller Microscope.**

scope, which, though perfectly efficient, are cheaper and more simple. One of these is represented in *fig.* 236., the details of which will be easily understood after the explanations given above, without further description.

496. **Mr. Varley's microscope.**—This artist has constructed instruments with provisions similar to those already described, but somewhat different in their form and details. He has, however, recently introduced a microscope, which claims the advantage of enabling the observer to examine living objects, such as animalcules, notwithstanding the inconvenience arising from their restless mobility causing them continually to escape from the field of view. The stage motion, with its appendages, contrived by Mr. Varley, enables the observer, without difficulty, to pursue the object.

He has also contrived a phial microscope, by which aquatic plants and animals can be conveniently observed.

497. **M. Nachet's microscopes.** — M. Nachet, of Paris, has acquired an European celebrity for the excellence of his instruments, and for the various inventions and improvements in their construction, by which he has extended their utility. He has constructed instruments in various forms, according to the uses to which they are to be applied and their price. For medical and chemical purposes, the body of the microscope slides in a vertical tube, the coarse adjustment being made by a rack and pinion, and the fine by a screw. The stage is firmly fixed under the object piece, at the top of a hollow cylinder, within which the illuminating apparatus and other appendages are included.

One of the most recent novelties due to this eminent artist, is a form of microscope by which two or more observers may, at the same time, view the same object, thus conferring upon the common microscope a part of the advantages which attend the solar microscope. This is accomplished by connecting two or more tubes, each containing its own eye piece, with a single tube containing an object piece; it has been already shown that the axis of the tube containing the eye piece, may be placed at any desired inclination with that which contains the object piece, by placing in the angle formed by the two tubes, a reflector, or reflecting prism, in such a position, that the pencils of rays proceeding from the object piece shall be reflected to the eye piece, without otherwise deranging them. It is evident, therefore, that if the rays proceeding from the object piece could be at the same time received by two or more reflectors, so placed as to reflect them in two or more directions, they might be transmitted along two or more tubes in these directions to two or more eye pieces, through which the same object might thus be viewed at the same time, and through the same object piece by two or more different observers.

A double instrument of this description is shown in *fig.* 237., where A is the object piece directed vertically downwards on the stage; above it is a case, containing a triangular prism which is so formed that the light reflected from its left side shall pass along the axis of the right-hand tube, and that reflected from its right side along the axis of the left-hand tube. Observers looking into eye glasses set in these tubes, would therefore both see the same object in precisely the same manner.

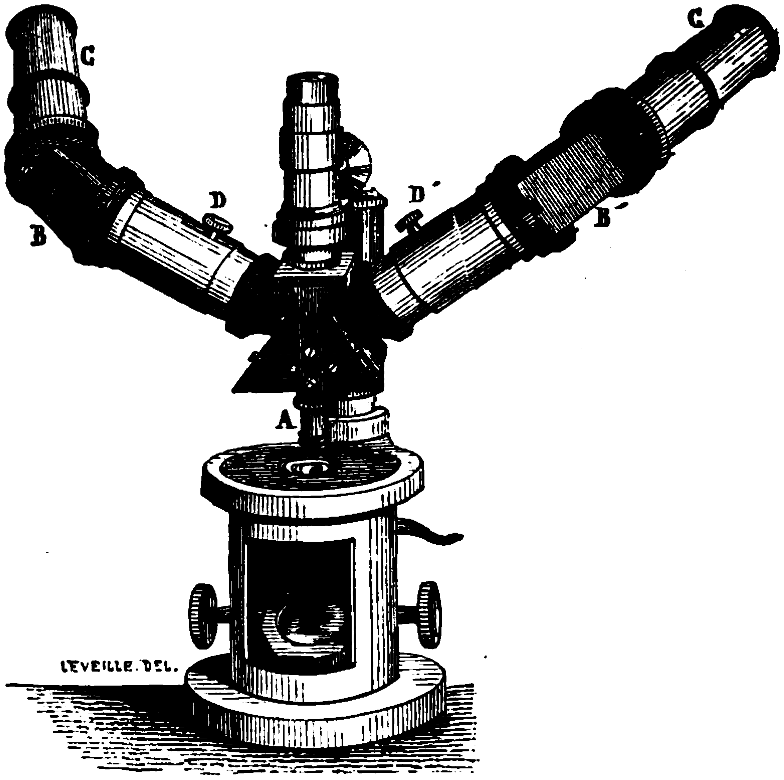


Fig. 237.—NACHET'S DOUBLE MICROSCOPE.

It may perhaps be objected, that the focus which would suit the eye of one observer, would not suit the other; the difference, however, between the focal adjustments of different eyes is always so inconsiderable, that it can be equalised by a small motion given to the tubes carrying the eye pieces.

Microscopes, as they are usually mounted, reverse the objects, the top appearing at the bottom, the right at the left, and *vice versâ*. This being found inconvenient in instruments used for dissection, where the motion of the hand and the scalpel of the operator would be reversed, expedients are provided by which the image is redressed, and the object viewed in its natural position. This is accomplished in the microscope represented in *fig.* 237., by two prisms fixed at B B′ in the tubes, which are placed at right angles to the lower prism A; by this second reflection, the reversed image of the first reflection, being again reversed, is made to correspond with the natural position of the object.

498. Nachet's binocular and stereoscopic microscopes.—An interesting variety of this form of instrument, which may be

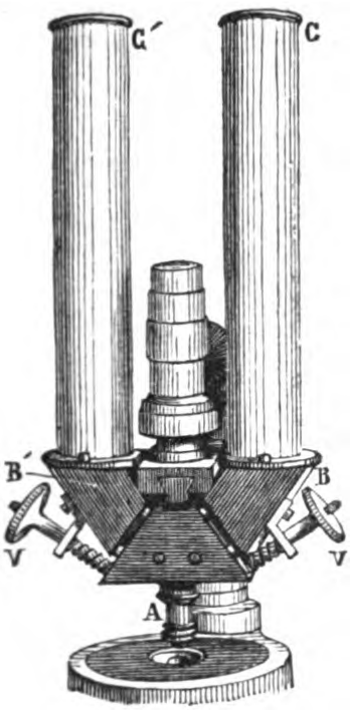


Fig. 238.

called a *binocular microscope*, is shown in *fig*. 238. In this case the two tubes, B C and B′ C′, containing the two eye pieces, are placed parallel to each other, the distance between them being regulated by the screws V V; if this distance be so adjusted as to correspond with the distance between the eyes of the same individual, the microscope may be used with both eyes, in the same manner as a double opera glass. This has the advantage of giving a stronger appearance of relief to the objects viewed, which is especially desirable for a certain class of objects, such as crystals.

The same maker has recently constructed microscopes of a like form, having the properties of the stereoscope.

499. **Triple microscope.** — A triple microscope, upon the principle above described, is shown in *fig*. 239., where A is the object piece, B the multiple prism, and C, C′, and C″ the three eye tubes.

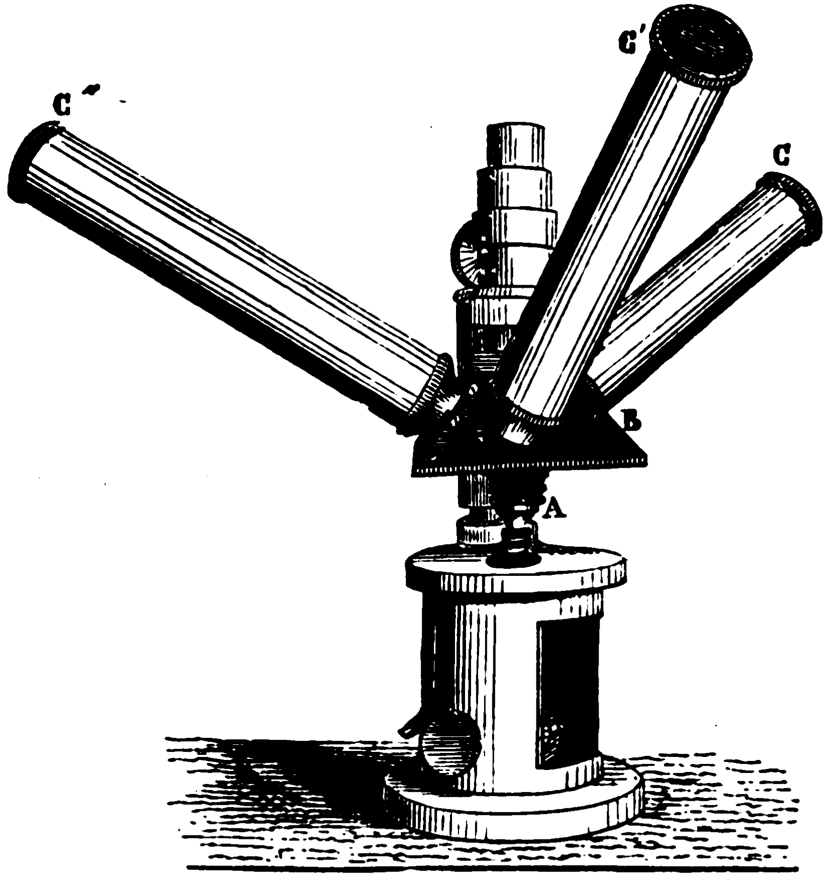


Fig. 239.

500. **Quadruple microscope.** — A similar instrument, with four eye tubes, including figures to illustrate the mode of observing with it, is shown in *fig.* 240.

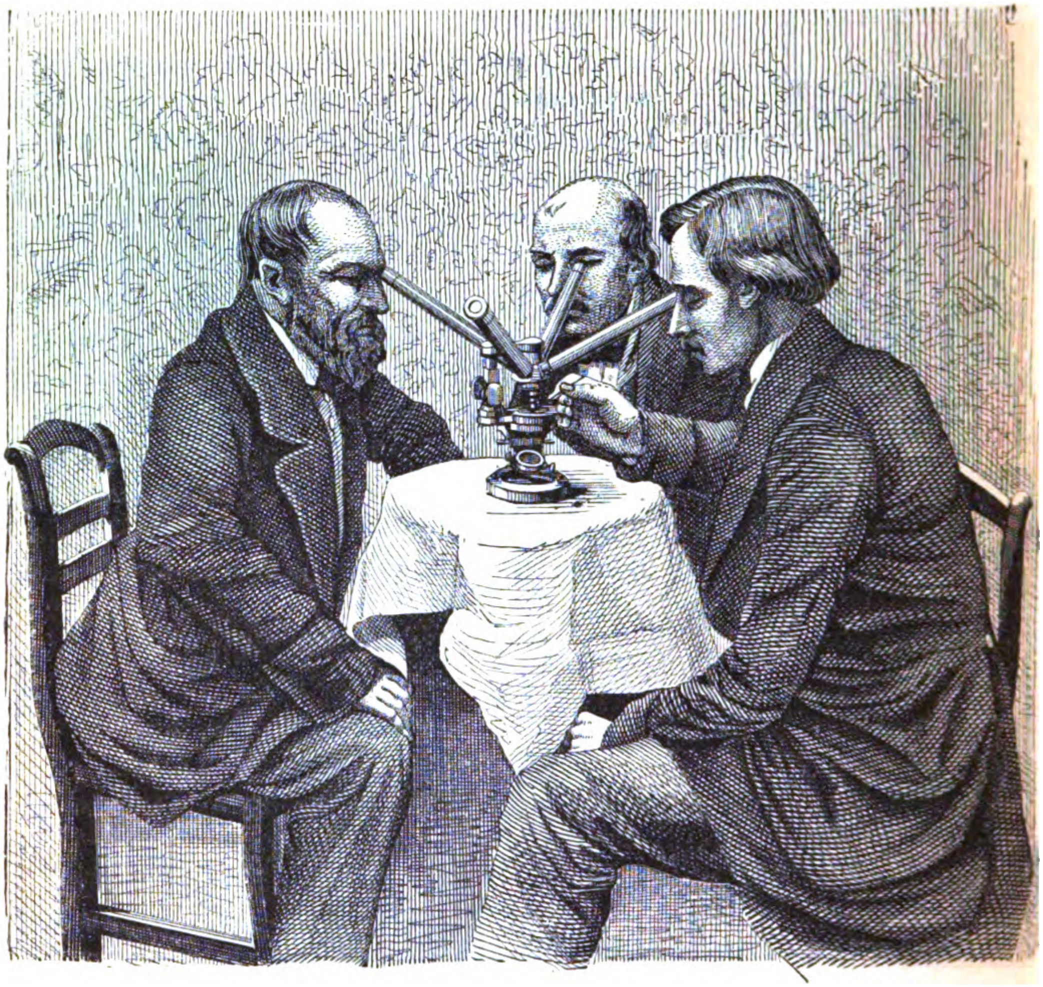


Fig. 240.

One of the advantages of this class of instruments is, that a professor and one or more of his pupils may view the process of a microscopic dissection which with a common microscope would be impossible, and to which the solar microscope would be inapplicable. Microscopic dissections, in general, can only be exhibited, to those who do not execute them, by their ultimate results. Any phenomena which are developed in their progress, can only be made known to others by description; and it is not necessary to say how imperfect such a mode of communication must be, compared with direct observation.

V. THE TELESCOPE.

501. **Principle of the instrument.** — The telescope is an instrument by means of which an object is viewed distinctly which

cannot be so viewed by the naked eye, by reason of its distance. The term is derived from two Greek words, τηλε (télé), *at a distance*, and σκοπεω, *I view*.

Its principle is identical with that of the compound microscope. An optical image of the object to be viewed is produced by means of a concave reflector, or a converging lens; and this image is then submitted to observation with a microscope composed of one or more converging lenses.

Telescopes consist, therefore, of two classes, Reflectors and Refractors; the image being produced in the former class by concave reflectors, and in the latter by lenses.

502. **The Gregorian reflecting telescope.**—A longitudinal section of this instrument is represented in *fig.* 241. A B is a

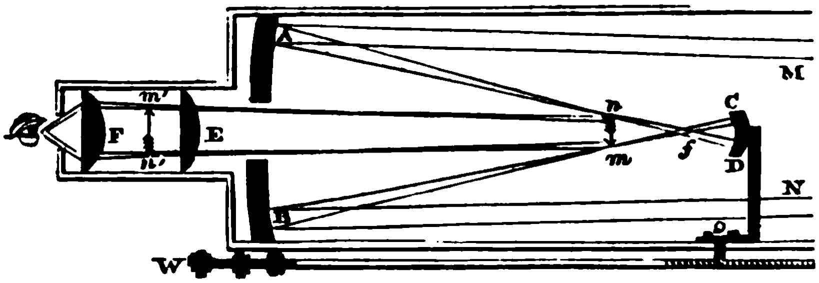


Fig. 241.

large concave speculum formed of an alloy of metals adapted to receive a high polish. A circular aperture is made in the centre, so that the reflecting portion of the speculum is that part only which is outside the circular aperture. A second concave speculum C D is placed with its concavity in the other direction, at a distance from A B greater than the focal length of the great speculum. The eye piece F is placed in a smaller tube inserted in the greater one opposite the opening of the great speculum.

The extremity of the great tube being open, and presented towards the object of observation, an inverted image of this object is formed at *m n* in the principal focus of the great speculum A B. This image forms an object for the small speculum C D, and another image is formed in the conjugate focus *m' n'*; this latter image being inverted with respect to *m n*, and therefore erect with respect to the object.

The pencils proceeding from C D are sometimes brought to a focus by the interposition of a converging lens E, called a field glass (470.), but this is not necessary.

The image *m' n'* is viewed by the eye glass F, which, as already explained, may be considered as a simple microscope.

The telescope is mounted with proper apparatus, by which it can be directed to the object, and by which its focus can be regulated.

503. **Cassegrain's reflecting telescope.** — A longitudinal section of this instrument is given in *fig.* 242. Its details are in all respects similar to the Gregorian reflector, except that the second speculum C D is convex, instead of being concave, and receives the pencils proceeding from A B before they come to a focus. It turns them back towards the eye piece, where an image is formed, as in the former case.

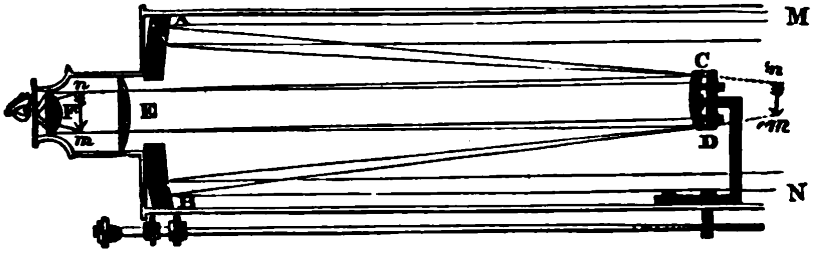


Fig. 242.

504. **Newtonian reflecting telescope.** — A longitudinal section of this instrument is represented in *fig.* 243., where A B is the great speculum which would form an image of the object at *m n* in its principal focus. But the pencils, before they arrive at that point, being received upon a plane reflector C D, placed at an angle of 45° with the axis of the telescope, the image is formed at *m′ n′* in a lateral tube inserted in the great tube, where it is viewed by an eye piece, as before explained. In this case the open end A of the great tube is directed towards the object, and the observer examines the object by looking in at the side of the telescope, in a direction at right angles to its length.

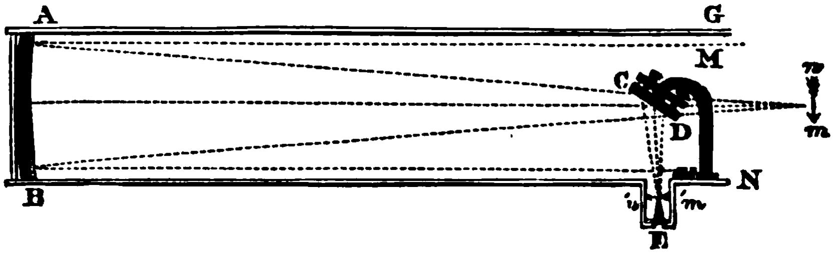


Fig. 243.

In all these cases, the central rays of the pencils directed upon the great speculum are lost. In the Gregorian and Cassegrain, the central portion of the speculum is removed, and in the Newtonian telescope the central rays are intercepted by the plane reflector C D.

505. **Herschel's telescope.** — The form of reflecting telescope which, until a recent epoch, had attained by far the greatest cele-

brity of any that had been constructed, is that which was erected by Sir W. Herschel, and used by him with such signal success, as to render his name memorable in the history of astronomical science. Herschel, after having constructed a great number of reflecting telescopes on the Newtonian principle, varying from seven to twenty feet in length, aided by the patronage of George III., completed in 1789 his celebrated telescope, forty feet in length, by which, on the very day it was completed, he discovered the sixth satellite of Saturn. The great speculum of this telescope measured nearly fifty inches in diameter, its thickness being three inches and a half, and its weight about half a ton. The open end of the telescope being directed to the point of the heavens under observation, and the speculum being fixed at its lower end, the observer is suspended in a chair, so as to be able to look over the lowest part of the edge of the opening. The speculum being a little inclined to the axis of the tube, the image is formed near the lowest point of the edge of the opening, where it is viewed by the observer with proper eye pieces.

The quantity of light obtained by this prodigious speculum, enabled Sir W. Herschel to use magnifying powers which greatly exceeded any which before his time had been applied. He was thus enabled, in examining the fixed stars, to apply in some cases a magnifying power of 6450.

This instrument is represented in *fig.* 244. (p. 368.).

The instrument is mounted on a platform which revolves in azimuth on a series of rollers. The telescope is placed between four ladders, which serve the double purpose of a framework for its support and a convenient means of approaching the superior end of the great tube. These ladders are united at the top by being bolted to a cross bar, to which the pulleys are attached. By one system of pulleys the telescope is raised or lowered; and by another the gallery or balcony in which the observer stands is also raised or lowered, so as to enable him to look into the tube. These pulleys are each worked by a windlass established on the platform below. The framing is strengthened by another system of diagonal ladders, as well as various masts and braces, which appear in the figure. The telescope is so mounted that it can be raised until its axis is vertical, so that an object in the zenith can be observed with it. The observer's gallery rests in grooves upon the ladders, and slides up and down easily and smoothly by the operation of the pulley, so that when the telescope tube is elevated, even to the zenith, the observer can ascend and descend at pleasure, by signals given to the man at the windlass. A small staircase is placed near the foot of one of the principal ladders, by which

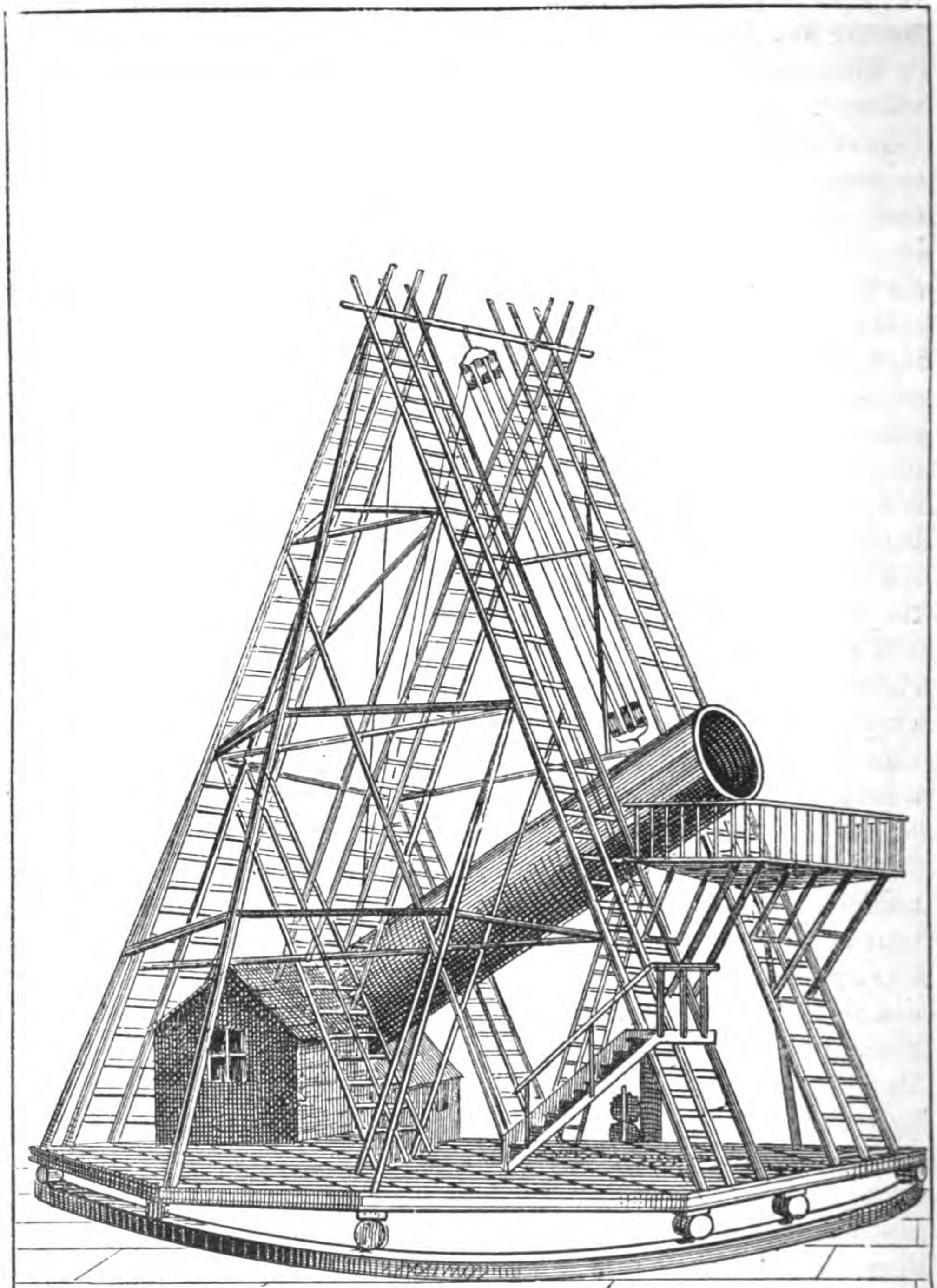


Fig. 244.—The Herschelian Forty-Foot Telescope.

observers can mount into the gallery ~~when it~~ is let down to its lowest point.

The total length of the telescope tube is 39 ft. 4 in., and its clear diameter 4 ft. 10 in. It is constructed entirely of iron. The great speculum is placed in the lower end of the tube, the apparatus for adjusting it being protected by the wooden structure which appears in the figure. The diameter of the speculum is 4 ft., and the magnitude of its reflecting surface is consequently 12·566 square feet. It contains 1050 lbs. of metal.

The axis of the speculum is so inclined to that of the tube, that its focus is at about two inches from the lower edge of the upper mouth of the tube, so that the observer, standing in the gallery with his back to the object, and looking over the edge of the tube towards the speculum, can direct an eye piece, conveniently mounted at that point, upon the image of the object of observation formed by reflection in the focus.

Three persons are employed in conducting the observations: the observer, who stands in the gallery; his amanuensis, who may either be in the gallery or in the wooden house below, receiving the dictation of the observer by a speaking tube; and the person who works the windlass.

506. **The lesser Rosse telescope.**—This instrument, with its mounting, is represented in *fig.* 245. The arrangements are so similar to those of the Herschelian instrument described above, that they will be easily understood from the plate without further description. The speculum is of 3 feet aperture, and 7·0686 square feet reflecting surface. The length of the telescope is 27 feet. It is erected upon the pleasure grounds at Parsonstown Castle, the seat of its illustrious constructor. The weight of metal in the speculum is about 13 cwt.

507. **The greater Rosse telescope.**—This stupendous instrument of celestial investigation, by far the largest and most powerful ever constructed, is represented in *figs.* 246. and 247., from drawings made for this work under the superintendence of his lordship himself. *Fig.* 247. presents a south, and *fig.* 246. a north view of the instrument.

The clear aperture is 6 ft., and consequently the magnitude of the reflecting surface is 28·274 square feet, being greater than that of Herschel's great telescope in the ratio of 7 to 3.

The instrument is at present used as a Newtonian telescope (504.); that is to say, the rays proceeding along the axis of the great speculum are received at an angle of 45° upon a second small speculum, by which the focus is thrown towards the side of the tube where the eye piece is directed upon them. Provision is,

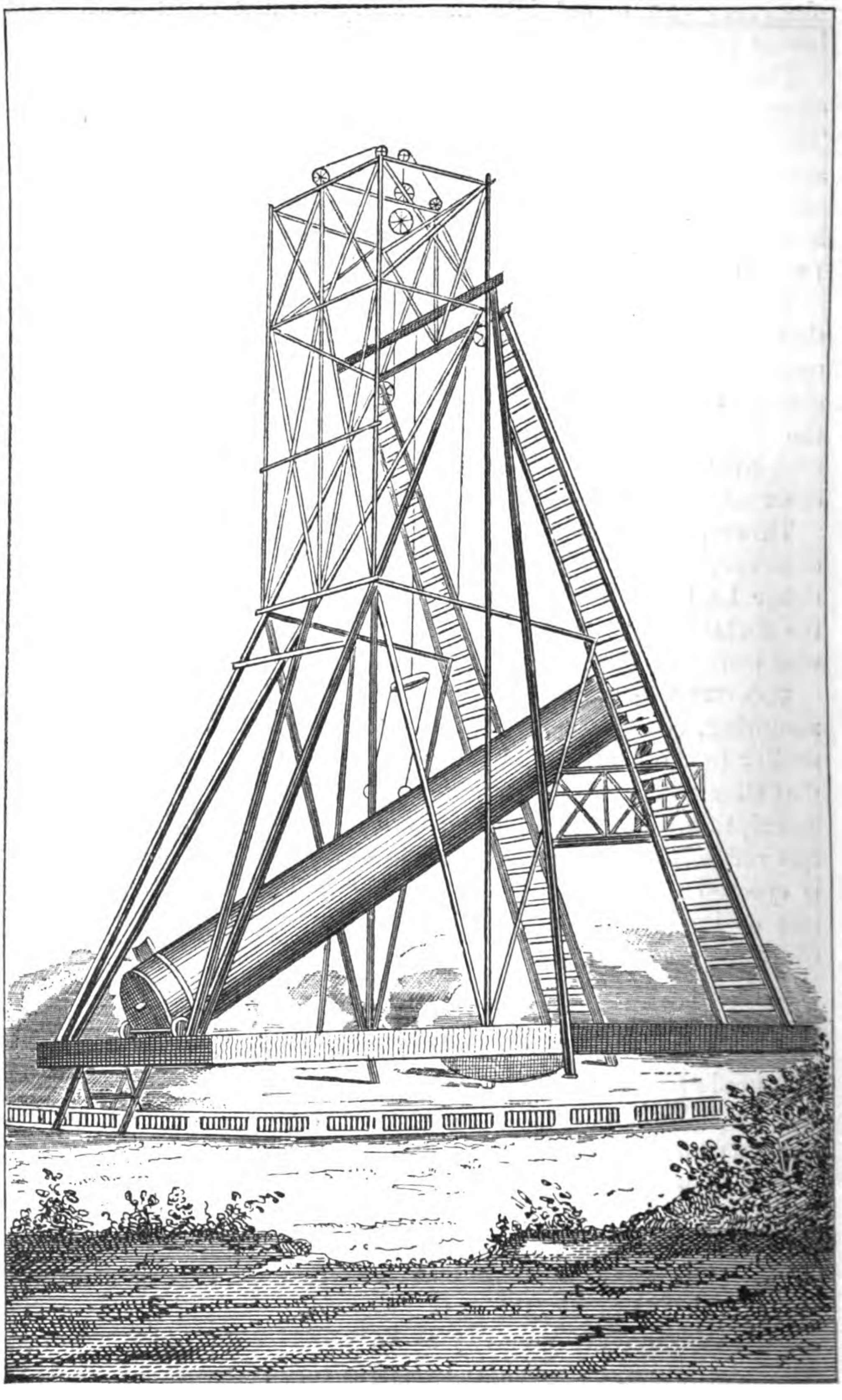


Fig. 245.—THE LESSER ROSSE TELESCOPE. FOCAL LENGTH, 27 FEET; APERTURE, 3 FEET.

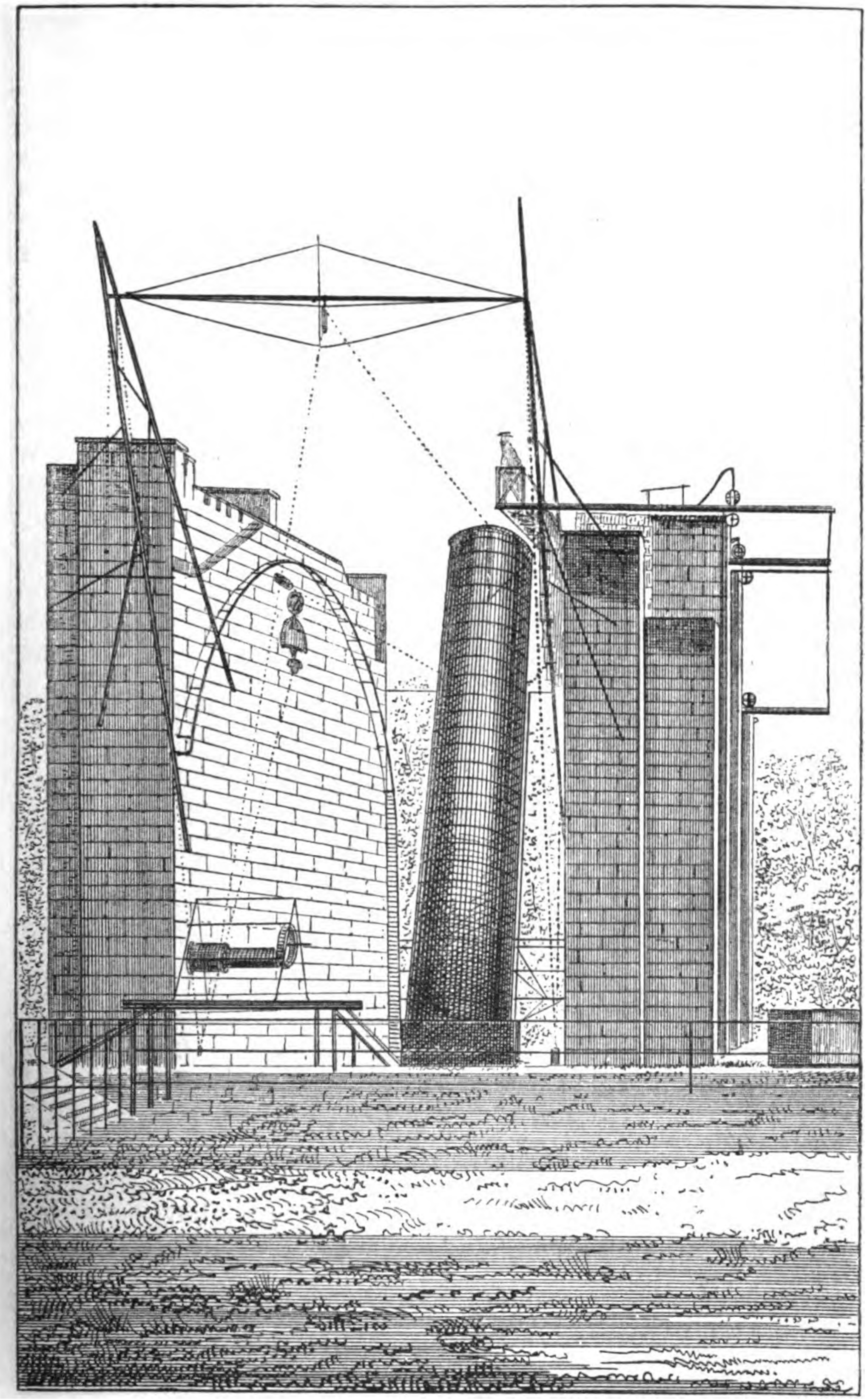


Fig. 246.—The Great Rosse Telescope. North End. Focal Length, 53 feet; Aperture, 6 feet.

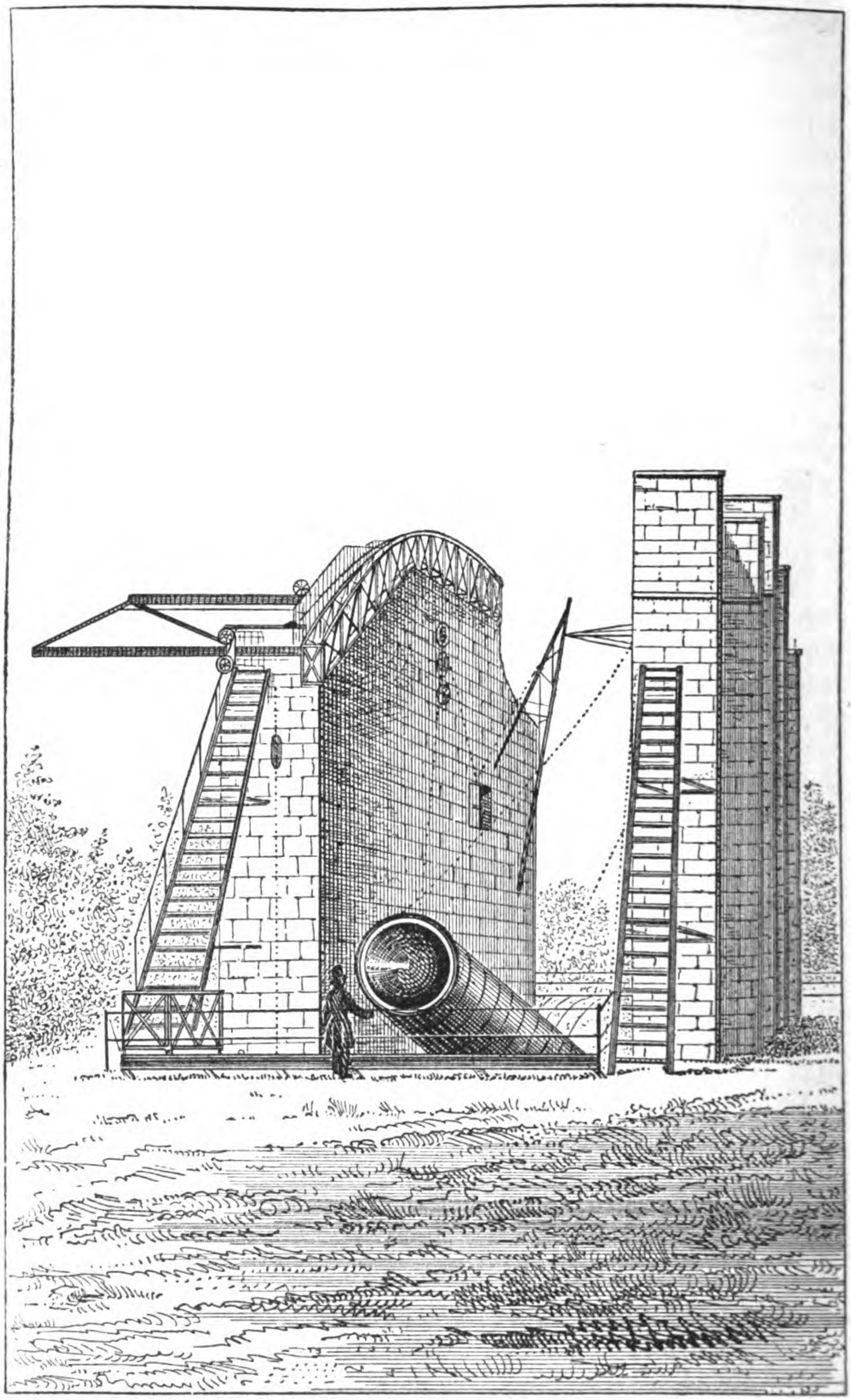


Fig. 247.—Great Rosse Telescope. South End.

however, made to use the instrument also as an Herschelian telescope.

The great tube is supported at the lower end upon a massive universal joint of cast iron, resting on a pier of stone work buried in the ground, and is so counterpoised as to be moved with great ease in declination. In all such instruments, when it is required to direct them to an object, they are first brought to the desired direction by some expedient capable of moving them more rapidly, and they are afterwards brought exactly upon the object by a slower and more delicate motion. In this case, the quick motion is given by a windlass, worked upon the ground by an assistant at the command of the observer. The slow motion is imparted by a mechanism placed under the hand of the observer.

The extreme range of the telescope in right ascension, when directed to the equator, is 1 hour in time, or 15° in space; but when directed to higher declinations, its range is more extensive.

The tube is slung entirely by chains, and is perfectly steady, even in a gale of wind.

When presented to the south, the tube can be lowered until it is nearly horizontal; towards the north it can only be depressed to the altitude of the pole. The apparatus of suspension is so arranged that the instrument may be worked as an equatorial, and it is even intended to apply a clockwork mechanism to it.

The horizontal axis of the great universal joint, by which the lower end is supported, carries an index pointing to polar distance, and playing on a graduated arc of 6 feet radius. By this means the telescope is easily set in polar distance. The same object is also attained, and with greater precision, by a 20-inch circle attached to the instrument.

Two specula have been provided for the telescope, one of which contains 3½, and the other 4 tons of metal, the composition of which is 126 parts by weight of copper to 57¼ of tin.

The great tube is of wood, hooped with iron, and is 7 feet in diameter, and 52 in length. The side walls, 12 feet distant from the tube, are 72 feet in length, 48 feet in height on the outside, and 56 feet on the inside. These walls are built in the plane of the meridian.

A strong semicircle of cast iron, about 85 feet in diameter, is firmly bolted to the inside face of the eastern wall, and is seen in *fig.* 246.; the telescope being connected with this circle by a strong racked bar, furnished with friction rollers attached to the tube, so that the observer with a handle near the eye piece can move it on either side of the meridian to the distance of about 7½°, or half an hour of right ascension, on either side of an equatorial star.

The stairs and galleries for the observers are supported by the western pier. The first gallery commands a view of objects at an altitude of 42°; it consists of a strong, light, prismatic framing, sliding between two ladders attached to the southern ends of the piers. It is counterpoised, and can be raised to any required position by a windlass. This gallery appears on the ground in *fig.* 247., between the two ladders, and the windlass by which it is elevated is shown in *fig.* 246. Three other galleries are provided at the summit of the western pier, which command the heavens to 5° below the pole; each of these are supported by two beams, which run between grooved wheels, and are drawn forward by an elegant piece of mechanism. These galleries hold twelve persons.

"I have enjoyed," says Sir David Brewster, "the great privilege of seeing this noble instrument, one of the most wonderful combinations of art and science that the world has yet seen. I have in the morning walked again and again, and ever with new delight, along its mystic tube, and in the evening, with its distinguished inventor, pondered over the marvellous sights which it discloses: the satellites, and belts, and rings of Saturn, — the old and new ring, which is advancing with its crest of waters to the body of the planet, — the rocks, and mountains, and valleys, and extinct volcanos of the moon,—the crescent of Venus, with its mountainous outline, — the systems of double and triple stars, — the nebulæ and clusters of stars of every variety of shape, and those spiral nebular formations which baffle human comprehension, and constitute the greatest achievement in modern discovery." *

508. **Lassells' telescope.** — This is a reflector, the speculum of which has a clear diameter of two feet, with twenty feet focal length. The speculum metal is an alloy of copper and tin, with a small proportion of white arsenic. Mr. Lassells uses sometimes a small two-inch speculum, and sometimes a prism, to deflect the image towards the eye glass. The deposition of dew upon the prism is prevented by attaching to it a case containing a small piece of heated lead. The telescope is erected under a revolving cupola of thirty feet in diameter, which carries a stage for the observer. With this instrument, which is the work of Mr. Lassells himself, he has discovered four members of the solar system; two satellites of Uranus, one of Saturn, and one of Neptune.

The instrument was originally erected at Mr. Lassells' residence near Liverpool. In the latter part of 1852 he removed it to Malta, to obtain the advantages of a finer climate and a lower latitude.

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* Brewster's "Optics," p. 499.

509. **Nasmyth's telescope.** — This instrument, invented by Mr. James Nasmyth, is a combination of the reflecting telescopes of Cassegrain and Newton. The rays reflected from the great speculum are received either upon a small speculum or prism placed in the axis of the tube, between the focus and the great speculum. By this they are reflected at right angles, and the image is formed in a tube inserted in one of the trunnions upon which the instrument turns. The image is then viewed in the usual way by an eye piece. The advantage of this arrangement is, that while the great tube is moved in altitude, the lateral tube in the trunnion is fixed. The observer can, therefore, survey the whole meridian, or any other vertical circle, without changing his position.

The instrument is moved in azimuth by means of a turn-table, like those used on railways for turning locomotive engines. The frame supporting the instrument, and the seat of the observer, are established upon a circular platform, which forms the upper surface of this turn-table, and by a simple and easy operation any desired azimuth can be given to it.

Every requisite motion, both in altitude and azimuth, can be imparted to the tube by the observer himself.

The length of the tube is twenty-eight feet, and its diameter fifty-four inches.

510. **The Galilean telescope. — Opera glass.** — This telescope, which takes its name from Galileo, by whom it was first used, is a refracting telescope, the principle of which is represented in *fig.* 248. A B is the object glass, in the principal focus of which, E, an inverted object of the image would be formed; but before the pencils arrive at this point, they are received by a divergent lens C D, which, destroying their convergence, causes them to enter the eye parallel, as they would if they proceeded from an object at a considerable distance.

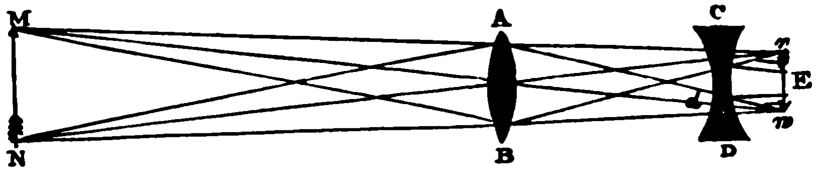

Fig. 248.

The general direction of the axes of the pencils, however, is not changed, and the eye consequently receives them as if they had proceeded from an object at the same distance from the eye as the image *m n* is from the eye glass C D. The apparent magnitude, therefore, of the object as seen with the eye glass C D, is measured by the angle which the image *m n* subtends at the centre of the lens C D; and the apparent magnitude of the object is equal to the angle which the same image subtends at the centre of the object glass A B.

If, therefore, we divide the focal length of the object glass by the distance of the eye glass from the image, we shall then obtain the magnifying power.

Let us suppose, for example, that the focal length of the object glass is fifty inches, that the focal length of the eye glass is one inch, and that the eye of the observer is adapted to the reception of parallel rays. In this case, the focal length of the object glass will be fifty times the distance of the eye glass from the image, and the telescope will magnify accordingly fifty times. But if the eye of the observer be adapted to the reception of diverging rays, then the eye glass C D must be removed further from the image than its focal length, and, consequently, the magnifying power will be less than it would be for an eye adapted to parallel rays; and if, on the contrary, the eye of the observer be adapted to converging rays, the eye glass must be moved near to the image, and the magnifying power will be greater.

In all cases, the distance of the eye glass from the object glass is equal to the difference between their focal lengths, for eyes adapted to parallel rays. It is a little less for short-sighted, and a little more for long-sighted eyes.

This form of telescope has long been disused for all purposes where very distant objects are observed. It is, however, still continued with great convenience where the objects of observation are nearer, as in the case of opera glasses, which are nothing more than Galilean telescopes.

These instruments have lately been mounted in pairs, so as to enable the spectator to use both his eyes, as with spectacles.

511. **The astronomical telescope.** — This is the name given to a refracting telescope, consisting of two convergent lenses, one used as an object lens, to form an image of the object to be observed, and the other as a simple microscope, to examine this image. The principle of this instrument has been already sufficiently explained in the case of the compound microscope, from which it differs in nothing but in the proportion of its parts. A B, *fig.* 249., is the object glass; an inverted image *m n* of the object M N is formed at its focus.

This image is viewed by the eye piece C D, which for eyes adapted to parallel rays is placed at a distance from *m n* equal to its focal length. The image *m n* is seen under an angle equal to that which it subtends at the centre of the eye glass C D, and its apparent magnitude being equal to the angle which it subtends at the centre of the object glass A B, it follows that the magnifying power of the instrument is found, by dividing the focal length of the object glass by the focal length of the eye glass. The image, as seen in this instrument, is always inverted with respect to the object; but as it is used for astronomical purposes, this is unimportant.

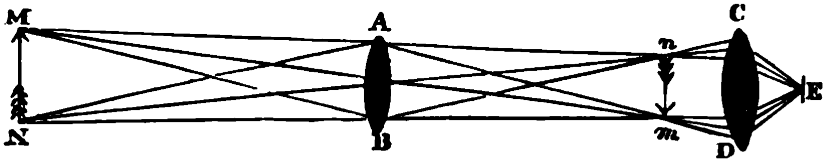

Fig. 249.

512. **Terrestrial telescope.**—When the telescope described above is applied to terrestrial objects, it exhibits them inverted. This is corrected by interposing between the eye and the image other lenses, by which a second image is formed, inverted with respect to the first, and therefore erect with respect to the object. This arrangement is represented in *fig.* 250., where A B is the object and *m n* the first image which is inverted. A convergent lens C D is placed before this image, at a distance equal to its focal length; consequently, the pencils proceeding from *m n*, after passing through C D, will emerge with their rays parallel. These pencils are received by another converging lens of equal focal length E F, by which they are again rendered convergent, and are made to form the image *m′ n′*, which is inverted with respect to *m n*, and erect with respect to the object. This image *m′ n′* is viewed by the eye glass G H in the usual manner.

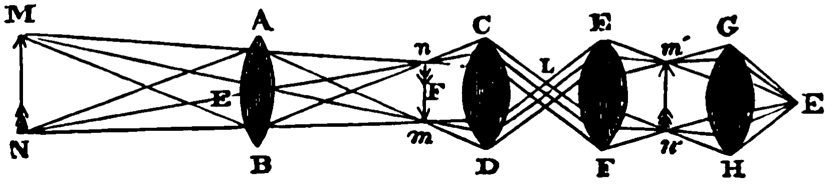

Fig. 250.

513. **Eye pieces.**—In the preceding exposition of the principle of the telescope it has been assumed, provisionally, that the optical image of the object produced, in the case of reflectors by the speculum, and in the case of refractors by the object glass, is examined by means of a single converging lens. Such a method is always practicable, but much greater distinctness of definition and freedom from aberration is produced, by using eye pieces composed of two plano-convex lenses.

Such eye pieces are of two kinds. One in which the image is placed beyond the two lenses, and their effect is therefore, by their combined action upon the pencils, to render them parallel: these are called *positive* eye pieces, and having been adopted by the celebrated Ramsden in the telescopes constructed by him, are sometimes designated by his name.

In the other class of eye pieces the lens which is more distant from the eye receives the rays converging from the object glass before they form an image, and, by increasing their convergence, the image is formed between the two lenses of the eye piece, and is viewed through the lens next the eye as a simple converging lens. This form is called the *negative* eye piece, and having been first adopted by Huyghens in his telescopes, is sometimes designated by his name.

514. **Positive eye piece.** — A positive eye piece, drawn upon a full scale, is shown in *fig.* 251., where L L and L′ L′ are the two

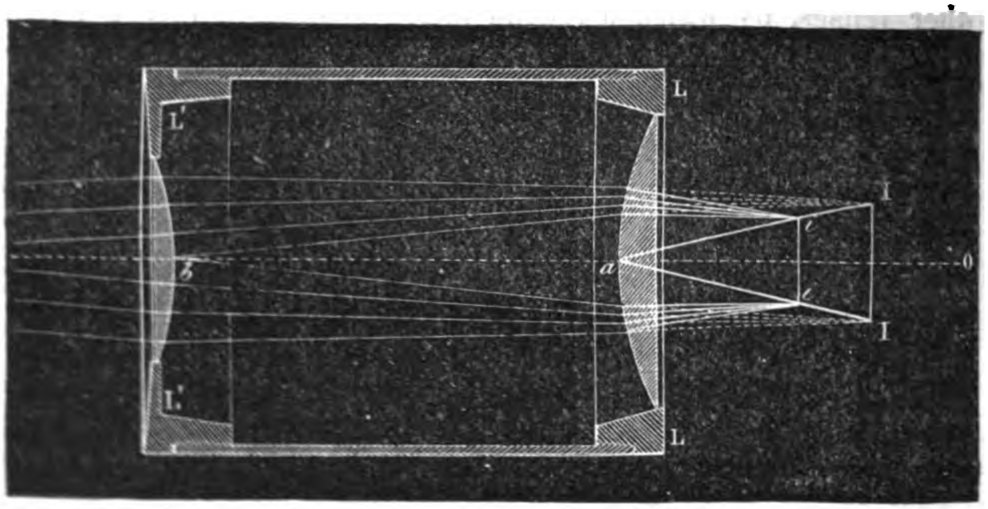

Fig. 251.

plano-convex lenses; the plane side of the latter being turned towards the eye, and that of the former towards the object glass. Their convexities are consequently turned towards each other. Let o a b be the axis of the telescope, and let *i i* be the image produced by the object glass at a distance from it equal to its focal length. The pencils of rays which diverge from *i i* will, after passing through the lens L L, be rendered less divergent, so that an imaginary image I I will be produced at a greater distance from L L than *i i*, and this image will be viewed by the eye glass L′ L′. The effect of the lens L L will be to bring rays upon L′ L′, which would otherwise pass beyond its edges, by which means it enables the observer to comprehend within his view a greater extent of the object. It is in this sense that the lens L L is said to augment the field of view, and is therefore [illegible] the *field* *glass*; the lens L′ L′ being called the *eye glass*, and their combination the *eye piece*.

It is evident from what has been stated that the distance of the field glass from the object glass in such an eye piece is greater than the focal length of the latter.

515. **Negative eye piece.**—A section of a negative eye piece drawn on a full scale is represented in *fig.* 252.

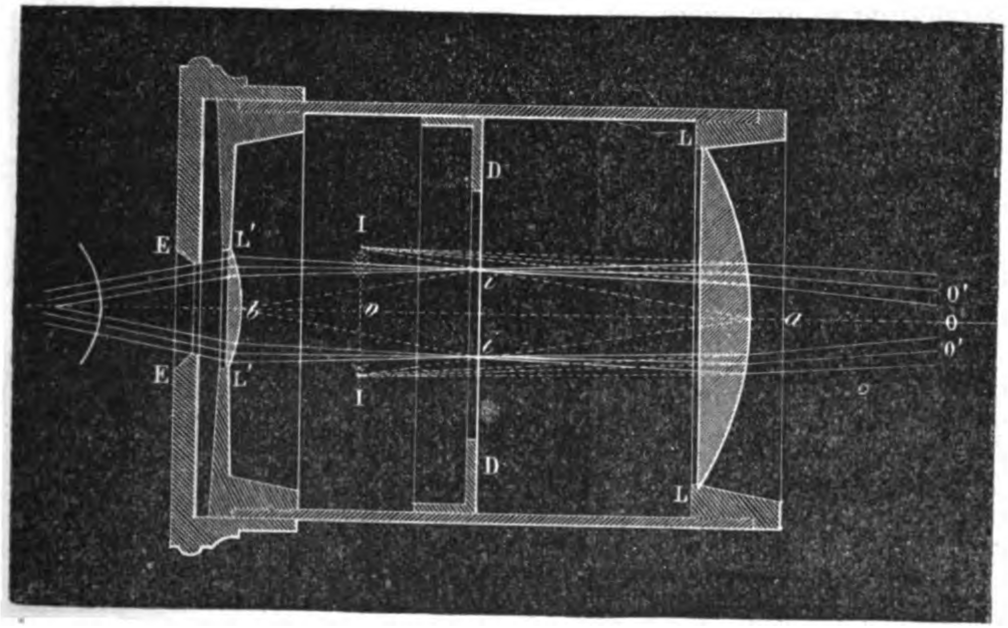

Fig. 252.

In this case the plane sides of both lenses are presented to the eye. The field glass L L, placed at a distance from the object glass less than the focal length of the latter, receives the pencils of rays O′ O′ before they form an image. Let I I be the image which the object would have produced, if the rays had not been intercepted by L L. This image will now be brought back and formed at *i i* nearer to L L, and will have somewhat less dimensions. From this change of position and magnitude a greater number of pencils proceeding from it will pass through the eye glass L′ L′; and the lens L L, having thus as before the effect of enlarging the field of view, is still called the field glass.

Negative eye pieces have been generally adopted by preference by the most eminent continental opticians, and are constructed in general in the manner shown in the figure. A diaphragm, D D, is interposed between the lenses at the point where the image formed by the field lens is produced, and another diaphragm, E E, is placed in front of the eye glass.

516. **Power of eye pieces.**—By means of the formulæ which have been already explained for the determination of the foci of

lenses, it will be easy to determine all the circumstances attending the application of eye pieces, whether positive or negative, when the curvatures of the lenses, their apertures, and the focal length of the object glass are severally given. It will, however, be more satisfactory here to give the practical rules which have been adopted by the most eminent makers, founded partly on theory and partly on practice, for determining the relations between the focal lengths of the several lenses and their relative distances, corresponding to any proposed magnifying power. We shall accordingly give these rules as applied to negative eye pieces.

Let $m =$ the magnifying power,
$\mathrm{O} =$ the focal length of the object glass,
$\mathrm{F} =$ the focal length of the field glass,
$\mathrm{E} =$ the focal length of the eye glass.

The following rules are those adopted:

$$m \times \mathrm{F} = 2 \times \mathrm{O};$$

that is, the magnifying power multiplied by the focal length of the field glass will be equal to twice the focal length of the object glass.

If, therefore, it be required, with a given object glass, to find the focal length of the field glass necessary to produce a given magnifying power, it is only necessary to divide twice the focal length of the object glass by the magnifying power.

If, on the other hand, it be required to find the magnifying power corresponding to a given field glass, it is only necessary to divide twice the focal length of the object glass, by the focal length of the field glass.

In all cases the focal length of the eye glass is one third of that of the field glass; that is,

$$\mathrm{E} = \tfrac{1}{3}\,\mathrm{F}.$$

The distance between the field glass and the eye glass is two thirds of F.

The aperture of the field glass $= \frac{1}{2}\,\mathrm{F}$.
The aperture of the eye glass $= \frac{1}{6}\,\mathrm{F}$.
$\mathrm{D\,D} = \frac{1}{3}\,\mathrm{F}$.
$\mathrm{E\,E} = \frac{1}{9}\,\mathrm{F}$.
The distance of E E from L′ L′ is $= \frac{1}{18}\,\mathrm{F}$, and
The distance of the eye from L′ L′ is $= \frac{1}{6}\,\mathrm{F}$.

In the following table, the magnifying powers produced by four classes of eye pieces, in general use in the telescopes constructed by Frauenhofer and Cauchoix, are given, which correspond to the several focal lengths of the object glasses given in the first column.

The focal lengths of the field glasses of these four eye pieces are as follows:—

I.	2·25 Inches.
II.	1·50 ,,
III.	1·00 ,,
IV.	0·666 ,,

Focal Length of Object Glass.	I.	II.	III.	IV.
Inches.				
20	,,	,,	40	60
30	,,	,,	60	90
42	,,	54	84	126
48	,,	64	96	144
54	48	72	108	162
60	54	80	120	180
72	64	96	144	216

These rules have been adopted in the construction of the great telescopes erected by Frauenhofer at Munich, Dorpat, Pulkowa, in the United States, and elsewhere, and by Cauchoix in various places; the principal being the great telescope of Sir James South, having 11¼ inches aperture and 18 feet focal length, and that of Mr. Cooper, of Sligo, having 12¼ inches aperture and 24 feet focal length.

The latter instrument is the largest refractor hitherto constructed, bearing a magnifying power of 1000. The Dorpat telescope by Frauenhofer has 14 feet focal length and 9 inches aperture.

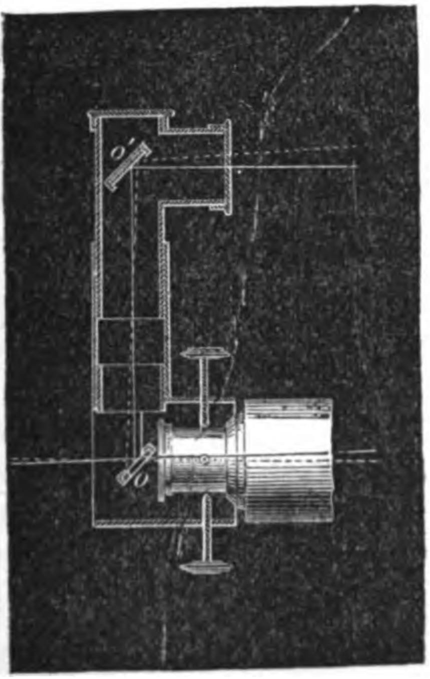

Fig. 253.

517. **Pouillet's method of determining the magnifying power of telescopes.** — This method, which is independent of any calculation founded upon the focal length of the lenses, consists in placing a white rule with black divisions at 50 or 60 yards distance from the instrument. In front of the eye glass, a little plane metallic reflector *o* (*fig.* 253.) is placed, pierced with a hole about the 10th of an inch in diameter to enable the eye to look through the eye glass. Near this is placed another plane reflector *o'*, parallel to the first. When the instrument is directed to the rule, the observer sees its image magnified by the telescope through the hole in *o*, and at the same time sees

the rule itself in its natural magnitude by double reflection from the two reflectors; the rays received upon o' being reflected to o, and thence to the eye. The two images of the rule, the one magnified and the other not, are thus seen superposed, and the observer can easily ascertain the relative proportion of their divisions. Thus, for example, if 20 divisions of the rule, as reflected by o, be equal to one division of the rule as seen through the telescope, the magnifying power is 20.

518. **Mounting of refractors.**—The apparatus by which these large instruments are directed to any required points of the firmament, are very various, according to their magnitude and the circumstances under which they are applied. As an example, we have given in *fig.* 254. a representation of the mounting adopted by Cauchoix for his largest class of instruments.

The instrument is placed in an angular bed *u u*, supported upon a framing, C, of adequate strength. It is moved through a certain angle laterally by a pinion V, which works in a curved rack M M. When a greater lateral change of direction is required than can be obtained by this rack M M, the object is accomplished by shifting the position of the entire stand by means of the castors B B B.

The instrument is moved vertically by means of a pair of winches *s s*, attached to the ends of an axle *r*, on which a pinion is fixed which works in a wheel fixed on the axle *t*, upon the ends of which are two pinions, in which two endless chains, *q q*, work; these endless chains pass over rollers at the top and bottom of the frame, and being attached to a sliding piece *o o*, raise it and lower it. This sliding piece *o o* is jointed to a frame D D, which is hinged upon the frame C, the latter being itself hinged upon the top of the stand at V. When the frame D is raised or lowered, the frame C is also necessarily raised or lowered. The object glass is inserted in the upper end of the great tube E, and the eye piece in the lower end of the small tube M. Beside the great telescope is a small telescope *z*, called a finder, the use of which is to enable the observer with greater facility to direct the great telescope to any desired object. Owing to its small field of view, this process would be attended with some difficulty and delay if no such aid were supplied; but the small telescope *z* can be at once directed to an object, and since its axis is parallel to that of the great telescope, the axes of both instruments will always be directed to the same point; so that when an object is brought into the centre of the field of the finder, it will also be in the centre of the field of the great telescope.

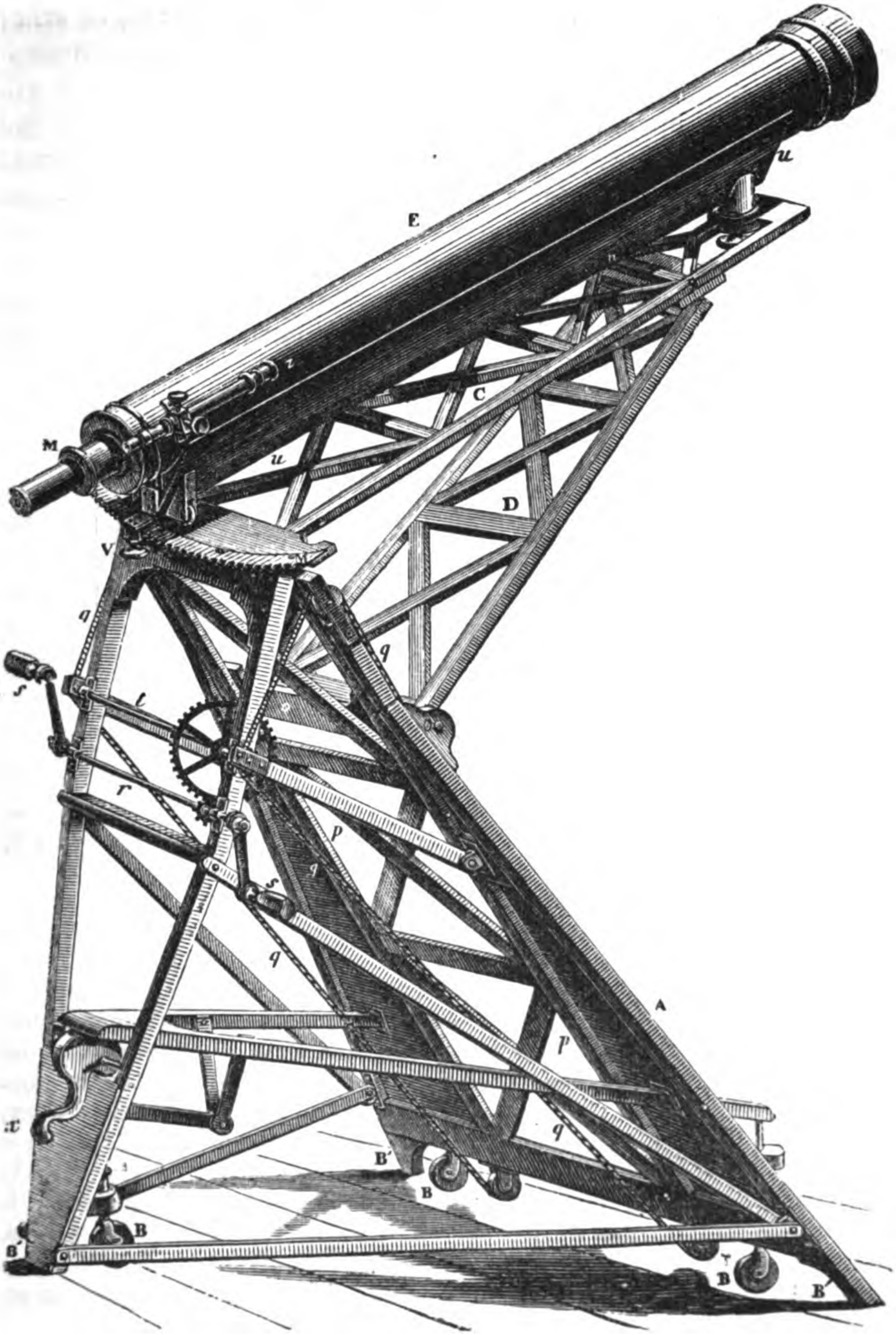

Fig. 254.—Cauchoix's Telescope Stand, Imperial Observatory, Paris.

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VI. The Magic Lantern.

519. The magic lantern is an optical instrument adapted for exhibiting pictures, painted on glass in transparent colours, on a large scale by means of magnifying lenses.
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It has been shown (156., *et seq.*) that when a picture, or other object, is placed in front of a convex lens, at a distance from it somewhat greater than its focal length, such picture or object will be reproduced upon a screen, placed at a certain distance behind the lens, that distance being greater, the nearer the picture in front of the lens is to its principal focus. This is the principle upon which the magic lantern is constructed.

520. **Common form.** — It varies in form and arrangement, according to its price and the circumstances under which it is used, but in general consists of a dark lantern, *fig.* 255., within which a strong lamp L is placed, having a bent chimney at the top, to allow the smoke and heated air to escape, while the light is intercepted.

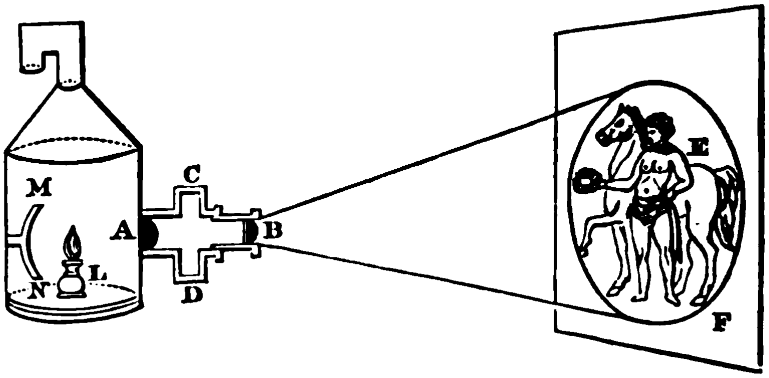

Fig. 255.

In front of the lamp, and on a level with its flame, a tube is inserted, in which a large convex lens A is fixed, by means of which the light of the lamp is condensed upon the picture placed opposite the lens A, by sliding it through a groove, C D. From this mode of fixing the picture, the latter has generally been called a "slider." In the tube thus projecting from the lantern, another tube is fitted sliding in it, as one tube of an opera glass slides in the other. At the end of this second tube a convex lens B is set, and the tube is so adjusted that the distance of B from the picture shall be a little greater than the focal length of the lens B. A large screen F, made of white canvas, which may be much improved by covering it with paper, is then placed at a distance from B, and at right angle to the axis of the lens. By properly adjusting the tube B, and the distance of the screen F, the picture upon the slider in C D will be reproduced at E upon the screen, on an enlarged scale.

It must be observed, however, that as the picture will be inverted, with relation to the object, it will be necessary to turn the slider in C D upside down, in order to have the picture on the screen in its proper position.

To increase the illumination of the slider, a concave reflector M N is usually placed behind the lamp, by which the light projected upon the lens A is increased. A better effect, however, may be produced by simply bending a sheet of white paper or pasteboard round the inside surface of the lantern.

521. **Magnifying power.**—With the same lantern, and the same slider, a picture of any desired magnitude can be produced.

To increase the picture, it is only necessary to push in the lens B, so as to bring it closer to the slider, and to remove the screen F to a greater distance. But it must be remembered that every attempt to enlarge the picture will not only be attended with greater indistinctness, owing to spherical aberration, and more appearance of colours at the edges of the figures, owing to chromatic aberration, but also the brightness of the picture will be greatly diminished, since it is evident that the greater the surface over which the light by which the slider is illuminated is diffused, the more faint, in the same proportion, will the picture on such surface be; and, since the magnitude of such surface increases in the same proportion as the square of its linear dimensions, it follows that when the picture has double the height or width, it will be four times less bright.

The body of the lantern should be large, so that it may not become inconveniently heated. The best oil should be burnt in the lamp, so as to diminish the smoke and disagreeable odour. The glass chimney of the lamp should be made as high as possible, and the wick should be of large calibre.

522. The pictures on the sliders should be as large as possible, in order to ensure sufficient illumination on the screen. With a given magnitude of picture on the screen, and a given force of lamp, the illumination will be proportional to the magnitude of the slider. If a small slider be used to produce a picture on the screen of a given magnitude, the confusion arising from both kinds of aberration will be greater than if a larger one were used.

523. There are two ways of exhibiting the pictures on a screen: in one, the lantern is placed in front of the screen, with the spectators; in that case the picture is seen by the light reflected from the screen, after having been projected upon it by the lantern.

Care should, therefore, be taken that no light shall penetrate through the screen, since all such light would be lost, and the picture on the screen would be proportionally more faint. A screen composed of muslin, or any other textile fabric, would in such case be defective, inasmuch as more or less of the light would penetrate it. The best sort of screen is one made of strong white paper, pasted on canvas, and stretched on a frame, as canvas is for a picture.

When the magic lantern is used for purposes of amusement, rather than those of instruction, it is generally found desirable to use a semi-transparent screen, the lantern being mounted on one side of the screen, and the spectators placed on the other, as shown in *fig.* 256. In this case, the screen should be made of white muslin or fine calico, stretched upon a frame, its transparency being increased by wetting it well with water. In some cases the muslin is prepared with wax or oil, which may be convenient to save the trouble of wetting it, but which in other respects does not answer the purpose better.

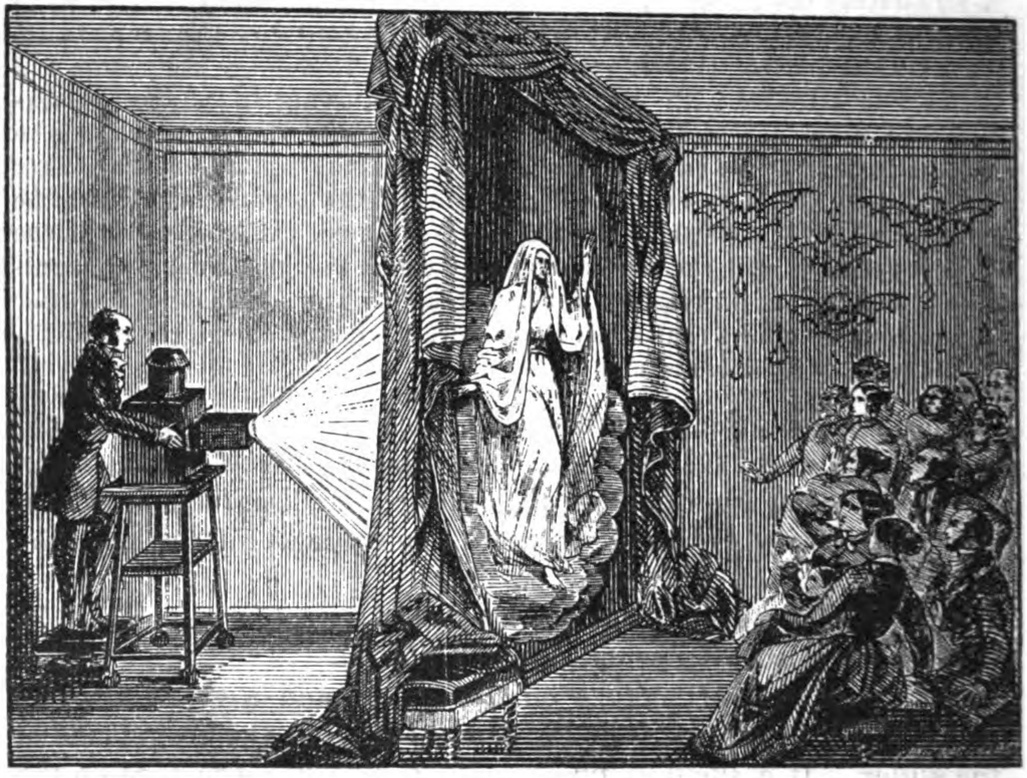

Fig. 256.

524. **Phantasmagoria.** — When the pictures are produced through a transparent screen, the exhibitor, being concealed from the spectators, may make them vary in magnitude; first gradually increasing, and then gradually diminishing. This is accomplished by moving the lantern gradually and alternately from and towards the screen, varying the focus during the motion, so as to render the picture upon the screen always distinct.

Let us suppose, for example, that the nozzle of the lantern is first placed in actual contact with the screen. The picture on the screen will then be exceedingly small, and the spectators, to whom the screen is invisible, will imagine the object to be at a great distance. Let the exhibitor then move back the lantern slowly from the screen, keeping the focus constantly adjusted, the picture on the screen will then be gradually enlarged, and the impression produced on the spectators will be, that its increased magnitude is produced by the gradual approach of the object towards them; and so complete is this delusion, that the rapid increase of magnitude of the picture actually startles even persons who are most familiar with the optical causes which produce the effect. It sometimes appears as if the object would approach, so as to come in actual collision with the spectator.

When the object seems thus to be brought near the spectator, it is made to retire gradually by moving the lantern towards the screen, the effect being produced by the gradual diminution of the image upon the screen, and this is continued until the nozzle of the lantern, coming again in contact with the screen, the object seems again to be lost in the distance, its magnitude being reduced to a mere point. The exhibitor seizes this moment to change the picture, displacing one slider by the introduction of another, a manœuvre which, when adroitly performed, will escape the notice of the spectators. The new picture is then exhibited in the same way.

Effects of this kind have been denominated "phantasmagoria," from the Greek words *φαντασμα* (phantasma), *a spectre*, and *αγοραομαι* (agoraomai), *I meet*.

525. **Dissolving views.**—Interesting and amusing effects are produced by placing two lanterns of equal power, so as to throw pictures of precisely equal magnitude on the same part of the same screen. A sliding cover is placed in front of the nozzle of each of the lanterns, and these are moved simultaneously in such a manner, that when the nozzle of one lantern is completely opened, that of the other is completely closed, so that, according as the former is gradually closed, the latter is gradually opened.

To illustrate this class of effects, which always create an agreeable surprise, let us suppose that two sliders are placed in the lanterns, one representing a landscape by day, and the other representing precisely the same landscape by night, and let the nozzle of that which contains the day landscape be opened, the other being closed: the picture on the screen will then represent the day landscape. If the covers of the nozzles be now slowly moved, so that that of the lantern which shows the day landscape shall be gradually closed, and that of the other shall be gradually opened, the effect on the screen will be that the daylight will gradually decline, the view assuming, by slow degrees, the appearance of approaching night. This gradual change will go on, until the nozzle of the lantern containing the day picture has been completely closed, and that containing the night picture completely opened, when the change from day to night will be accomplished, the picture on the screen being then a night landscape.

The optical effect produced by two lanterns working together, called *dissolving views*, with which the public has been rendered familiar at several of the public institutions in London, depends on the alternate opening and closing of the nozzles of two lanterns, in the manner here described. The mistiness and confusion which is exhibited in the gradual disappearance of the one view, and the gradual appearance of the other, arises from the circumstance of the nozzles of both lanterns being partially open at the same moment, so that both views, faintly illuminated, are projected upon the screen at the same time. The mixture of their outline and colours produces the mistiness and confusion, with which all spectators of such exhibitions are familiar. According as the nozzle of the lantern, which contains the disappearing view, is more and more closed, and that which contains the appearing view more and more open, the latter becomes more and more distinct, and becomes perfectly so, when the one lantern is completely closed, and the other is completely opened.

526. **Illumination of pictures by gas and electric light.**— For family and school purposes, a good lamp is the most convenient means of illuminating the sliders; but where exhibitions are produced before larger and adult audiences, other and more effectual means of illumination are resorted to. For several years, the lanterns by which dissolving views, and other effects, have been produced in the public exhibitions in London, have been illuminated by the oxy-hydrogen light. This light proceeds from a ball or cylinder of lime, rendered incandescent, or white hot, by the flame of a blow pipe, from which a mixture of oxygen and hydrogen gases, in the proportion in which these gases produce water, issues.

It might be imagined that the light produced by a piece of solid matter like lime, however intensely heated, could never be brilliant enough to produce a strong illumination; nevertheless, the light radiated from the lime in this case, was the most intense artificial light which had ever been produced, until the invention of another, which we shall presently notice.

In the oxy-hydrogen lanterns, the cylinder of lime is mounted so as to occupy the place of the flame of the lamp in the axis of the lenses. The flame of the blow pipe is projected upon that side of it which is presented towards the lenses, and since the lime, though it does not undergo combustion, is gradually wasted by the action of the flame, it is kept in slow revolution by clockwork, connected with the axis upon which it is supported, so as to present to the flame successively different parts of its surface.

This method of illumination, though still continued, is greatly surpassed in splendour by that of the electric light, which has recently been applied to the magic lantern by M. Dubosc, the successor of M. Soleil, the celebrated Paris optician.

The electric light is produced by bringing two pieces of charcoal, previously put in connection with the poles of a voltaic battery, nearly into contact; the volta current will then pass from one to the other, the ends of the charcoal thus nearly in contact becoming incandescent, and emitting the most brilliant artificial light which has ever yet been produced.

The method of mounting this illuminating apparatus in the lantern is shown in *fig.* 257.

The wires H K, being connected with the poles of the battery, are attached to two pieces of metal, the negative wire H communicating with the upper pencil of charcoal, *c*, and the positive wire K with the lower charcoal pencil, *a*.

The points of the pencils being nearly in contact, the light will be produced in the manner just explained.

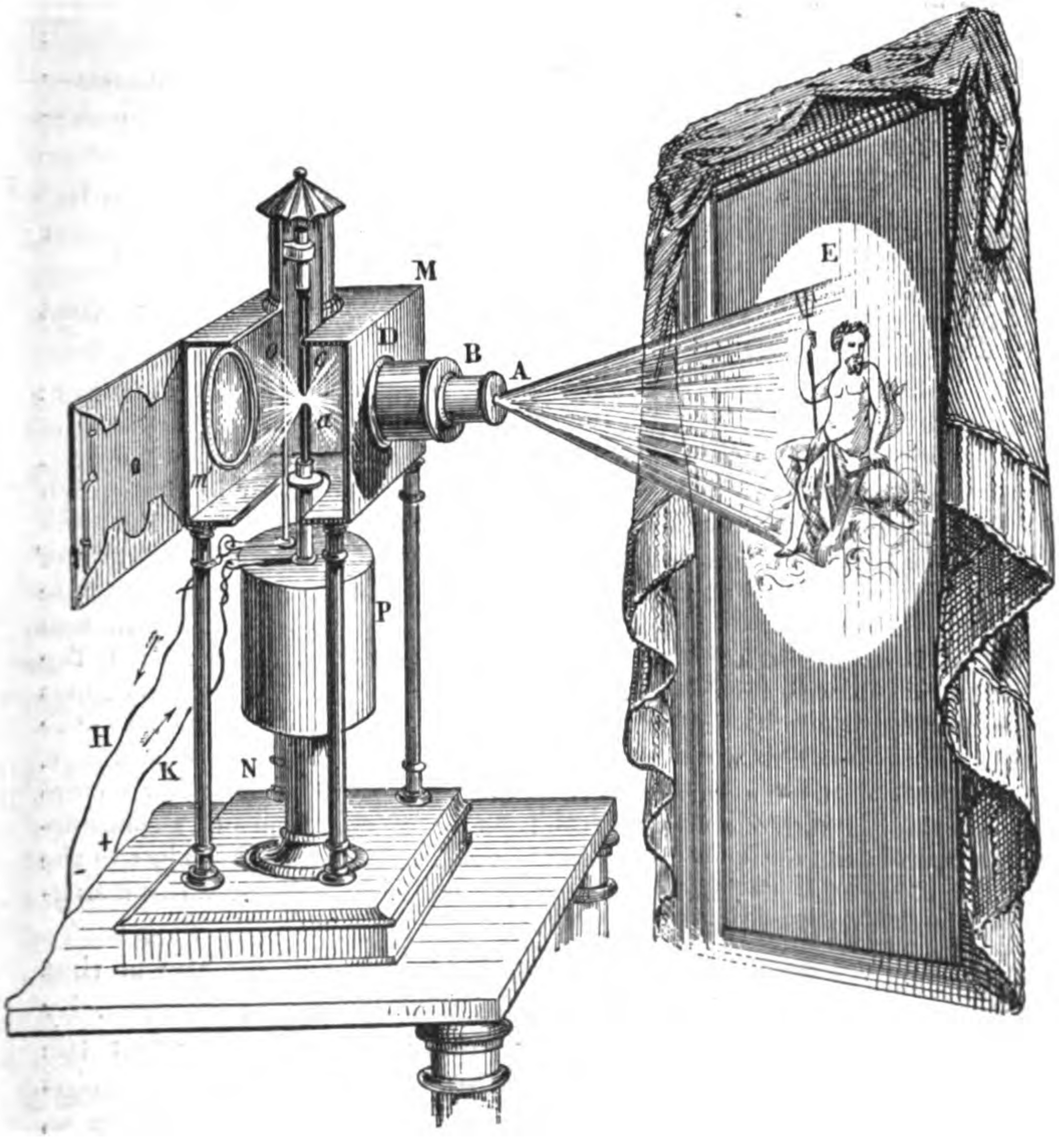

Fig. 257.

Although the charcoal does not, properly speaking, undergo combustion, it is gradually wasted, and when the points would thus become separated, the current would be suspended, and, therefore, the light would cease. To prevent this, and to maintain the illumination, an apparatus consisting of clockwork is provided in the case P, by which the charcoal pencil, *a*, is kept nearly in contact with the pencil, *c*. The clockwork is so constructed that its motion is governed by the current.

M. Dubosc has contrived means, by which a single electric light will serve to illuminate at the same time two lanterns, placed side by side for exhibition. This is accomplished by placing the light between two reflectors, so inclined that each reflects it in the direction of the axis of one of the lanterns.

VII. The Solar Microscope.

As an instrument for popular and general instruction, the solar microscope holds a high place. Until recently, its use has been restricted in these climates, by the circumstance of bright sunshine, and a room having a suitable aspect, being conditions indispensable for its performance. But by the substitution of the oxy-hydrogen light, and, more recently still, of the electric light, the utility and pleasure, derivable from this instrument of popular illustration, have been immensely extended.

527. The principle of the solar microscope is the same as that of the magic lantern already explained.

The instrument consists of two parts, essentially distinct one from another: the first, the illuminating; and the second, the magnifying part. Since it is desired to exhibit a very enlarged optical image of a very minute object, and since the light which is spread over the image can only be that which falls on the object, it is evident that the brightness of the image will be more faint than that of the object, in the exact proportion in which the surface of the former is greater than that of the latter. To illustrate this, let us suppose that the object exhibited is an insect, a quarter of an inch in length, and that it is magnified 40 times in its linear dimensions, the length of the optical image will then be 10 inches, and its surface will be 1600 times greater than that of the object. The light, therefore, which illuminates the object, supposing the whole of it to be transmitted to the optical image, being diffused over a surface 1600 times greater, will be 1600 times more faint. But, in fact, the whole of the light never is transmitted, a considerable part of it being lost in various ways in passing from the object to the screen. The necessity, therefore, for very intense illumination in this instrument must be evident.

If these conditions were not borne in mind, it might appear that a magic lantern might be converted into such a microscope, by merely increasing the magnifying power of the lenses; but the light of the lamp, which is sufficient to illuminate a picture magnified 10 or 12 times in its linear, and, therefore, from 100 to 144 times in its superficial dimensions, would be utterly insufficient, if it were rendered 1600 times more feeble.

528. **Illuminating apparatus.** — The illuminating apparatus of the solar microscope consists of a large convex lens, upon which a cylindrical sunbeam of equal diameter is projected. This lens causes the rays of such a sunbeam to converge to a point, and they are received upon the object to be exhibited before their convergence to a focus, and at such a distance from the focus, that the entire object shall be illuminated by them. In fact, the rays may be considered as forming a cone which is cut at right angles to its axis by the slider upon which the object is fixed.

Let C C, *fig.* 258., be the condensing lens; let F be the focus to which the rays would be made to converge, but being intercepted by the slider S S, they are collected upon the small circular opening o o in the slider, and in this circular opening the small microscopic object to be exhibited is mounted between two thin plates of glass.

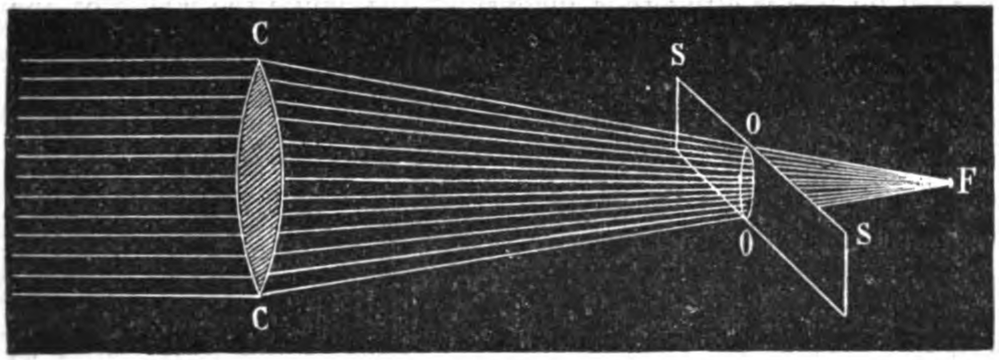

Fig. 258.

Now, it is evident that the intensity of the light thus projected upon the object, will be greater than that with which it would be illuminated without the interposition of the lens c c, in the exact proportion of the surface of the lens c c to the surface of the circular opening o o. Thus, for example, if the diameter of the lens c c be 5 inches, and the diameter of the opening o o half an inch, the diameter of the lens will be 10 times, and, therefore, its surface 100 times greater than that of the opening o o. In that case the object would be illuminated with a light just 100 times more brilliant than if the sun's light fell directly upon it, without passing through the lens c c.

It is found convenient in some cases to condense the light by means of two lenses. The cone of rays proceeding from c c might be received upon another condensing lens, by which its convergence might be increased. The advantage of this arrangement is that the distance of the object from c c, and therefore the length of the microscope, is rendered less than it otherwise would be.

There is, however, one practical inconvenience to be guarded against in this arrangement. The lens c c, which condenses the sun's light upon the object, also condenses its heat, and if the same object be exposed in the instrument for any considerable time, it would thus be injured or destroyed. This inconvenience may be obviated by the interposition of certain media, which, while they are pervious to the sun's light, are impervious to its heat; such media are said to be athermanous.*

By the interposition of such a medium, the object may be prevented from receiving any increased temperature whatever.

It happens that water, which is the most convenient medium for this purpose, is very imperfectly pervious to heat, and is rendered almost completely athermanous by dissolving in it as much alum as it is capable of holding in solution. The object, therefore, is perfectly protected from the effects of heat, by placing between the slider and the condensing lens a cell, consisting of two parallel plates of glass, fixed at about an inch asunder, and filled with such a saturated solution of alum. The light intercepted by this is altogether inconsiderable, while the whole of the heat is stopped by it.

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* From the Greek negative α (a), and θέρμη (thĕrmè) *heat.*

529. **Magnifying apparatus.**—The magnifying part of the solar microscope consists of an achromatic lens, or combination of lenses, of very short focal length; this being brought before the object, at a distance from it a little greater than its focal length, will produce a highly magnified optical image of the object, upon a screen placed at a proper distance before it.

In the case of the magic lantern, it is not indispensable to incur the expense of achromatic lenses, and even the expedients to correct the spherical aberration are but little attended to. The magnifying powers used in that instrument not being great, and the object exhibited not requiring extreme accuracy of delineation, the expense which would be incurred in producing large lenses free from the aberrations is not necessary. But in the case of microscopic objects, where great magnifying powers are applied, lenses in which the aberrations are not corrected would produce images so confused and indistinct as to be altogether useless. Achromatic combinations, therefore, in which the spherical aberrations are also corrected, are in this case indispensable.

As in the magic lantern, the same lenses may be applied, so as to produce different magnifying effects. If the distance of the lenses from the object were so great as twice their focal length, the image would be projected upon the screen at a distance in front of the lens also equal to twice its focal length, and would in that case be exactly equal to the object, and consequently there would be no amplification at all. As the lenses, however, are moved nearer to the object, the distance at which the image would be formed and its magnitude would be increased, and this increase would go on without practical limit, until the distance of the lens from the object would become equal to its focal length, in which case the image, having been enlarged beyond bounds, would altogether disappear.

In practice, therefore, the focus of the lens is brought to such a distance from the object, that the image upon the screen shall have a magnitude sufficient for all the purposes of exhibition. It is not desirable, however, in any case, to push the amplifying power of the instrument too far, because the illumination of the image in that case becomes inconveniently faint; and if there be any causes of aberration uncorrected in the lenses, whether spherical or chromatic, their effects will be rendered more apparent.

530. **Adjustments.** — In the mounting of the instrument, provisions are necessary for varying, within certain limits, the distance of the object, as well from the illuminating as from the amplifying lenses. If the object be very minute, it is necessary that it should be illuminated with proportionate intensity; and, therefore, that it should be moved very near to the focus of the illuminating lens, c c. If it be larger, this position would, however, be unsuitable, inasmuch as the light would be collected upon a small part of it, to the exclusion of the remainder. In that case, therefore, the object must be brought farther in advance of the focus, F, of the illuminating lens, so as to intersect the cone at a point of greater section, and thus to receive a light which, though less intense, will be diffused over its entire surface.

The amplification required will be greater in proportion as the object is smaller. For very minute objects, therefore, the amplifying lens must be brought nearer to the object, and the screen must be removed farther from it, while for larger objects the arrangement would be the reverse.

531. **Screen.** — All that has been said on the subject of the screen in the case of the magic lantern will be applicable to the solar microscope, except that, in this case, the method of showing the object through a transparent screen is objectionable, because of the light which is lost by it, and for other reasons; and, besides, it is useless, that method of exhibition being adapted only for phantasmagoria, and other similar subjects of amusement.

532. In what has been explained above, it has been assumed that a beam of solar light is thrown upon the condensing lens c c, in the direction of its axis. Now it is evident that it could never happen that the natural direction of the sun's rays would coincide with that of the axis of the tube of the microscope; for, that axis being necessarily horizontal, or nearly so, the sun, to throw its rays parallel to it, should be in the horizon. Some expedient, therefore, is necessary, by which the direction of a sunbeam can be changed at will, and thrown along the axis of the tube.

The obvious method of accomplishing this is by means of a plate of common looking-glass; such a plate, being conveniently mounted in front of the condensing lens, may always have such a position given to it that it will reflect the sunbeam which will fall upon it in the direction of the axis of the tube.

But since, by reason of its diurnal motion, the sun changes its position in the heavens from minute to minute, the position of the reflector, which at one time would throw the light in the proper direction, would cease to do so after the lapse of a short interval. A proper provision must be made, therefore, by which the position of the reflector may be changed from time to time with the motion of the sun in the firmament, so that it shall always reflect the light in a proper direction.

533. **Mounting.** — A perspective view of the solar microscope, mounted in the most efficient manner, is given in *fig.* 259.; but

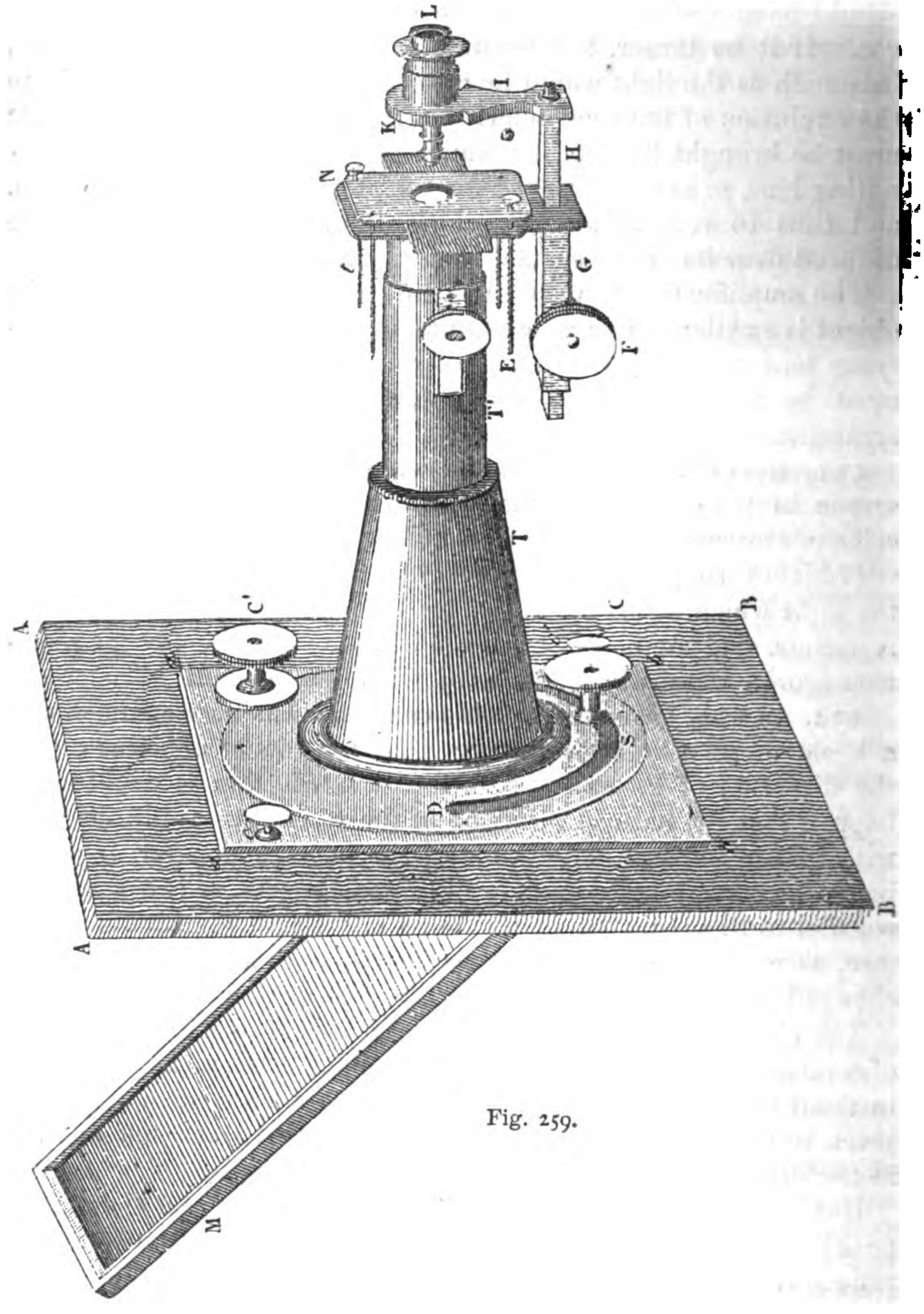

Fig. 259.

the principle of its performance will be more easily understood by reference to the sectional diagram in *fig.* 260., where C C is the condensing lens, and H H the mirror which receives the sun's light, and reflects it in the direction of the axis of the tube.

This mirror turns on a hinge, by which it may be inclined at any desired angle to the axis of the tube; and a provision is also made by which it can be turned round the axis, so that its plane may be presented in any desired direction to the sun: a smaller condensing lens is interposed, upon which the rays, converging from C C, are received, and by which, with increased convergence, they are projected upon the opening O O in the slider S S, in which the object is mounted.

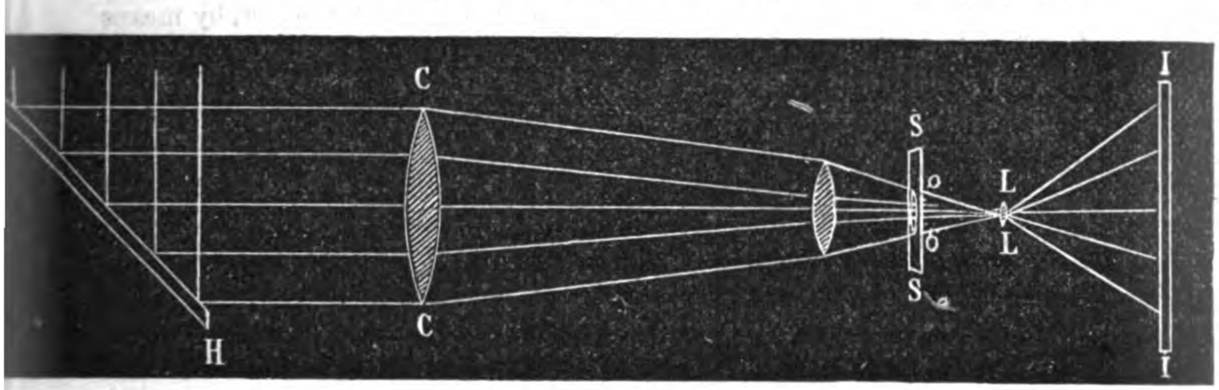

Fig. 260.

The tube in which the slider S S is inserted, and which carries the smaller condenser, slides within another tube, in the end of which the greater condenser C C is set. By this arrangement, the section of the cone of light, which falls upon the opening O O, may be varied, according to the magnitude of the object.

The amplifying lens, or lenses, L L, are conveniently mounted in a tube, which can be moved within certain limits to or from the object, so as to accommodate the focus to the position of the screen I I, upon which the image is projected.

After these explanations, the reader will have no difficulty in comprehending the instrument, as shown in perspective in *fig.* 259.

A board, A A B B, is pierced by a large circular aperture, the diameter of which is a little greater than that of the larger condensing lens; a square brass plate, *a a b b*, to which the microscope is attached, is screwed upon this board in such a position that the condensing lens shall be concentric with the hole in it, and, consequently, that the axis of the instrument shall be at right angles to the board.

The plane mirror M, by which the light of the sun is reflected along the axis of the instrument, is mounted outside the board A A B B, moving on a hinge, as already described; and screws are provided at C C', by means of which its inclination to the axis of the microscope can be varied at pleasure, and also by which it can be turned round the axis, the screw which governs its motion moving on the circular opening S D. By these means, whatever be the position of the sun in the heavens, such a position can always be given to the plane of the mirror, that the light may be reflected along the axis of the microscope.

The great condensing lens is set in the larger end of the conical tube T, and the lesser in the end of the cylindrical tube T'; the latter tube being moved within the former by an adjusting screw, which appears at its side. By the second condensing lens, the light is collected upon the opening in the slide, which is held between two plates N, pressed together with spiral springs.

The tube T′ consists of two parts, one moving within the other, like those of the telescope.

The amplifying lenses are mounted in a brass ring, K, carried by the upright piece, I, so that its optical axis shall coincide with that of the illuminating apparatus. This optical part can be moved to and from the object, by means of a rack and pinion, F, attached to the piece H, which slides in the box G.

The structure and principle of the instrument being understood, it only remains to explain the method of using it.

The room in which the operations are conducted should have sufficient depth to allow the space between the microscope and the screen, which is necessary for the formation of an image of the required magnitude. This space will vary with the magnifying power required, but in general 10 or 12 feet beyond the nozzle of the instrument is sufficient. The room should be rendered as dark as possible, to give effect to the image, which, however well illuminated, is always incomparably less bright than would be objects receiving the light of day. The window shutters should therefore be carefully closed, and all the interstices between them stopped. If the room be provided with window curtains, they should be let down and carefully drawn. In a word, every means should be adopted to exclude all light, except that which may enter through the microscope.

An opening being provided in a convenient position in one of the window shutters, corresponding in magnitude with the aperture in the board A A, B B, the latter is screwed upon the window shutter so that the two openings shall coincide. The mirror M will then be outside the window shutter, while the instrument and its appendages will be inside. The window selected should, of course, be one having such an exposure that the sun's rays can be reflected by the mirror in the direction of the axis of the tube.

To adjust the instrument, remove the piece N, which supports the slider, so that the light may pass unobstructed to the amplifying lens. By varying the position of the reflector M, by means of the milled heads C C′, a position will be found in which a uniformly illuminated disc will appear on the screen; this disc may be rendered more clear and distinct by adjusting the instrument by means of the rack and pinion attached to the tube.

When these preliminary adjustments are made, the piece N is replaced, and an object inserted in it; the instrument being then more exactly focussed, a distinct image of the object, upon a large scale, will be seen on the screen.

The management of the instrument will vary with the nature of the object. If it be a very transparent one, a strong light thrown upon it would cause it almost to disappear. The light, therefore, in such case, must be so regulated as to produce the image in the most favourable manner, which may always easily be accomplished by moving the tube T′ in and out of the tube T, until the desired result is obtained.

When the experiments are continued for any considerable interval, it will be necessary, from time to time, to accommodate the reflector M to the shifting position of the sun, which may always be done by the milled heads C C′. This adjustment, however, might be superseded by mounting the mirror M upon an apparatus called a Heliostat, the effect of which is, to make the mirror move with the sun, by means of clockwork. Such an apparatus, however, is expensive, and the adjustment above described is attended with no great inconvenience or difficulty.

VIII. THE GAS AND PHOTO-ELECTRIC MICROSCOPES.

534. A large proportion of the utility and pleasure derivable from the solar microscope is in these climates lost by the uncertainty and infrequency of sunshine. The invention of the oxy-hydrogen light, already described (526.), has rendered this interesting instrument independent of the sun. Its application to the solar microscope is in all respects similar to its use in the magic lantern. It is placed at a certain point in the axis of a large converging lens, at a distance from the centre of the lens greater than its focal length; the lens by this means renders the rays diverging from the lime convergent, and they are generally received as in the solar microscope upon a second converging lens of smaller diameter, by which they are collected upon the object to be illuminated.

535. A mechanism of clockwork is usually provided to keep the pencil of lime in a state of slow revolution, so that it shall be evenly worn over its entire surface.

536. **The photo-electric microscope.** — A great improvement, however, has been more recently introduced by the substitution of the electric for oxy-hydrogen light. The application of this expedient to the magic lantern has been already explained, (526.), but in its application to the microscope, several provisions have been introduced to insure its uniformity and continuance, which merit notice.

537. **Its illuminating apparatus.** — The illuminating apparatus of the photo-electric microscope in its most improved form is represented in *fig.* 261., where *i* is a large converging lens, which corresponds with the great illuminating lens c c, (*fig.* 260.), in the solar microscope.

The pencils of charcoal *a* and *b*, which produce the light, are so mounted that the point of greatest splendour shall be accurately in the axis of the lens *i*, and that their extremities are kept constantly at that distance from each other which produces light of the greatest brilliancy. For this purpose provisions are made by which the points of these pencils, according as they are worn away by the action of the voltaic current are moved towards each other, but at the same time are prevented from approaching each other closer than that limit of distance, which gives the light the greatest splendour.

The pencils *a* and *b* are fixed in sockets *c* and *d*, which are urged towards each other by spiral springs in the same manner as the candles in carriage lamps are pushed upwards. These springs, however, are controlled and prevented from acting upon the pencils of charcoal, except when the light declines in splendour by the distance between the charcoal points becoming too great. This is accomplished by the following ingenious expedient: —

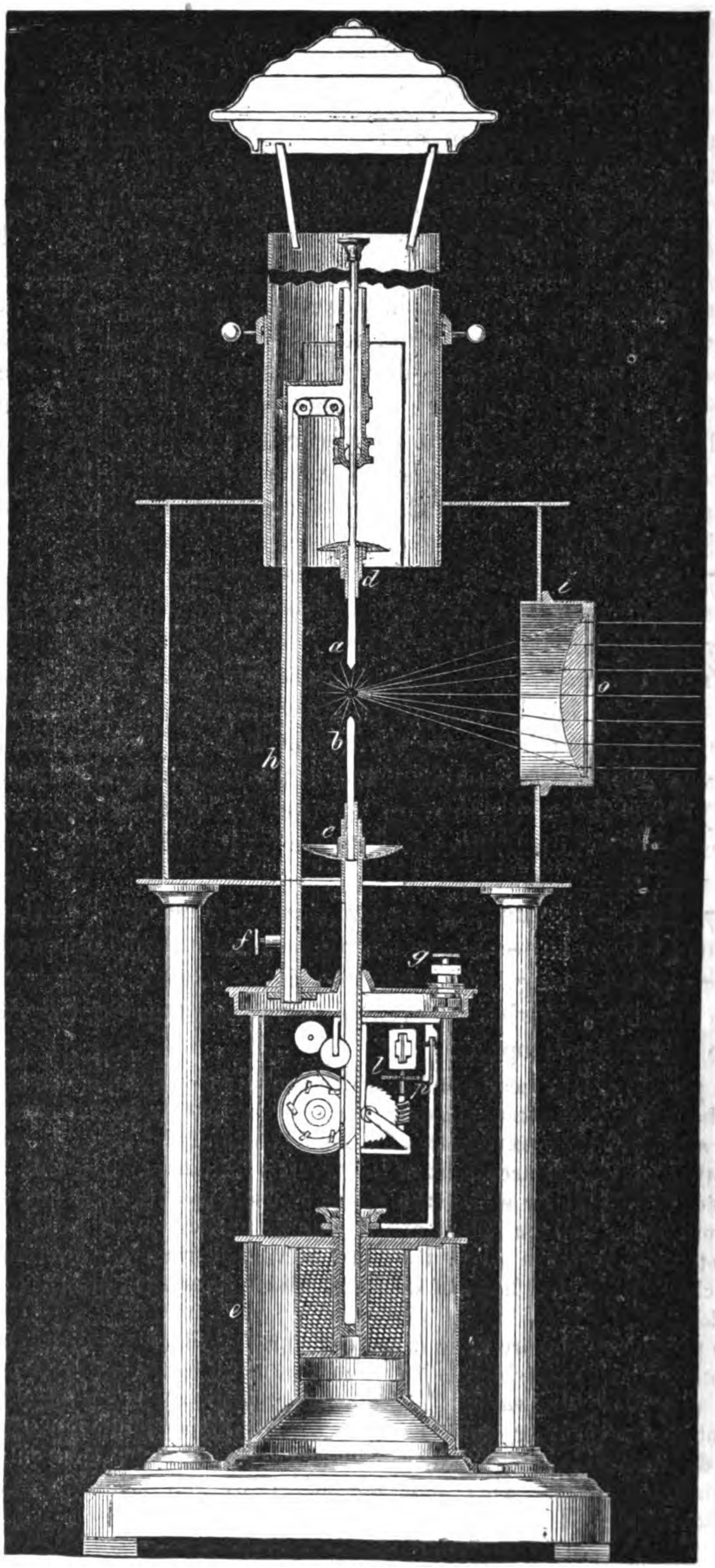

Fig. 261

The springs are controlled by a detent which is placed in connection with the contact piece of a sufficiently powerful electro-magnet, the wire coil of which is placed in the circuit of electric current which passes between the charcoal points. So long as this current flows with sufficient intensity, the light produced will have the necessary splendour, and the electric magnet will be rendered so powerful that it will hold the contact piece upon its poles, and the piece, so long as it is thus held, will keep the detent in such a position as to prevent the springs from acting on the charcoal pencils. But when by reason of the waste of the charcoal by the action of the current, the distance between the points of the pencils is unduly increased, the light declines in splendour, and the current passing with difficulty between the pencils, loses some degree of its intensity. At the same time the electro-magnet loses in a proportionate degree its attracting force, and, letting go the contact piece, allows the springs to act upon the pencils and move them towards each other. As they approach each other the current is re-established, the splendour of the light restored, and the electro-magnet receiving its attractive force draws to it the contact piece, and stops the action of the springs upon the pencils.

The conducting wires of the voltaic battery, which usually consists of from 60 to 100 pairs upon Grove's or Bunsen's system, are connected with the charcoal pencils by the screws *f* and *g*, the positive pole being usually connected with *g*, and the negative with *f*. From *g* the current is carried by a conducting wire to the coils of the electro-magnet *e*, after passing through which it is conducted to the support *c* of the lower charcoal pencil *b*. The current from the negative wire at *f* is carried through the tube *h*, insulated at its lowest part over pulleys to the socket *d*, and thence to the charcoal pencil *a*. If the two pencils *a* and *b* be separated from each other by a certain limit of distance, the current will be wholly suspended. As they approach each other it will flow with an intensity increasing as the distance between the points of the pencils is diminished; and, as before stated, the flow of the current is attended with a strong evolution of light, the splendour of which attains a maximum degree when the pencils are placed at a certain distance asunder.

A side view of the mechanism by which the pencils are moved, is given on a larger scale in *fig.* 262., where *a* is the axis upon which are placed several wheels, some of which are fixed so as to move with the axis, and the others are merely held on it by friction. The barrel *b*, containing the mainspring, which is the moving power, is fixed upon the axis *a*, while the two pulleys *c* and *d* are held upon it by friction only. Upon the pulley *c* is rolled the chain, by which the lower pencil *b* (*fig.* 261.), is moved, and upon the pulley *d* is rolled the chain which moves the pencil *a*. It will be easily perceived, from *fig.* 261., how these chains, after having passed over the intermediate pulleys, are connected, one with the pencil *b*, and the other with the pencil *a*. When the mainspring in *b* (*fig.* 262.), has been liberated by the electro magnet the pencils are moved towards each other;

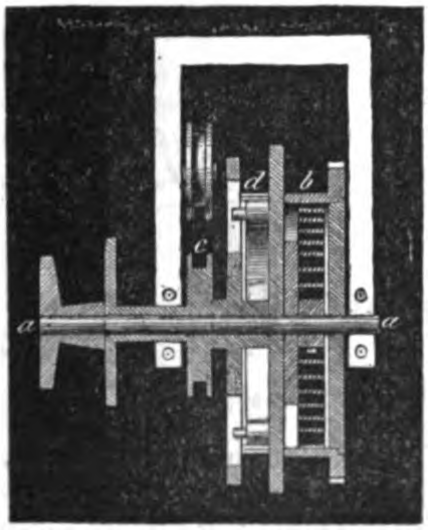

Fig. 262.

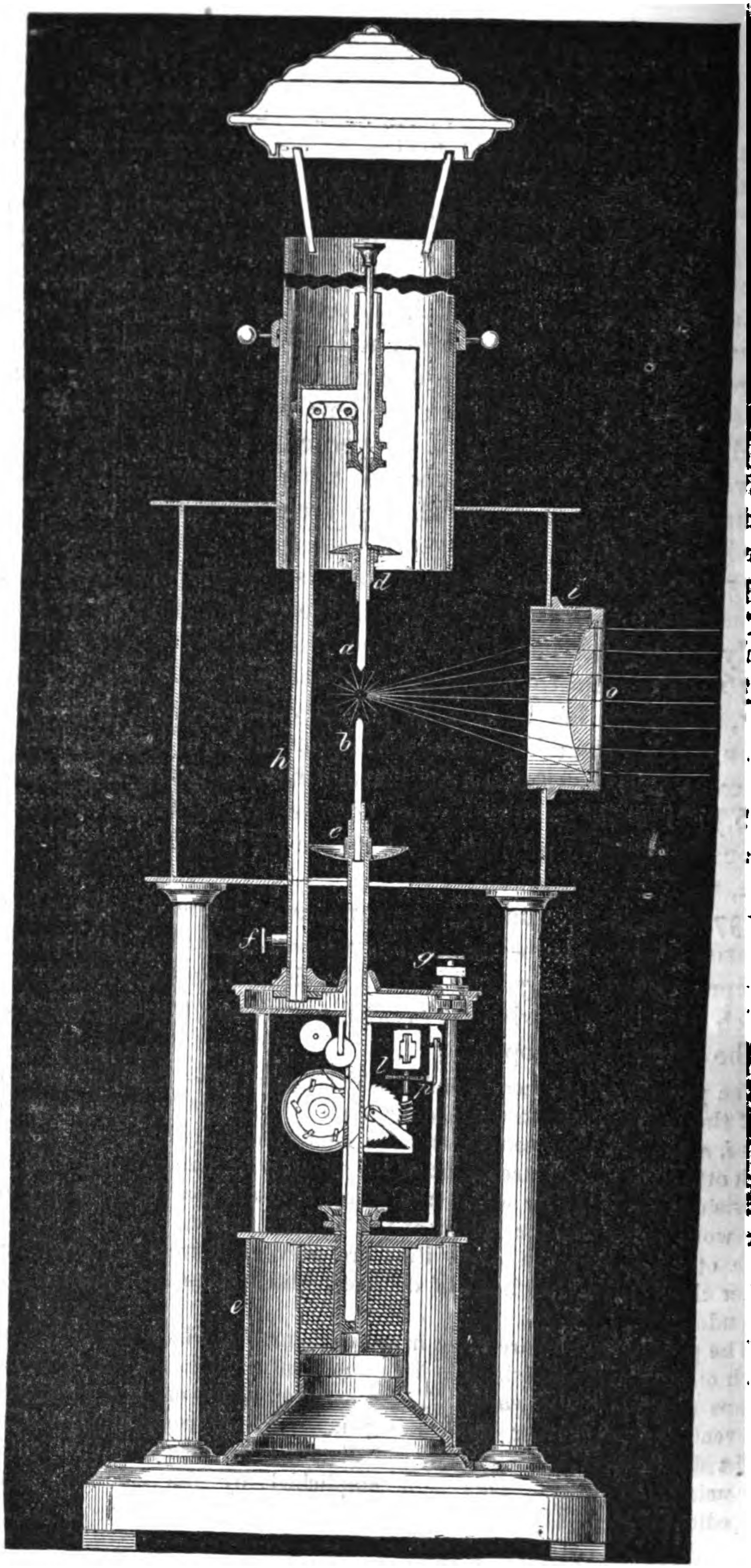

Fig. 261.

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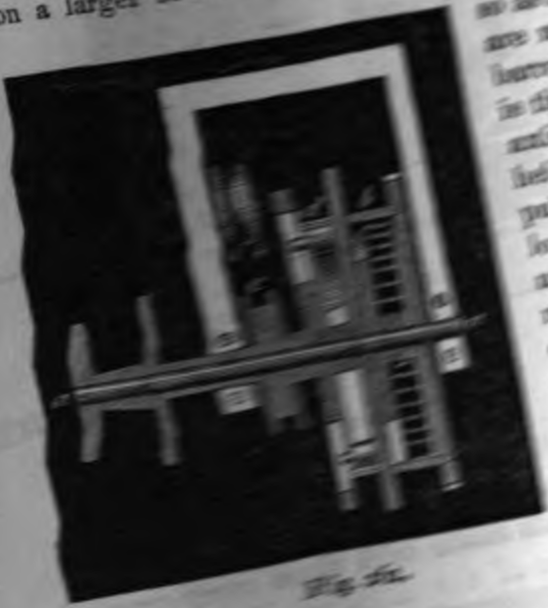

Fig. 261.

but since they are not equally worn away, they must not be moved towards each other through equal spaces, since in that case the focus of light would be removed from the axis of the illuminating lens *i*, (*fig.* 261.). The provision by which M. Dubosc has contrived to regulate this motion of the pencils, is as follows: — The wheel *d* has a variable diameter. It is made of two plates, one carrying six radii, jointed near the centre, whose free extremities render the circumference of the wheel greater or less according as they are less or more inclined. The other plate has six oblique slits, in which six pins move attached to the free extremities of the radii. Thus, by turning this latter plate, the radii are opened or closed, and the circumference rendered greater or less. A watch spring fixed to one of these pins, and making about a turn and a half, supports itself upon the others and forms the groove of the pulley. The chain attached to the other extremity of this spring keeps it in its place.

A provision is made by which the relative diameters of the two pulleys shall be regulated, according to the relative rate of waste of the two charcoal pencils. When the upper pencil wastes faster than the lower, the diameter of the pulley upon which the chain which moves it is coiled, is rendered greater than that of the other pulley, and when the lower pencil wastes faster, the contrary relation is established. When the pencils are brought to the proper distance, the detent connected with the contact piece of the electro-magnet stops the motion of the pulleys, and the splendour of the light is maintained steady and uniform.

538. **Experiments performed with it.** — All the experiments made with a common solar microscope can be reproduced with this apparatus. If it be desired, for example, to make the well known experiments upon the decomposition and recomposition of light, the usual apparatus of prisms and their accessories are presented to the pencil of light issuing from the lens *i* (*fig.* 261.), and in like manner all the phenomena of diffraction, inflection, and polarisation, can be experimentally illustrated.

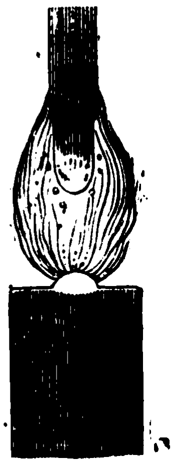

Fig. 263.

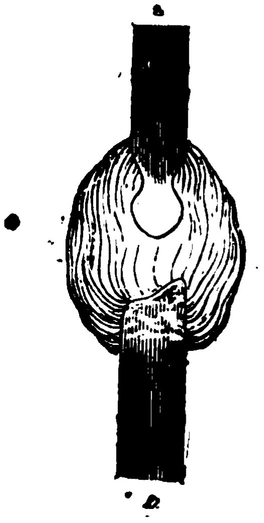

Fig. 264.

Among the most curious experiments made with this apparatus are those in which the magnified image of the electric light itself is thrown upon the screen. In this manner we are enabled actually to see the ponderable molecules of the charcoal, passing between the points of the pencils, as shown in *fig.* 263, and if we take for the positive pole a small charcoal cup, in which are placed successively small pieces of the substances upon which experiments are to be made, such as platinum, gold, silver, &c., they will be observed to be successively liquefied and vaporised, producing flames of various and beautiful colours. These several flames may be analysed in the usual way by prisms, and nothing can be more curious and interesting than the difference found to prevail between the physical character of their lights and those of the corresponding tints of solar light.

IX. The Camera Obscura.*

539. This is an instrument of extensive utility in the arts of design; by it the process of drawing is reduced to that of mere tracing, and its use has of late been greatly extended by its application in the art of photography.

We have already explained (156. *et seq.*) that if a convex lens, or any equivalent optical combination, be presented to a distant object, such as a landscape, an inverted image of that object, with its proper outline and colours, will be produced at the principal focus of the lens. Let us suppose, for example, that the window-shutters of a chamber being closed, so as to exclude the light, a hole be made in them, in which a convex lens is inserted: let a screen made of white paper be then placed at a distance from the lens, equal to its focal length, and at right angles to its axis; a small picture will be seen upon the screen, representing the view facing the window to which the axis of the lens is directed; this picture will be delineated in its proper colours, and all moving objects, such as carriages or pedestrians, the smoke from the chimneys, and the clouds upon the sky, will be seen moving upon it with their proper motions. The picture, however, will be inverted, both vertically and laterally, the sky being below and the ground above; trees and buildings will have their tops downwards; vehicles will move with their wheels, and pedestrians with their feet, upwards; objects on the right of the landscape will be on the left of the picture, and *vice versâ;* and all motions will be reversed in direction, objects moving to the left appearing to move to the right, and those which fall, appearing to rise.

This remarkable optical phenomenon was discovered in about the middle of the sixteenth century, by Baptista-Porta, a Neapolitan philosopher, and it was not long before it assumed a variety of forms, more or less useful; the name *camera obscura* was given to it from the circumstances explained above.

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* Two Latin words, signifying "a dark chamber."

540. **Methods of mounting.** — A great variety of forms have been given to this instrument, varying according to the circumstances under which it is applied, one of the most simple of these is shown in *fig*. 265.

The lens, L, is inserted in an opening in the top of a rectangular box, the height of which is made to correspond nearly with its focal length, the bottom of the box is placed at a convenient height to serve the purpose of a desk or table for the draughtsman; a sheet of drawing paper being placed upon it will receive the optical picture of such distant objects as may be found in the direction of the axis of the lens. The lens is set in a tube, which slides in the opening made in the box, so that by moving it more or less upwards or downwards, the instrument may be brought into focus, and a distinct picture produced upon the paper; an opening is made in the box, at that side of it towards which the bottom of the picture is turned; the draughtsman introducing through this opening the upper part of his person, lets fall over him a curtain, suspended from the upper edge of the opening, so as to exclude all light from the box, save that which proceeds from the lens at the top. Thus placed, the draughtsman can trace the outlines of the picture.

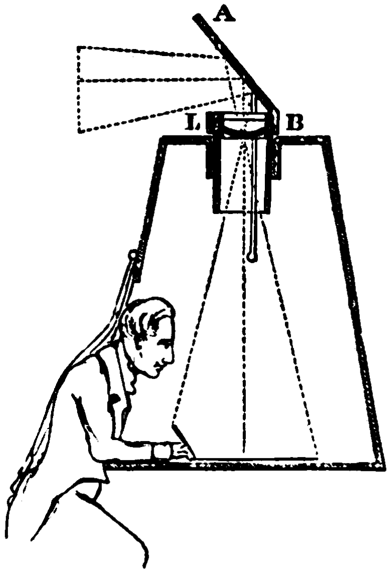

Fig. 265.

But in the case here supposed, the axis of the lens being vertical, the picture would be that of the firmament. To obtain a picture of any part of the surrounding landscape, a plane mirror, A B, is fixed upon a hinge at B, and is regulated in its position by a handle which descends into the box, so that the draughtsman can give it any desired inclination. The effect of this mirror is indicated in the figure by the rays, which, falling upon it, are reflected downwards to the lens. It will be evident, from what has been already explained in (142.) *et seq.*, that when this reflector is properly adjusted, a picture of the landscape before it will be reflected towards the lens L B, and by it projected upon the desk of the draughtsman.

The oblique mirror A B, and the lens L, are sometimes replaced with advantage by a prism, such as that represented in *fig.* 266. The face, *a c*, of this prism, at which the rays coming from the landscape enter, being convex, these rays are affected exactly as they would be if they entered the convex surface of a lens; when they fall upon the plane surface of *a b*, of the prism, they will be reflected from it, according to what has been explained in (123.); thus reflected, they will fall upon the other side, *c b*, of the prism; this side is ground concave, but its concavity being less than the convexity of the side *a c*, the effect of the two sides upon the rays will be the same as that of a meniscus

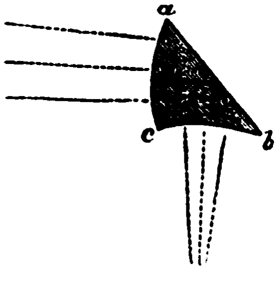

Fig. 266.

lens, one side of which has the convexity *a c*, and the other the concavity *b c*. In such a lens the convexity prevailing over the concavity, the effect will be that of a convex lens.

The curvatures of the two sides of the prism are so regulated that its focal length shall correspond with the height of the box.

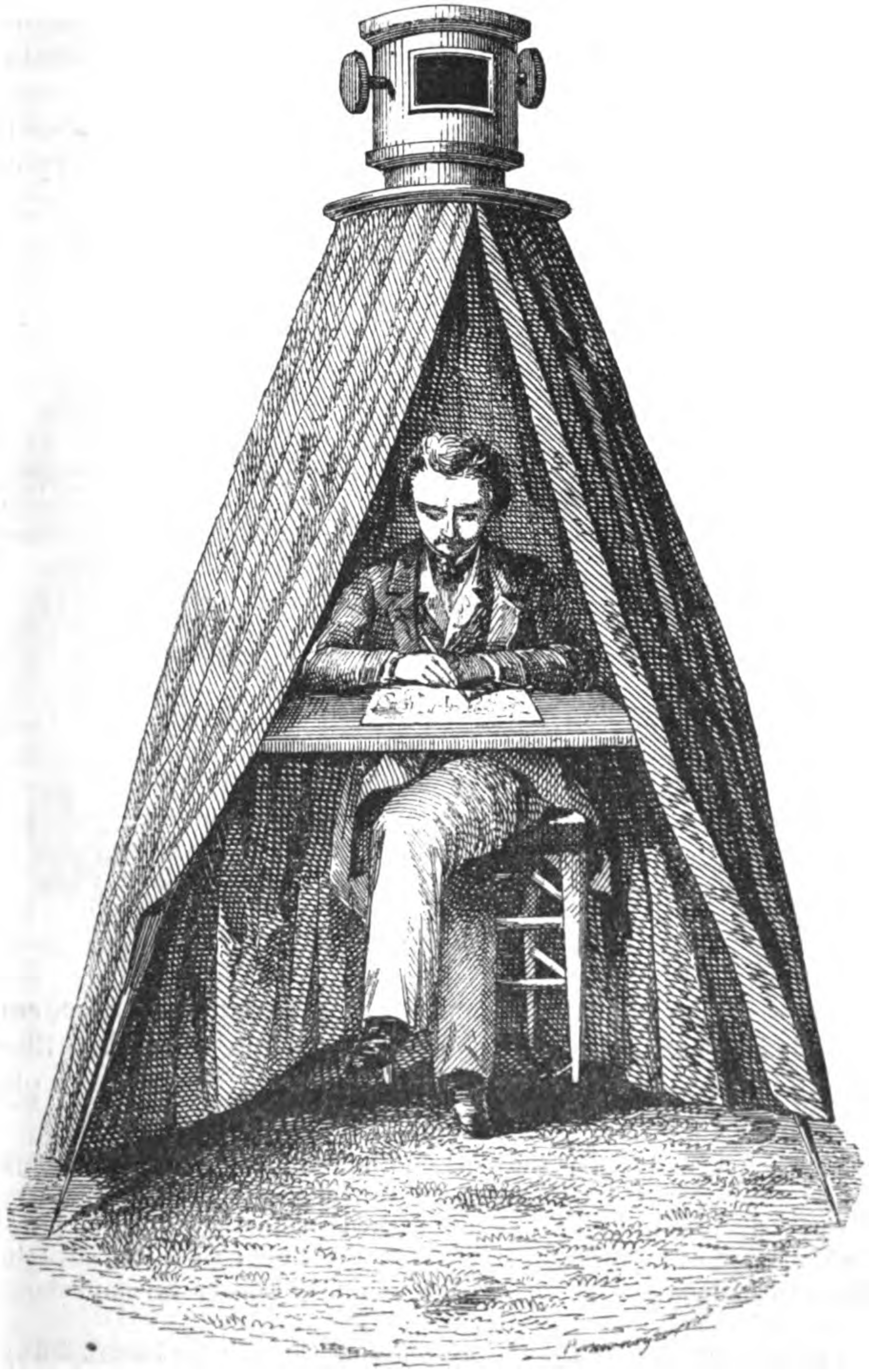

Fig. 267.

One of the methods of mounting a camera constructed with such a prism, is shown in *fig.* 267. The prism is mounted in a case, upon a horizontal axis, and its inclination is regulated by milled heads, like the heads of screws, on the outside; the case on which it is mounted has an opening through which the rays proceeding from the landscape are admitted; and it can be turned round its vertical axis, so that the opening can be presented in any direction to the surrounding landscape. The apparatus is supported by a triangle, and the draughtsman is surrounded by a curtain, forming a tent, from which the light is sufficiently excluded; the height of the tent, relatively to the table, is of course regulated according to the focal length of the prism.

541. **Portable camera.** — Another variety of mounting for cameras is shown in *fig.* 268. This, which is one of the most portable forms of the instrument, consists of a rectangular case, composed of two parts, one of which slides within the other like a drawer; in one end is placed the lens B, in the other a plain mirror M, inclined at an angle of 45° to the top of the box.

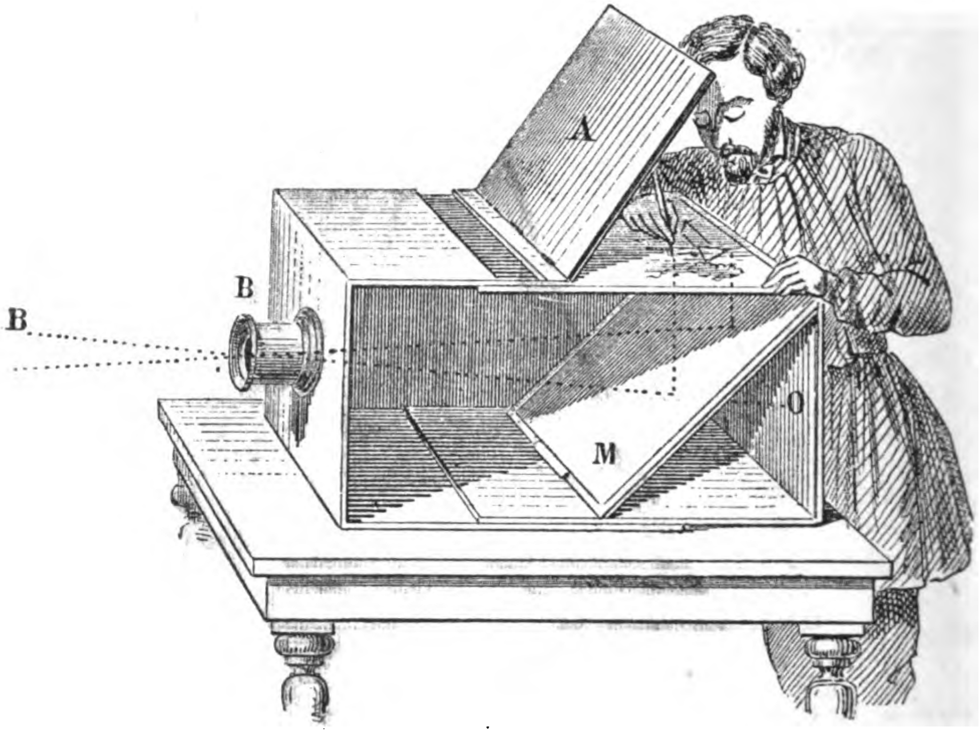

Fig. 268.

Over this mirror is a lid A, movable on hinges, under which in the opening is set a square plate of ground glass; the lid A is provided with arrangements by which it can be fixed at any desired inclination with the plate of ground glass, so as to shade the latter from the light; sides are sometimes provided to exclude the lateral light, which may also be accomplished by throwing a dark coloured cloth over the box.

The rays which produce the picture, entering through the lens B, fall upon the mirror M, by which they are reflected upwards, to the plate of ground glass N, on which they produce the picture. The instrument is brought into focus by drawing out the end O of the box, until the picture appears with sufficient distinctness on the glass N.

A leaf of tracing paper, being laid upon the glass, the picture is seen through it, so that it can be traced with facility and precision.

542. **Camera for photography.**—The form of camera usually applied for photography is represented in *fig.* 269.; it is more simple in its construction than those already described, neither the prism nor the oblique mirror being used.

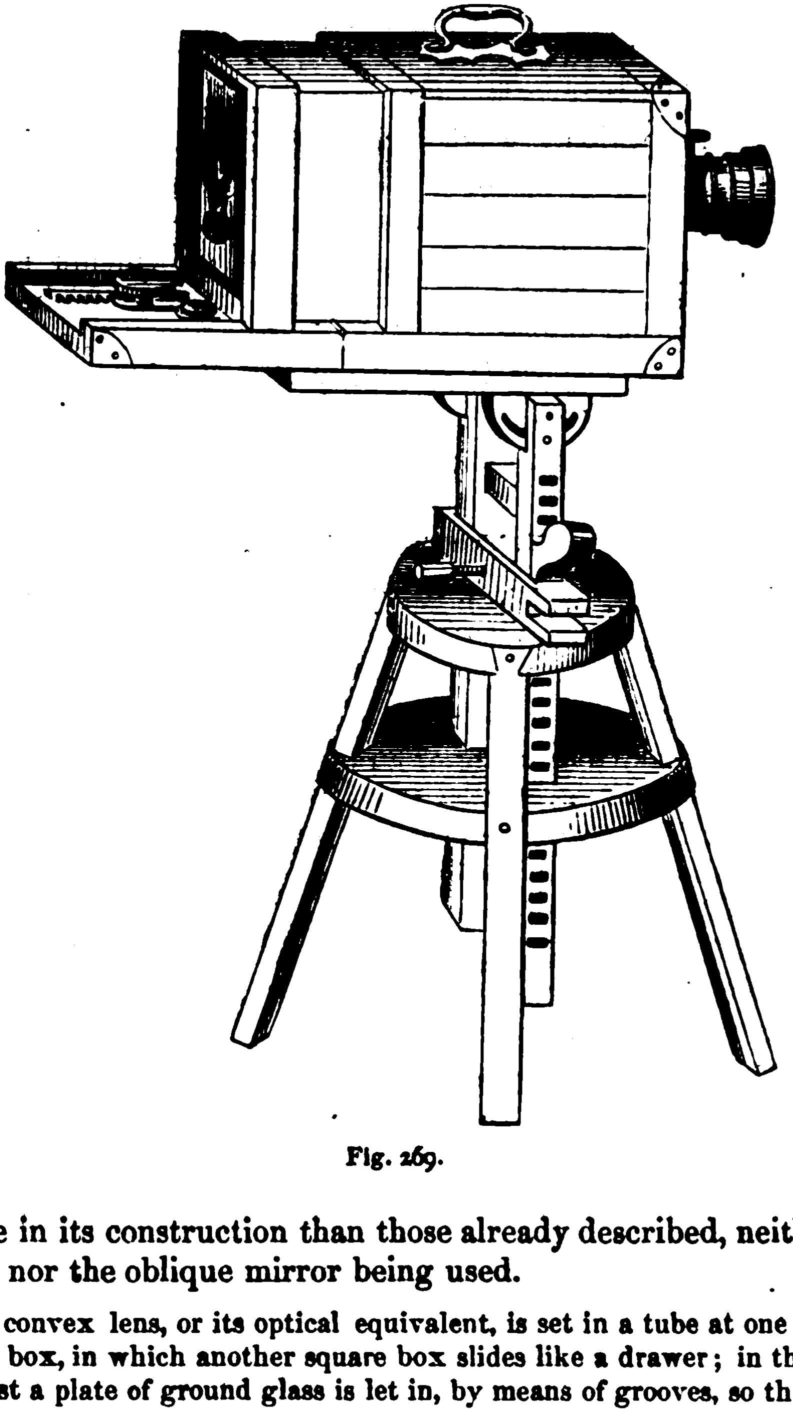

Fig. 269.

The convex lens, or its optical equivalent, is set in a tube at one end of a square box, in which another square box slides like a drawer; in the end of this last a plate of ground glass is let in, by means of grooves, so that it can be inserted and removed at pleasure; the instrument is brought into focus either by sliding the one box within the other, or by a rack and pinion in the groove. When the picture is distinctly delineated upon the ground glass, the latter is drawn out, and a case containing the daguerreotype plate or photographic paper is inserted in its place. The paper or plate being, in the first instance, screened from the reception of the picture, by a plate of metal or board let into a groove in front of it. When all is prepared for the operation, this screen is suddenly raised by the operator, and the picture allowed to fall upon the prepared paper or plate; and being allowed to continue there a certain number of seconds, more or less according to the brightness of the light, the screen is again suddenly let down, and the case containing the paper or plate is withdrawn from the groove, and the paper or plate is submitted to certain chemical processes by which the picture is brought out and rendered permanent.

The cameras which are adapted to photography, require to be constructed with greater attention to optical precision, than those which are used for other purposes in the arts. The focal length of the lenses being much shorter, optical expedients must be adopted for the removal of spherical aberration, which are not necessary in other applications of the instrument. The nature of photography also renders it necessary that the lenses should be achromatic, or nearly so.

X. THE CAMERA LUCIDA.

This instrument, which takes its name by contrast from the camera obscura, is one of the many gifts of the genius of Dr. Wollaston to the arts. Like the camera obscura, its chief, though not its only use, is to enable a draughtsman, by the mere process of tracing, to make a drawing of an object.

543. **Method of applying it.** — The observer places upon its table, a sheet of drawing paper, and the instrument being placed level with his eye, he looks into it, and sees the object to which it is directed, and at the same time sees, in the same direction, the sheet of paper which is upon his table, so that in fact, the object to be drawn, or its optical image, is seen projected and depicted on the paper.

If he take in his hand a pencil, and direct it to the paper, as if he were about to write or draw with it, he will see his own hand and the pencil directed to the paper, upon which the object is already optically delineated; and he will consequently be able, with the utmost facility and precision, to conduct the point of the pencil over the outlines of the object and those of every part of it, so as to make as correct a drawing of it as could be made by the process of tracing, in which a picture, placed under semi-transparent paper, is traced by a pencil moving over its outlines.

To present the principle of this contrivance under its most simple point of view, let A B, *fig.* 270., be an object which would be seen by the eye of an observer at E, under the visual angle A E B, and let P P be a sheet of paper, placed upon a horizontal table before the observer. Now let a piece of plane glass, one half of which is silvered on the lower surface, be placed at an angle of 45° with the direction in which the object A B is seen, so as to intercept the view of it from the eye at E; the rays A E and B E, which encounter the silvered part of the glass, and which previously proceeded to E, will now be reflected to *o*; still, however, retaining the same divergence, so that they will enter the eye E′ of the observer, supposed to look downwards at *o*, as if

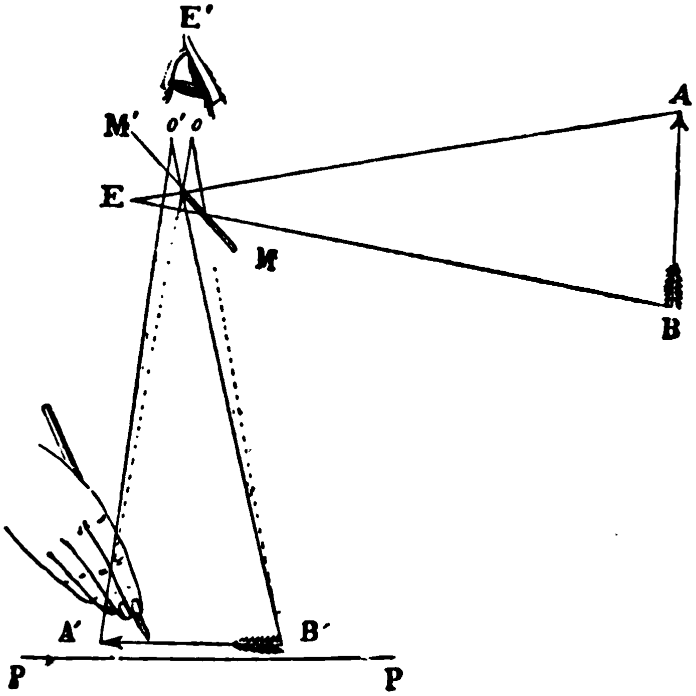

Fig. 270.

they had proceeded from A′ B′. In this manner the observer, looking from E′ towards the table, will see an image of the object at A′ B′, the point A′ of the image which corresponds with the top of the object being nearest to him, and the point B′, which corresponds with the bottom, being farthest from him; so that, in effect, the image will appear inverted.

Now suppose two lines, A′ *o*′ and B′ *o*′, drawn from the extremities of the image A′ B′, to a point *o*′ very near to *o*, and so as to pass through that part of the glass M M′ which is not silvered. An eye looking from *o*′ would then see the part of the paper upon which the image A′ B′ is projected, and would also see a pencil held in the hand of the draughtsman directed to the paper.

If the distance between the points, *o* and *o*′ be less than the diameter of the pupil of the eye, the observer looking down from E′ will see at the same time, and in the same position, the image A′ B′ and the part of the paper corresponding with it,—for he will see the image by the rays which converge to *o*, and the paper by those which converge to *o*′; the effect, in short, will be that he will see the image as if it were actually projected upon the paper.

If the eye be advanced towards the mirror, so far as to cause the limiting ray A′ *o* to graze the lower edge of the pupil, the paper will be altogether intercepted by the silvered part of the glass M M′, and the observer, though still seeing the image of A B reflected on the glass, will no longer see it on the paper, and for the same reason, he will see neither his hand nor the pencil; and he cannot, of course, make the drawing.

If, on the contrary, the eye be moved from the glass so far as to cause the limiting ray A′ *o* to graze the upper edge of the pupil, the image of A B reflected from M M′ will altogether disappear, and nothing but the hand and the pencil will be seen, these last being visible through the unsilvered part of the glass.

544. It is evident, therefore, that in order to enable the eye to see the entire image projected on the paper, it must be held in such a position, that while the limiting ray B′ *o*′, shall pass within the lower edge of the pupil, the limiting ray A′*o* shall pass within its upper edge. That this may take place, it is necessary that the distance between the points *o* and *o*′ shall not exceed the diameter of the pupil, and that the eye be steadily held, so that *o* and *o*′ shall be both within the pupil.

Since the average diameter of the pupil is two tenths of an inch, it follows that the distance between the points *o* and *o*′ should not exceed that limit, and that any displacement of the head, which would displace the eye through the space of two tenths of an inch, would remove from view the pencil or image, partly or wholly.

It will be easy from these considerations to appreciate the difficulty of using this instrument, and the necessity for practice and patience from those, who expect to acquire facility and expertness in its management.

545. **Method of correcting inversion.**—The inversion of the object produced by the reflector M M′, being inconvenient, a modification of the instrument was contrived, which gives an erect image; this is accomplished by the easy and obvious expedient of subjecting the rays proceeding from the object to two successive reflections, the first of which, as described above, would give an inverted image, which being itself inverted by the second, gives an erect image of the object.

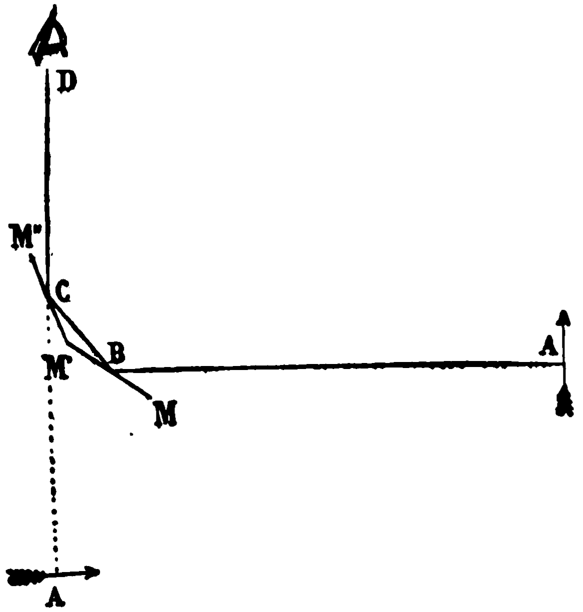

Fig. 271.

This is effected by two plane reflecting surfaces M M′ and M′ M″ (*fig.* 271.), placed at an angle with each other of 145°; the one, M M′, being inclined at

22½° with a horizontal line, and the other at the same angle with the vertical line. A ray A B, coming horizontally from the object, will fall upon M M′ at an angle of 22½°, and being reflected at the same angle, will fall upon M′ M″, still at the same angle, being reflected from it in the vertical direction C D. An object A, after the second reflection, will therefore be seen erect upon a level surface, before a draughtsman who stands with his face towards A, and stooping over the reflector M′ M″, sees the image of A in it.

In some forms of the instrument, the reflections are made by a prism, on the principle explained in (123.). Thus, if one reflection only be used, a rectangular prism is applied, as shown in *fig.* 272., the ray A B from the object entering the face of the prism perpendicularly, and being reflected at B to the eye at C.

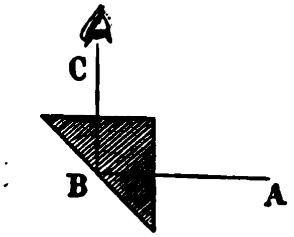

Fig. 272.

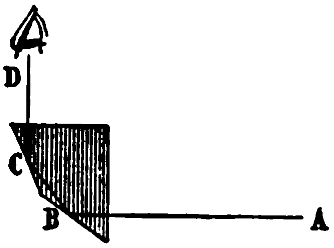

Fig. 273.

If two reflections be used, a quadrangular prism, having two angles of 67½°, one right angle, and one of 135°, is applied, as shown in *fig.* 273. The course of the ray from the object to the eye being A B C D.

In the preceding cases we have supposed the observer to see the object by reflection, and the paper and pencil directly; but it is evident that the conditions may as easily be reversed, so that the object may be seen directly, and the paper and the pencil by reflection. Thus we may suppose the plane mirror M M′ in *fig.* 270., to be silvered in the upper instead of the lower surface, and the observer looking from E horizontally to see the object directly through the unsilvered part, while he sees the paper and pencil by the reflection from the silvered part.

This method is in many cases found more convenient than that first described.

546. **Amici's camera.**—In some forms of the instrument, the observer looks at the object through a small hole made in a plane reflector, placed at an angle of 45° in the direction of the paper, the diameter of the hole being less than that of the pupil. In this case, while the object is seen directly through the hole, the paper and pencil are seen by reflection from the surface of the reflector surrounding the hole; this is the form of the camera lucida applied to the microscope by Professor Amici.

547. **Magnitude of picture.**—Whatever be the form of the camera, the visual magnitude of the image projected on the paper as seen by the eye applied to the instrument, is the same as the visual magnitude of the object seen directly, and this will be the case at whatever distance from the camera the paper may be

placed. It follows from this, that the actual magnitude of the picture projected on the paper will be greater or less, according to the distance of the paper from the camera, and that consequently the observer, by regulating the distance of the paper, can obtain a picture of the object on any scale he may desire.

To render this more apparent, let C, *fig.* 274., be the place of

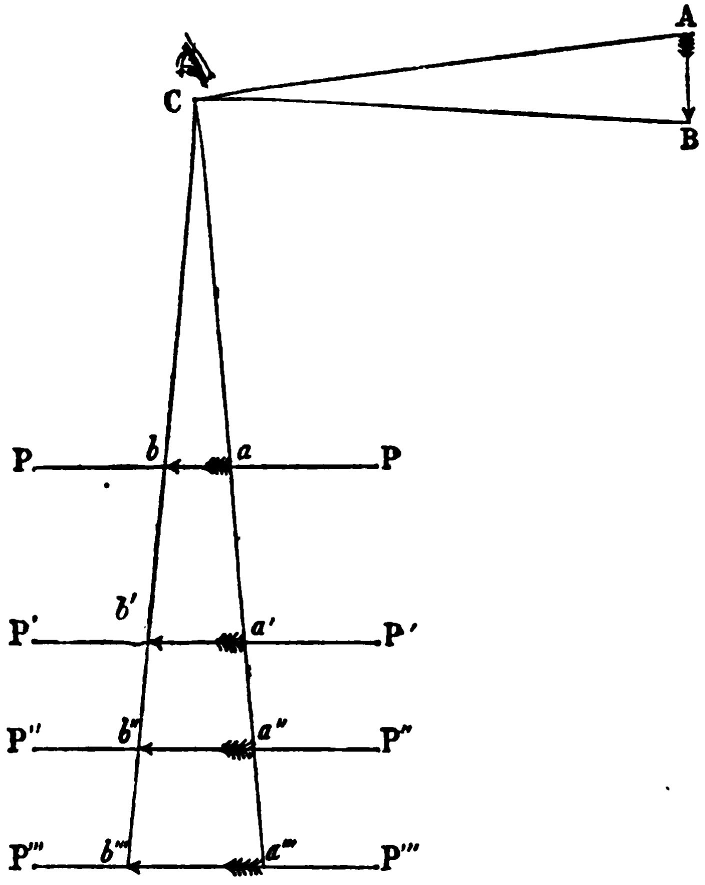

Fig. 274.

the camera, and A B the object, whose visual angle will therefore be A C B. If the paper be placed at P P, the lines C *a* and C *b*, drawn to the extremities of the image upon it, will make the angle *a* C *b* equal to A C B, so that the visual angle of the image *ab*, will be equal to that of the object A B.

If the paper be now removed to P′ P′, the visual lines C *a*, C *b*, continued to it at *a′ b′*, will still be those which mark the extremities of the image, whose visual magnitudes will therefore be measured by the same angle. But the space which the image covers on the paper at P′ P′, or, what is the same, the actual length of the optical picture on the paper will be greater than at P P, in the proportion of *a′ b′* to *a b*, or, what is the same, to the distance of P′ P′ to that of P P from C.

In the same manner it will appear that if the paper be successively moved to greater distances, such as $P''P''$, and $P'''P'''$, the picture will be magnified in its linear dimensions, in the exact proportion in which its distance from the camera is increased.

548. **Application to microscope.** — One of the most recent and beautiful applications of the camera lucida, is its adaptation to the compound microscope, by means of which, details and lineaments of objects, so minute as to escape ordinary vision, are depicted with a precision and fidelity only surpassed by the results of photography.

The instrument is fixed upon the eye piece of the microscope in such a manner that, while the observer looks directly through the eye glass at the object, he sees the paper and pencil by reflection, the latter being placed upon the table before him. Supposing the axis of the microscope to be horizontal, the paper and pencil will be reflected from a plane mirror placed at an angle of 45° with the vertical, the reflecting side being turned downwards.

The instrument may be so arranged that the paper may be seen directly, and the object by reflection. In this case, the mirror is also placed at 45° with the vertical; but the reflecting side is presented upwards. The rays, proceeding through the eye glass from the object, are reflected upwards and received by the eye of the observer, which, looking downwards, views the paper directly.

In *figs*. 275. and 276. is shown the arrangement, by which the observer O views the object directly through a small hole in the oblique reflector, which is fixed upon the eye piece, while he sees the paper and pencil by two reflections, the first from the back of the prism P, and the second from the oblique reflector M *m*. The effect is to project the image of the object seen in the microscope V, upon the image of the paper seen in the reflector M *m*.

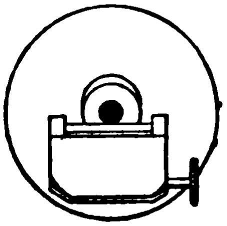

Fig. 275.

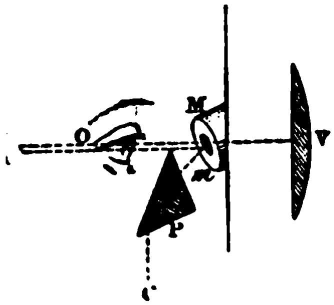

Fig. 276.

The prism P is interposed in this case to render the image of the hand and pencil erect. A front view of the prism and eye piece is shown in *fig*. 275., and a side view in *fig*. 276.

In *fig.* 277. an arrangement is shown by which the object is seen by reflection, and the paper directly.

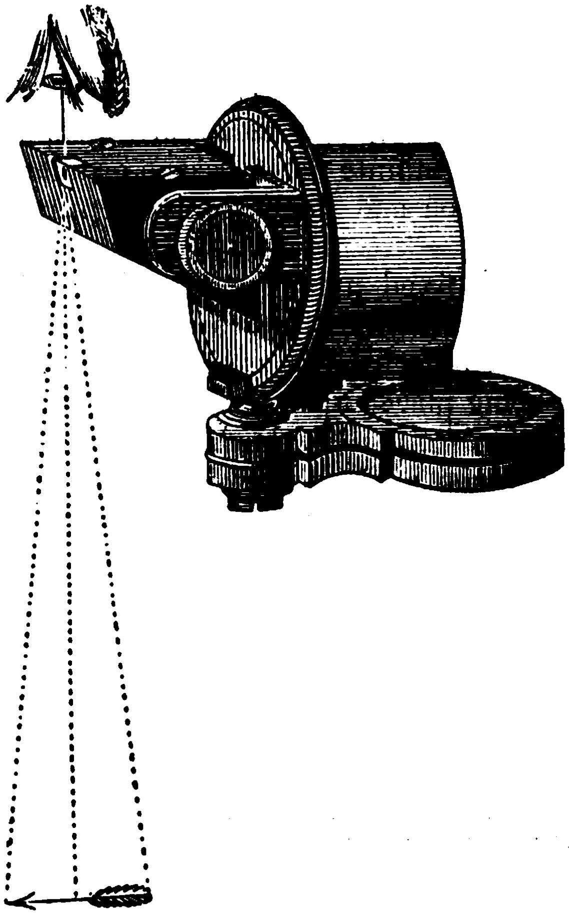

Fig. 277.

In this case the rays issuing from the eye piece of the microscope are reflected twice successively from the two sides of the prism, which are inclined to each other at an angle of 135°, as explained in (123.).

According to what has been explained in (547.), the observer can vary the magnitude of the picture on the paper by varying the distance of the paper from the prism, without varying the magnifying power of the microscope; and in this way he can make a tracing of the object on any desired scale.

XI. THE STEREOSCOPE.

549. **Surprising effects of the instrument explained.**—The surprise excited by the impressions of perspective and relief produced by the stereoscope have never, as we think, been fully or adequately explained. This emotion of astonishment does not merely arise, as is commonly supposed, from the fact that such impressions are stronger than those produced by the best executed drawings or paintings, but that, paradoxical as it may seem, they are actually in many cases stronger and more vivid, than any which could be produced by the objects themselves. In a word, the stereoscope has the property of exaggerating the natural effects of perspective and relief. To comprehend this it will only be necessary to revert for a moment to the principles upon which the effects of vision are based.

The mind judges of the relative position, form, and magnitude of visible objects by comparing their apparent outlines and varieties of light and shade with the previously acquired impressions of the sense of touch. The knowledge that such and such visual appearances and optical effects are produced by certain varieties of form, position, and distance having been already acquired, it substitutes with the quickness of thought the cause for the effect. The continual repetition of such acts, which are necessarily repeated as often as the sense of vision is exercised, and the extreme rapidity with which all such mental operations are performed, render us unconscious of them, and we imagine that shape, distance, and position are the immediate subjects of visual perception, instead of being consequences deduced from a set of perceptions of a wholly different kind.

550. **Causes of visual perspective and relief.**—In drawing and painting the effects of perspective and relief are therefore reproduced, by transferring to the canvas the same outlines and the same varieties of light and shade, which the objects delineated really present to the eye, and when this has been accomplished with the necessary degree of fidelity and precision, the same impression of distance, perspective, and relief is produced, as that which would be received from the immediate view of the objects themselves which are delineated.

551. **Effects of binocular parallax.**—In certain exceptional cases, however, a class of visual phenomena is manifested which are quite independent of mere outline and varieties of light and shadow, and which no effort of art can transfer to canvas. Inasmuch, also, as these phenomena, like those already mentioned, are optical effects of distance, form, and position, they become, like

the others, indications by which the mind judges of the relative forms and positions of the objects which produce them. Phenomena of this class are manifested, when the objects viewed are placed so near the observer, as to have sensible binocular parallax. The aspects under which they are seen in this case by the two eyes, right and left, are different. Certain parts are visible to each eye which are invisible to the other, and the relative position in which some parts are seen by one eye, differs from those in which the same parts are seen by the other eye. This difference of aspect and apparent position, arises altogether from the different position of the two eyes in relation to the objects. It is a phenomenon, therefore, which can never be developed, in the case of objects whose distance bears a large proportion to the distance between the eyes, because there is no sensible difference between the aspects under which such objects are viewed by the one eye and the other. The phenomenon, therefore, can only be manifested in relation to objects, whose distance from the observer is a small multiple of the distance between the eyes.

To render this more clear let us imagine a bust presented to an observer at a distance of a few feet, the face being turned obliquely so that one side is presented more to view than the other. Supposing the side which is turned towards the observer to be on his right, it is evident that the nose will intercept, more or less, the view of the side of the face which is on his left, but the part which it thus intercepts will not be the same for both eyes. It will evidently intercept more from the right than the left eye. On the other hand, the right eye will see a part of the right side of the bust, which will be concealed from the left eye by the projecting parts of the face.

It therefore appears that the two eyes, right and left, will have different views of the bust; so that if the observer were to make an exact drawing of the bust with his left eye closed, and another exact drawing of it with his right eye closed, these drawings would not be identical. One of them would show a part of the bust on the extreme right, which would not be exhibited in the other, and the latter would show a part on the extreme left, which would not be included in the former. Moreover, a part of the cheek and the eye would be shown in the drawing made with the right eye closed, which would not appear in the drawing made with the left eye closed.

Two such views of the same object are shown in *figs.* 278. and 279., the former being the view presented to the left and the latter to the right eye.

Now it is evident that when such an object is looked at with both eyes open, the two different visual impressions here described are simultaneously perceived, and they become to the mind, like the other visual impressions already described, signs and indications of the actual forms which produce them.

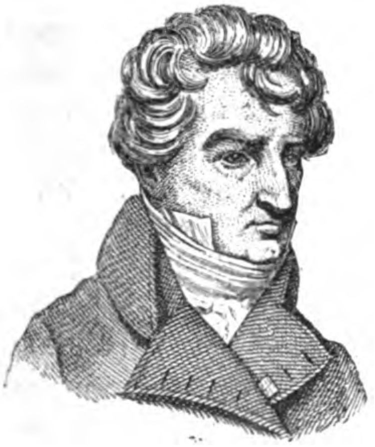

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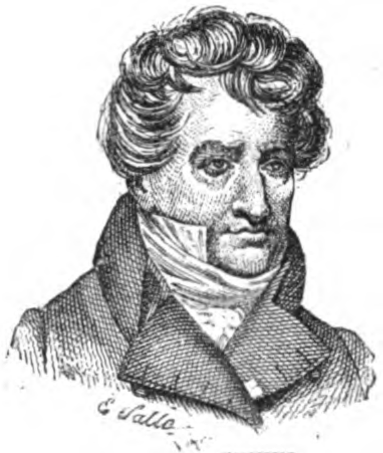
Fig. 278.



CUVIER

Fig. 279.

When objects, therefore, can be viewed at distances small enough to be attended with a sensible degree of binocular parallax, their perspective and relief are perceived, not only by the outlines and varieties of light and shade, which are the common indications of perspective and relief at all distances, but also by the class of binocular phenomena which we have just described.

Hence it follows that the perception of relief, and generally of form and relative position in objects whose proximity is sufficient to produce binocular parallax, is much stronger and more vivid than those whose distances, rendering the binocular parallax evanescent, leaves nothing but the outlines and the varieties of light and shadow, by which the mind can form a judgment of form, relative distance, and position.

But, since binocular parallax is reduced to the very small amount of half a degree at the distance of 24 feet, it is clear that it can only enter into the conditions by which we perceive perspective and relief, in the case of a very limited class of objects, and is not at all applicable to objects in general whose forms and perspective we habitually contemplate.

552. **Principle of the stereoscope.** — After what has been explained of the two different views which a near object presents, when looked at successively with the one eye and the other closed, the principle of the stereoscope will be easily understood.

A bust being placed before a competent draughtsman, as above described, at a distance sufficiently small to produce considerable binocular parallax, let him make two exact drawings of it, one with the right eye closed, and the other with the left eye closed. These two drawings will then represent the object as it is actually seen, when the optic axis of each eye is directed to it. Let us suppose that, by some optical expedient, the two drawings thus made can be so presented to the two eyes, that the optic axes, when directed to them, shall converge at the same angle as when they are directed to the object itself. In that case each eye will obtain the same view which it would obtain if the object itself were placed before it, and the visual perception must necessarily be the same as would be produced by the object looked at with both eyes open.

553. **Origin of the name.** — Now the optical expedient by which this is accomplished is the *stereoscope*, a name derived from two Greek words, στερεόν (stereon), *a solid object*, and σκοπέω, (skopeo) *I look at;* inasmuch as the effect is such as to make the observer imagine that a really solid object (in the geometrical sense of the term), instead of a flat surface, is placed before him.

Various optical combinations have been proposed and contrived, for the purpose of producing this effect upon two such drawings as we have here described. In some the visual rays proceeding from the pictures are thrown into the requisite direction by reflection, and in others by refraction.

554. **Wheatstone's reflecting stereoscope.** — In the first form given to the instrument by Professor Wheatstone, its inventor, the visual rays proceeding from the two pictures were deflected by two plane reflectors placed at a right angle, so that in entering the eyes they proceeded as if they had diverged from a common point, at which the object represented by the pictures would therefore appear to be placed.

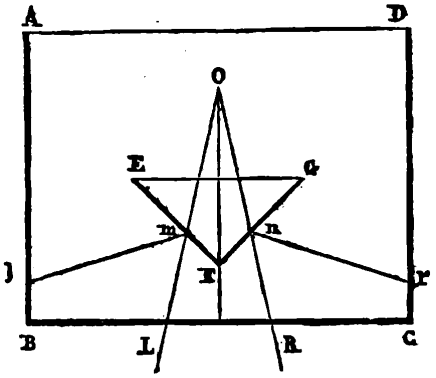


Fig. 280.

Let A B C D (*fig.* 280.) be the ground plan of a rectangular box, open upon the side A D so as to admit the light. Let R and L be two eye holes made in the side B C, at a distance apart equal to the distance between the eyes of the observer. Let E F and F G be two plane mirrors placed at right angles to each other. Let a drawing of an object seen with the right eye, the left being closed, be attached to the inside of D C at *r*, and another made from the object seen with the left eye, the right being closed, be in like manner attached at *l* to the inside of A B. Supposing the eyes of the observer to be placed at the holes R and L, the right eye will see by reflection the drawing *r* in the direction R *n*, and the left eye will see the drawing *l* by reflection in the direction L *m*. If the lines L *m* and R *n* be imagined to be continued backwards, they will meet at a certain point O behind the reflectors; and if the drawings *r* and *l* be made to correspond with the views which the right and left eyes would have respectively of the object itself, which they represent, placed at O, the impression produced by the two drawings thus seen will be precisely the same as those which would be produced on the right and left eye respectively by the object itself seen at O.

555. **Sir David Brewster's lenticular stereoscope.**—In this, which is the form of the instrument to which the public in general in all countries have given the preference, the visual rays proceeding from the two pictures are deflected and made to diverge from the desired distance, by means of two eccentric double convex lenses.

These are formed by cutting a double convex lens A B C D (*fig.* 281.), into two semi-lenses B A D and B C D, in the direction of a plane B D, passing through the centre of the lens. The two eccentric lenses are then cut out of these, so that their diameters A E and C E shall be the semi-diameters of the original lens. It will be evident that a section of the original lens, made by a plane passing through A E C at right angles to its surface, will have the form represented at A E C (*fig.* 282.), and consequently that the two eccentric lenses A E and C E will have their thickest part at E, and their thinnest at A and C. While the geometrical centres of these lenses are at *o* and *o*′, their optical centres are at the thickest point E of the radius.

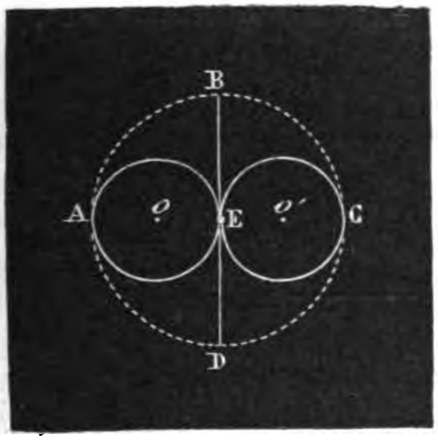


Fig. 281.

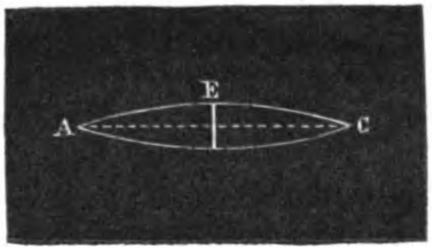


Fig. 282.

Now suppose these two lenses to be set with their edges A and C towards each other in two eye holes whose distance apart is equal to that of the eyes, and let two objects, P and P′ (*fig.* 283.), be placed before them at a distance equal to their common focal length. According to the properties of lenses already explained, pencils of rays diverging from P and P′, and passing through the lenses, will be, after refraction, parallel respectively to lines drawn from P and P′, through the optical centres E and E′ of the lenses. Thus the visual ray P *p* will, after refraction, issue in the direction *p* L, and the ray P′ *p*′ will issue in the direction *p*′ R, so that the points P and P′ will be seen in the directions L *p* and R *p*′ converging to the point O.

Now if P be a picture of an object as it appears to the left eye, and P′ a picture of it as it appears to the right eye, these two pictures will be brou together at O by the refraction of the lenses, and the eyes will see the combined pictures at O exactly as they would see the object itself if it were placed there.

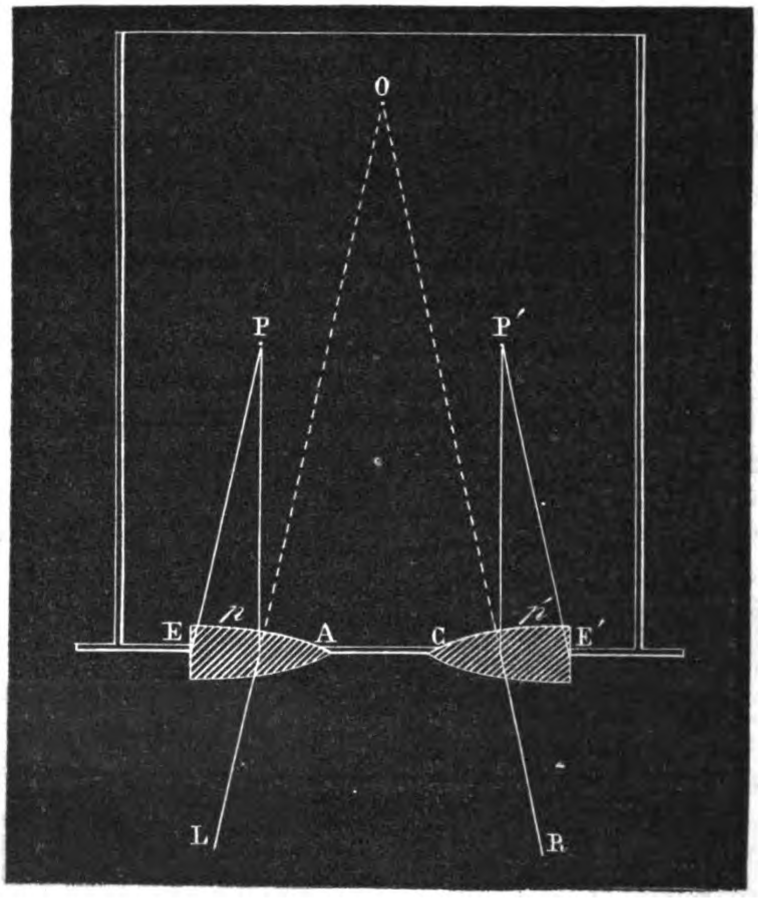


Fig. 283.

An advantage incidental to this arrangement is, that the convexity of the lenticular eye pieces A E and C E′, may be such as to produce any desired magnifying effect, within practical limits, upon the two pictures.

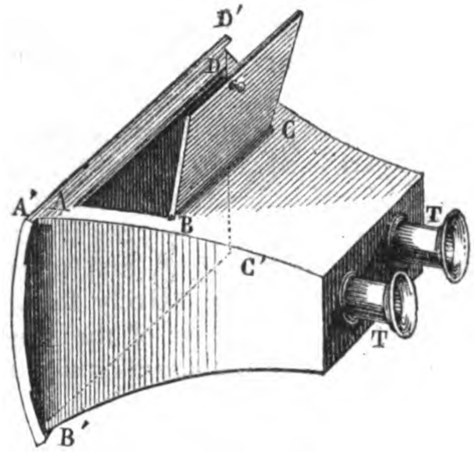


Fig. 284.

The tubes containing the eye glasses A E and C E′, are made to draw in and out so as to be adapted to different eyes; and they are fixed by pins, which pass into slits made in them, in that position in which the deflected rays have the proper degree of divergence.

The form in which this lenticular stereoscope is usually constructed, is shown in *fig.* 284. The pictures are either opaque or transparent. If they are opaque, they are illuminated through an opening A B C D, covered by a hinged lid, the inside surface of which is coated with tinfoil so as to reflect light upon the pictures. If they are transparent, the base of the instrument A′ B′ C′ D′ has a plate of ground glass set in it, which allows a diffused light to pass through the pictures.

556. **Method of obtaining stereoscopic pictures.**—In what has been stated above, it has been assumed that two drawings of the same object can be produced, differing one from another precisely as the two views of the same object would differ, when viewed by the right and the left eye successively, subject to a given degree of binocular parallax. Now, the difficulty, if not the total impracticability, of accomplishing this, with the extreme precision which is indispensable, by any process of hand-drawing, will be apparent; and if the stereoscope were dependent on such a process, the most remarkable effects manifested by it would never have been witnessed. Fortunately, however, contemporaneously with this beautiful optical invention, another, still more remarkable, was in progress of improvement. Photography lent its powerful aid to the stereoscope, and supplied an easy and perfectly accurate and efficient means of producing the right and left monocular pictures. If two lines be imagined to be drawn from the object inclined to each other at the angle which measures the proposed binocular parallax, two photographic instruments placed one on each of these lines, at the proper distance from the object, will produce the two desired pictures; or the same instrument would do so, placed successively in the directions of the two lines.

The stereoscopic pictures are accordingly produced by this method either upon daguerreotype plates, photographic paper, or glass. On daguerreotype plates they are necessarily opaque; on glass they are transparent; and on paper may be either opaque or transparent, according to the thickness and quality of the paper.

Since the greater number of stereoscopic pictures represent views of objects which must be so distant from the observer as to have no sensible binocular parallax, it may be asked how it is that stereoscopic effects, so remarkable as those which are manifested by such pictures, can be produced. If the stereoscopic effects be the consequences of binocular parallax, and of that alone, how can such effects be produced by pictures of objects, which have no such parallax?

557. **How the effects of relief are produced.**—This brings us back to a statement made in the commencement of this notice, that the appearance of perspective and relief produced by the stereoscope is, in most cases, exaggerated, as compared with that

produced by an immediate view of the objects themselves, and that it is consequently such as can never be perceived when the objects themselves are looked at; and that hence arises the sensation of surprise that such stereoscopic effects never fail to excite.

If we desire to obtain a pair of stereoscopic pictures of any object of considerable magnitude, a palace or a cathedral, for example, we take a position at such a distance from it as will enable us to obtain, in the camera obscura of the photographic apparatus, a picture of it on a sufficiently small scale. Supposing, then, two lines to be drawn from the centre of the object to the place selected for the camera, making with each other an angle equal to the amount of binocular parallax, which is necessary to produce the stereoscopic effect of perspective and relief; let two photographic instruments be then placed one on each of these lines, with their optic axes in the directions of the lines respectively, and therefore converging towards the same point of the object, and let the distances of their object glasses from that point be equal. The optical pictures which they will produce will in that case be those which would be seen by two eyes, right and left, having a distance apart equal to the distance between the object glasses of the two photographic instruments.

When the pictures are thus produced on a small scale they are placed in the stereoscope, the eye glasses of which will have the effect of causing them to be viewed in lines converging at the same angle, as that formed by the optic axes of the two photographic instruments by which the pictures were produced.

558. **Natural relief greatly exaggerated.**—It will be manifest, then, that the impression produced by the view of such pictures in the stereoscope will be such, as could never be produced by the immediate view of the objects themselves, inasmuch as they could never be seen with any such degree of binocular parallax, as that which has been given to them by the relative position of the two photographic instruments. This parallax will be greater than the natural binocular parallax of the object, in the same proportion as the distance between the centres of the object glasses of the two photographic instruments, is greater than the distance between the eyes. Thus if, in taking such a pair of stereoscopic views of a building, the distance between the photographic instruments is 50 inches, the parallax thus produced will be greater than the natural binocular parallax in the proportion of 50 to 2½ or 20 to 1, and so far as the perception of perspective and relief depends on binocular parallax, that which is produced from viewing the pictures of the building in the stereoscope, will be 20 times more strong and vivid than that which is produced

by the view of the building itself, seen from the station at which the pictures are taken.

It is then rigorously true, that the surprise and admiration excited by the stereoscope, does not arise from the truth of the picture which it presents, but from the strong exaggeration of perspective and relief which it exhibits. It is very true that no art of the draughtsman or painter could produce any such effects; but it is equally true that no such effects could be produced by the objects themselves.

Among the most interesting and instructive as well as surprising effects of the stereoscope, are those which it exhibits when stereoscopic views of geometrical solid figures are exhibited in it. The variety of these is endless. But since no mere verbal description could convey any adequate idea of them, we can only invite the reader's attention to this class of objects.

XII. THE KALEIDOSCOPE.

559. **Origin of the name.**—This pretty optical toy, named from three Greek words, *καλόν εἶδος* (kalon eidos), *a beautiful form*, and *σκοπέω* (skopeo), *I see*, was invented by Sir David Brewster, for the purpose of creating, in indefinite number and variety, beautiful forms, and exhibiting them so that they may be copied and rendered permanent.

560. **Structure of the instrument.**—Two oblong slips of looking-glass, A *a c* C and A *a b* B (*fig.* 285.), are placed edge to edge at A *a* inclined to each other at an angle of 60°. Thus placed, they are fixed in a tube of tin or brass of corresponding size, an end view of which is shown in *fig.* 286., where the circle A C B represents the tube, and A B and A C the edges of the plates of glass. One end of the tube is covered by two discs of glass, between which broken

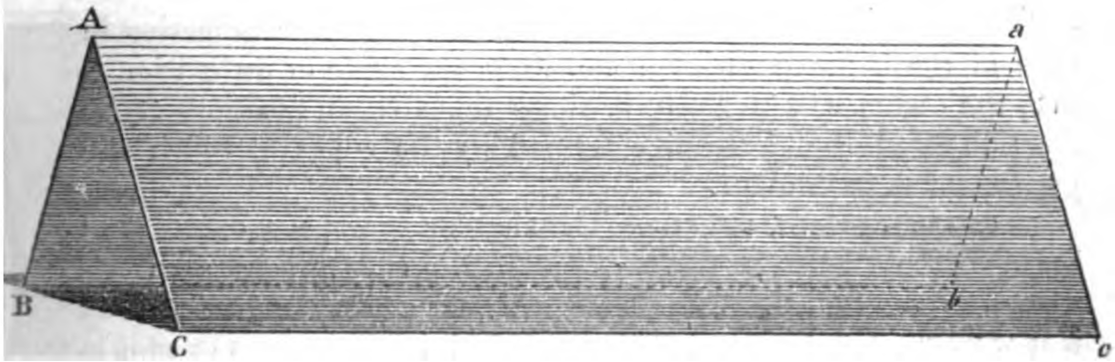


Fig. 285.

pieces of coloured glass or other transparent coloured objects are placed loosely, so that they can fall from side to side, and take an infinite variety of casual arrangements. The external disc is ground glass, to prevent the view of external objects disturbing the effect. The other end of the tube is covered by a diaphragm, with a small eye hole in its centre, through which the observer looks at the coloured objects contained in the cell at the other end. He not only sees these objects, but also their reflection in each of the inclined glasses; and when the angle of inclination is 60°, the object will be seen five times repeated, in positions regularly disposed round the line formed by the edges at which the glasses touch each other.

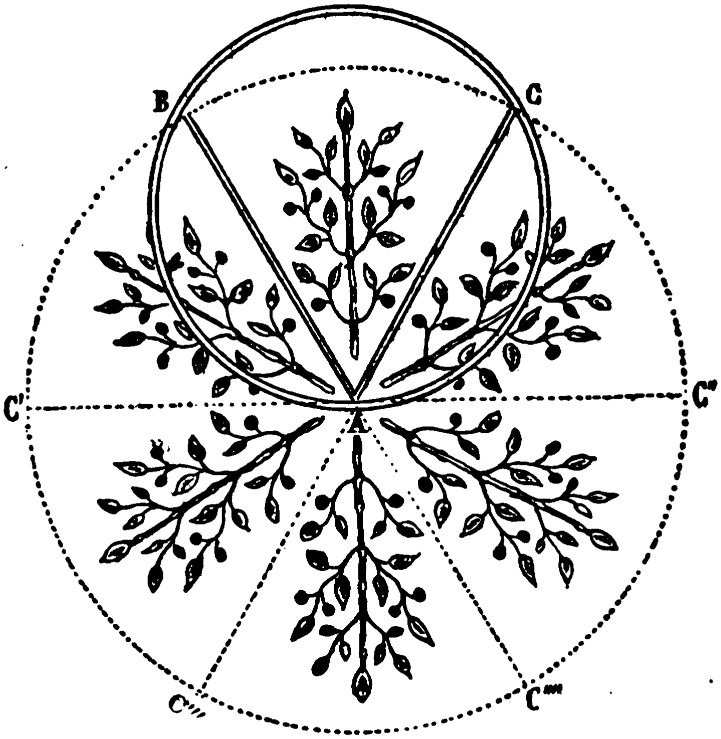


Fig. 286.

561. **Its optical effect.** — The angular space, B A C, included between the glasses, and every object within it, will be seen reflected in each glass. Thus B A C will be seen in the glass B A, as if it were repeated in the space B A C′ and in the glass A C, as if it were repeated in the space C A C″. But this is not all. The reflection B A C′ becomes an object before the glass A C, and being reflected by it, is reproduced in the space C″ A C⁗, and the reflection C A C″ being reflected by the glass A B, is reproduced in the space C′ A C‴. Thus, besides the view of the objects themselves which are between the glasses, and which would be seen if there were no reflection, the observer will see the four reflections, two,

C A C″ and C″ A C⁗, to the right, and two, B A C′ and C′ A C‴, to the left.

But the reflection C′ A C‴ is again reflected by the glass A C, and is seen in the space C‴ A C⁗, and at the same time the reflection C″ A C⁗ is reflected in the glass A B, and is also reproduced in the same space C‴ A C⁗. Thus it appears that this space C‴ A C⁗ receives the reflection of both glasses.

The observer, looking through the eye hole of the kaleidoscope, sees a circle whose apparent diameter, C C‴, is twice A C, the breadth of the reflector. This circle is divided into six angular spaces, two of which are the first reflections, and other two the second reflections of the inclined glasses. The other two consist of the actual space included between the glasses, and a similar space opposite to it which receives at once the third reflection of both glasses

Since looking glasses never reflect *all* the light incident upon them, these reflections will not be as vivid as the direct view of the space B C; nor will they, compared one with another, be equally vivid. The reflections B C′ and C′ C″ will be less vivid than the object B C, but more so than the second reflections C′ C‴ and C″ C⁗. The third reflection C‴ C⁗ would be less vivid than the second C′ C‴ and C″ C⁗, if it proceeded only from one glass, as do the latter. But it must be remembered that being the combined reflection of both glasses, the loss of brightness by the multiplied reflections of each glass is to some extent compensated.

562. **Varieties of form.** — We have here supposed that the glasses are inclined at 60°, but they may be inclined at any angle which is an aliquot part of 360°. Thus if they are inclined at 90°, the circular space or field of view round A will be divided into four angular parts, and the same observations are applicable. If the glasses are inclined at an angle of 45°, the field of view will be divided into eight equal angular spaces, seven of which will be filled by the reflections.

From what has been here explained, the unequal brightness of the spaces seen in the kaleidoscope will be understood. If, as is most common, the angle of the glasses be 60°, this is perceptible; but if it be 45°, the repeated reflections so reduce the brightness as to impair the beauty of the effect.

Such being the optical principle of the instrument, it remains to explain some practical conditions which are necessary to the due development of the phenomena. Let A C E and B C E, *fig.* 287., be the two mirrors, C E being their line of junction or common intersection. If the object be placed at a distance, as at N N, then there is no position of the eye at or above E which will give a symmetrical arrangement of the six images shown in *fig.* 286.; for the corresponding parts of the one will never join the corresponding parts of the other. As the object is brought nearer and nearer, the symmetry increases, and is more complete when the

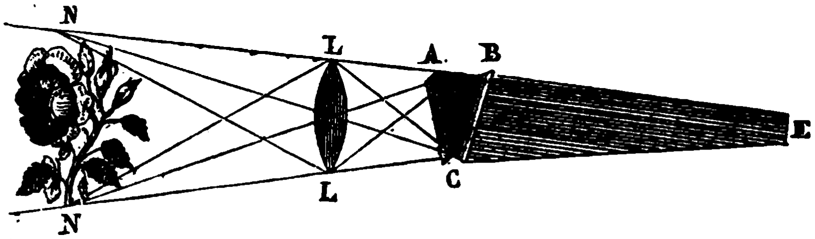


Fig. 287.

object N N is quite close to A B C, the ends of the reflectors. But even here it will not be perfect, unless the eye is placed as near as possible to E, the line of junction of the reflectors.

563. **Conditions of symmetry.**—The following, therefore, are the three conditions of symmetry in the kaleidoscope: —

I. That the reflectors should be placed at an angle which is an *even* or an *odd* aliquot part of a circle, when the object is regular and similarly situated with respect to both the mirrors; or an even aliquot part of a circle, when the object is irregular.

II. That out of an infinite number of positions for *the object* both within and without the reflectors, there is only one position where perfect symmetry can be obtained, namely, by placing the object in *contact* with the ends of the reflectors, or between them.

III. That out of an infinite number of positions for the *situation of the eye*, there is only one where the symmetry is perfect, namely as near as possible to the angular point, so that the whole of the circular field can be distinctly seen; and this point is the only one at which the uniformity of the reflected light is greatest.

In order to give variety to the figures formed by the instrument, the objects, consisting of pieces of coloured glass, twisted glass of various curvatures, &c., are placed in a narrow cell between two circular pieces of glass, leaving them just room to tumble about while this cell is turned round by the hand. The pictures thus presented to the eye are beyond all description splendid and beautiful, an endless variety of symmetrical combinations presenting themselves to view, and never again recurring with the same form and colour.

564. **Application of object lens to it.**—"For the purpose of extending the power of the instrument, and introducing into symmetrical pictures external objects, whether animate or inanimate," says the inventor, "I applied a convex lens, L L, *fig.* 287., by means of which an inverted image of a distant object N N, may

be formed at the very extremity of the mirrors, and therefore brought into a position of greater symmetry than can be effected in any other way. In this construction the lens is placed in one tube, and the reflectors in another, so that by pulling out or pushing in the tube next the eye, the image of objects at any distance can be formed at the place of symmetry. In this way flowers, trees, animals, pictures, busts, may be introduced in symmetrical combination. When the distance E B is less than that at which the eye sees objects distinctly, it is necessary to place a convex lens at E, to give distinct vision of the object in the picture." *

XIII. Polarising Photometer.

565. **Babinet's polarising photometer.** — M. Babinet has recently invented a photometer of great sensibility, depending on the polarisation of light. A perspective view of this apparatus is given in *fig*. 288., and a section by a plane passing through the axes of the tubes in *fig*. 289. Two tubes, a long one, *a b*, and a shorter one, *d c*, are fixed at an angle of 70° 50′, which is twice the polarising angle of glass. At the point where the axes of the

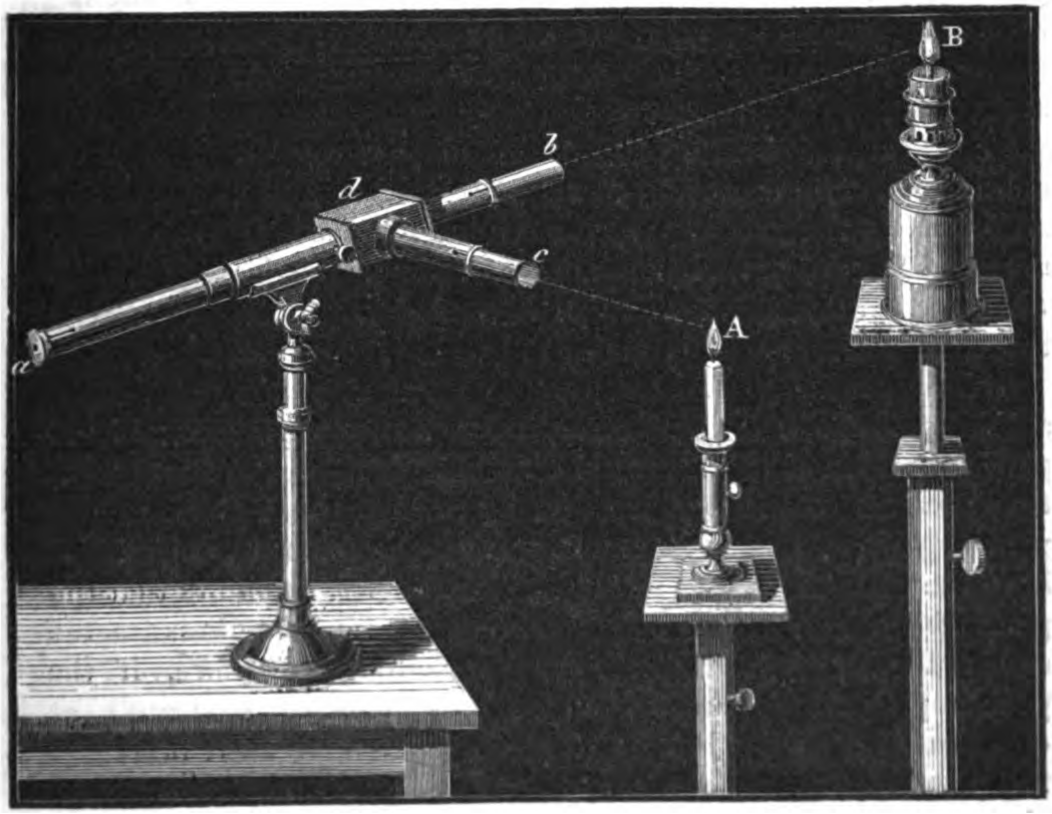


Fig 288.

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* Brewster's "Optics," p. 444.

tubes intersect, a bundle of twelve plane glass plates B (*fig.* 288.), dividing into two equal parts the angle A B D, formed by the axes of the tubes, is fixed. Two discs of ground glass *d*, *f*, are placed in the tubes at right angles to the axes, which have the effect of diffusing the light transmitted along them, and presenting to the eye the appearance of an illuminated disc. At *e e′* is placed a plate of quartz of double rotation, and at C a double refracting achromatic prism, by which the coloured images produced by the quartz *e e′*, are observed. The apparatus being disposed, as shown in *fig.* 288., two lights, A, B, being placed in the direction of the tubes, the light emitted from A being reflected by the glass plates at B, (*fig.* 289.), at an angle of 35° 25′, the reflected rays will be transmitted along the axis of the tube *a b*, and the pencil thus transmitted being polarised, and passing through the plate *e e′*, will, when viewed through the double refracting prism C, be seen with two complementary tints; bluish red and green, for example.

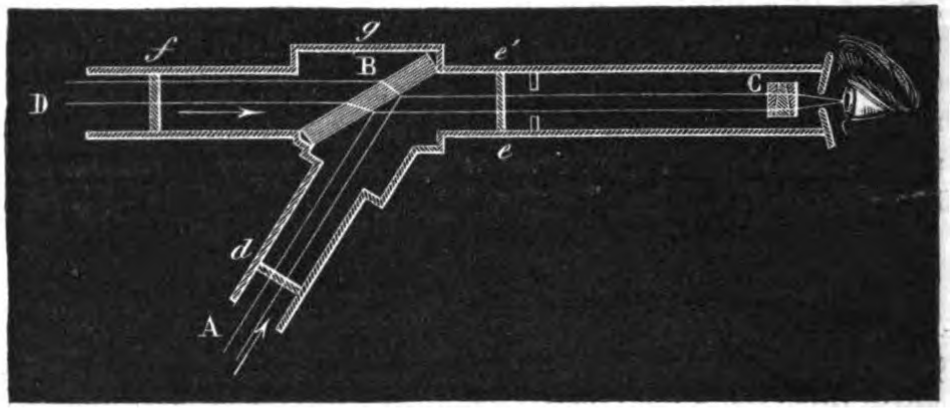

Fig. 289.

The light which proceeds from the lamp B, passing along the axis of the tube *a b*, and falling upon the posterior side of the plates B, will in part be transmitted through them, and the portion thus transmitted will be polarised by refraction. But since two pencils polarised, one by reflection and the other by refraction, have their planes of polarisation at right angles, it follows that the plate of quartz, *e e′*, will be coloured by the two pencils refracted and reflected, with complementary colours. Therefore, if the pencils proceeding from A and B, and arriving separately at opposite sides of the bundle of plates B, have the same intensity, the plate of quartz *e e′* will appear white. But on the contrary, if the two lights A and B have different intensities, the complementary pencils transmitted by B to *e e′*, will also have different intensities, and the light transmitted by the quartz will have complementary tints more or less pronounced.

In the application of the instrument to determine the relative tensities of two lights, their distances from the tubes *b* and *c* are varied until the light transmitted by the quartz *ee′* appear white. In that case it will follow that the illumination of the semi-transparent discs *d* and *f* has the same intensity, and consequently the absolute intensities of the two lights A and B will be, according to what has been proved, in the inverse proportion of the squares of their distances from the plates *d* and *f*.

The advantage which this photometer possesses is, that it reduces the determination of the relative intensities, to the power possessed by the eye to distinguish between tints of colour, instead of degrees of brilliancy. The eye is so constituted that it will perceive a small difference of tint in two juxtaposed objects, with much greater facility and certainty, than it would distinguish between two degrees of brightness, differing very little from each other, the objects being illuminated by lights of the same tint.

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Note. — This Index refers to the numbers of the paragraphs, and not to the pages.

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THE END.

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